Cornell University Mathematics Department Senior Thesis

REPRESENTATION THEORETIC EXISTENCE PROOF FOR FISCHER GROUP Fi_{23}

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ABSTRACT

In the first section of this senior thesis the author provides some new efficient algorithms for calculating with finite permutation groups. They cannot be found in the computer algebra system MAGMA, but they can be implemented there. For any finite group G with a given set of generators, the algorithms calculate generators of a fixed subgroup of G as short words in terms of original generators. Another new algorithm provides such a short word for a given element of G. These algorithms are very useful for documentation and performing demanding experiments in computational group theory.

In the later sections, the author gives a self-contained existence proof for Fischer's sporadic simple group Fi_{23} of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ using G. Michler's Algorithm [11] constructing finite simple groups from irreducible subgroups of $\mathrm{GL}_n(2)$. This sporadic group was originally discovered by B. Fischer in [6] by investigating 3-transposition groups, see also [5]. This thesis gives a representation theoretic and algorithmic existence proof for his group. The author constructs the three non-isomorphic extenstions E_i by the two 11-dimensional non-isomorphic simple modules of the Mathieu group $\mathcal{M}_{23}$ over $F = \mathrm{GF}(2)$. In two cases Michler's Algorithm fails. In the third case the author constructs the centralizer $H = C_G(z)$ of a 2-central involution z of E_i in any target simple group G. Then the author proves that all conditions of Michler's Algorithm are satisfied. This allows the author to construct G inside $\mathrm{GL}_{782}(17)$. Its four generating matrices are too large to be printed in this thesis, but they can be downloaded from the author's website [10]. Furthermore, its character table and representatives for conjugacy classes are computed. It follows that G and Fi_{23} have the same character table.

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Contents

0. Introduction

A simple group G is called sporadic if it is not isomorphic to any alternating group A_n or any finite group of Lie type, see R. W. Carter [3]. Until recently there was no uniform construction method for the known twenty six simple sporadic groups. In [11] a uniform construction method is given for constructing finite simple groups from irreducible subgroups of $\mathrm{GL}_n(2)$. For technical reasons it cannot construct the two largest known sporadic simple groups because there is no computer which can hold all their elements. But the other twenty four known sporadic simple groups can be constructed by Michler's Algorithm. In this thesis it is applied to provide a new self-contained existence proof for Fischer's sporadic simple group Fi_{23}.

In 1971 B. Fischer [5] found three sporadic groups by characterizing all finite groups G that can be generated by a conjugacy class $D = z^G$ of 3-transpositions, which means that the product of two elements of D has order $1, 2$, or 3. He proved that besides the symmetric groups S_n, the symplectic groups $\mathrm{Sp}_n(2)$, the projective unitary groups $\mathrm{U}_n(2)$ over the field with 4 elements and certain orthogonal groups, his two sporadic simple groups Fi_{22} and Fi_{23}, and the automorphism group Fi_{24} of the simple group Fi_{24}' describe all 3-transposition groups, see [6]. For each 3-transposiiton group $G = \langle D \rangle$ Fischer constructs a graph $\mathcal{G}$ and an action on it. As its vertices he takes the 3-transpositions x of D. Two distinct elements $x, y \in D$ are called to be *connected* and joined by an edge (x, y) in $\mathcal{G}$ if they commute in G. He showed that each of the groups considered in his theorem has a natural representation as an automorphism group of its graph $\mathcal{G}$. Unfortunately, Fischer's proofs are only published in his set of lecture notes of the University of Warwick [6]. See also [1], for a coherent account on Fischer's theorem.

In [6] Fischer gave the first existence proof for Fi_{22}, Fi_{23}, and Fi_{24}, by constructing the three graphs on which the groups act as automorphisms. Eighteen years later, M. Aschbacher proves in [1] the existence of Fi_{24} and hence also Fi_{22} and Fi_{23}, using a quotient of the normalizer of a cyclic subgroup of order 3 in the Monster simple group M, see [1], p. 5. However, both approaches do not allow specific calculations with elements in these groups nor do their methods generalize to arbitrary finite simple groups.

The purpose of this thesis is to provide a new existence proof for Fi_{23}, which has two advantages over the previous proofs. Fischer's proof doesn't generalize to all simple groups, because not all simple groups can be described by 3-transposition groups. Aschbacher's proof obviously doesn't generalize nor does it provide access to explicit computation with elements in the resulting groups, because the Monster group M doesn't have a permutation representation or a matrix representation of small enough degree; the smallest known faithful permutation representation of M wouldn't fit into any modern super-computer, and the smallest degree of matrix representation of M is about $183,000$. Dealing with dense $183,000$ by $183,000$ matrices is practically impossible for currently existing computers.

The new proof uses representation theoretic and algorithmic methods, mainly based on the Algorithm 2.5 of [11], which is also stated in section 2. The second part of Algorithm 2.5 of [11] is not repeated in this thesis, because it is identical to Algorithm 7.4.8 of [12]. Using this algorithm, the author constructs Fi_{23} as a subgroup of $\mathrm{GL}_{782}(17)$. From the 782-dimensional matrix representation, the author also constructs a faithful permutation representation of Fi_{23} of degree 31671, by which we can actually compute with elements and subgroups and check the performed calculations using the high performance computer algebra system MAGMA. The character table and representatives for conjugacy classes of Fi_{23} are also computed by means of this permutation representation.

The other strong point of this algorithmic proof is that this method generalizes to all simple groups (not having Sylow 2-subgroups which are cyclic, dihedral or semi-dihedral), see [12]. In [9] the author and G. Michler construct Fi_{22} and Conway's sporadic group Co_2 simultaneously. The author and G. Michler are also working on the simultaneous contructions of Co_1, Janko's sporadic group J_4, and Fi_{24}'. In [12] and recent work to which the author's joint article [9] with Michler and also the further work belongs, G. Michler and coauthors construct all known sporadic groups except the Baby Monster and the Monster. These methods allow a systematic search for simple groups.

Here is the summary of each section. Section 1 contains the description and MAGMA implementation of the author's short-word algorithms. In section 2, the author states Michler's Algorithm 2.5 of [11], which is a main tool of the construction of Fi_{23} in this thesis.

Section 3 shows the author's construction of the extensions of the Mathieu group $\mathcal{M}_{23}$ by its two non-isomorphic irreducible representations V_1 and V_2 of dimension 11 over GF(2). In this thesis it is shown that there is a uniquely determined non-split extension E of $\mathcal{M}_{23}$ by V_1. Micher's Algorithm 2.5 in [11] is applied to E for the construction of Fi_{23} in the next sections. However, its application to the split extensions E_1 and E_2 of $\mathcal{M}_{23}$ by V_1 and V_2 does not lead to any result.

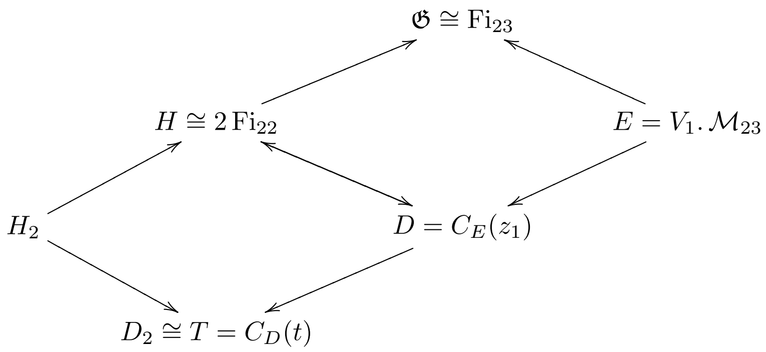
Section 4 contains the author's construction of the centralizer H (unique up to isomorphism) of a 2-central involution z_1 of E in any target simple group G. In particular, it is shown that $Z(H) = \langle z_1 \rangle$ and $H/Z(H) \cong \mathrm{Fi}_{22}$. Taking a 2-central involution z_1 in E and calculating $D = C_E(z_1)$ the author finds a suitable normal subgroup Q in D which enables him to construct a group H with center $Z(H) = \langle z_1 \rangle$ of order 2 such that $H/Z(H) \cong \mathrm{Fi}_{22}$. It is shown that $E, D = C_E(z_1), H$ satisfy all conditions of Algorithm 2.5 of [11], stated in Algorithm 2.1 in section 2.

In order to construct $H \cong 2\,\mathrm{Fi}_{22}$, the author quotes a result on Fischer's sporadic simple group Fi_{22} from his joint article [9] with G. Michler. This was necessary because the implementation of Holt's Algorithm into MAGMA was not able to construct a central extension of Fi_{22} by a cyclic group of order 2. Thus, the author constructs another amalgam $H_2 \leftarrow D_2 \rightarrow D$, where $D_2 = C_D(t)$ for some involution t and $H_2 = 2H(\mathrm{Fi}_{22})$, see Propositions 4.1 and 4.2 and Theorem 4.3. The free product $H_2 *_{D_2} D$ with amalgamated subgroup D_2 has a 352-dimensional faithful irreducible representation over GF(17), whose corresponding matrix group $\mathfrak{H}$ is proved to be isomorphic to the 2-fold cover $2\,\mathrm{Fi}_{22}$ of Fi_{22}, see Theorem 4.5.

In section 5 the author finally constructs the simple target group $\mathfrak{G}$ as a matrix group inside $\mathrm{GL}_{782}(17)$, by applying Algorithm 7.4.8 of [12] to the amalgam $H \leftarrow D \rightarrow E$. In particular, it has been shown that $\mathfrak{G}$ has a same character table as Fi_{23} as stated in [4]. The amalgam $E \leftarrow D \rightarrow H$ has a unique compatible pair of degree 782 over GF(17) which is not multiplicity-free at the D-level, see Theorem 5.1. So the author uses Thompson's Theorem 7.2.2 of [12] in the application of Step 5(c) of Algorithm 7.4.8 of [12]. It is shown that the free product $H *_D E$ with amalgamated subgroup D has exactly one (unique up to isomorphism) irreducible representation of degree 782 over GF(17) satisfying the Sylow 2-subgroup test, see Theorem 5.1. The corresponding matrix group $\mathfrak{G}$ in $\mathrm{GL}_{782}(17)$ is proved to be a simple group of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ which has a 2-central involution

$\mathfrak{z}$ such that $C_{\mathfrak{G}}(\mathfrak{z}) \cong H$, see Theorem 5.1. Furthermore, the author constructs a faithful permutation representation of $\mathfrak{G}$, and then the character table of $\mathfrak{G}$. It agrees with the one of Fi_{23} as stated in the Atlas [4].

The following diagram summarizes the author's construction of Fi_{23} given in sections 4 and 5.



All the performed demanding calculations for the given existence proof were only possible because of the implementations of the author's new algorithms described in the first section. They are very general and can be used in the course of mathematical research in computational group theory. The author's code works well in the computer algebra system Magma. For documentation of the performed calculations it is often crucial to get short-word generators for a certain subgroup of a group G with a given set of generators: $G = \langle g_1, g_2, \dots, g_n \rangle$. Let S be a subgroup G. We want to get a generating set for S as short words in terms of the original generators $g_1, g_2, \dots, g_n$.

For example, $S = \langle g_1 g_3 g_6,\ g_2,\ g_4 g_5 \rangle$. We want relatively small number of generators, and the lengths of words to be short.

Magma's inverse word map function (often) provides a generating set in terms of given generators, but unfortunately consisting of terribly lengthy words. Reiner Staszewski, who was a former research assistant of Professor Michler, developed a stand-alone algorithm for finding short-word generators. Paul K. Young, a current graduate student of the Mathematics Department of Cornell University, polished the idea and implemented the algorithm in Magma. Since Young's implementation had some problems when dealing with groups of large order (or permutation groups of large degree), the author modified and added several new ideas. Thus the author produced a relatively efficient implementation of the resulting algorithm.

Besides this short-word-generator algorithm, another algorithm for getting a short word for an element of a group with given generators is also presented in the first section of this thesis. For each algorithm, the description and its Magma implementation are given.

These new algorithms are indeed crucial in the author's construction of Fi_{23}; they are not just for documentation.They provide a successful method for constructing the 2-fold cover $2\,\mathrm{Fi}_{22}$ where Holt's Algorithm failed.

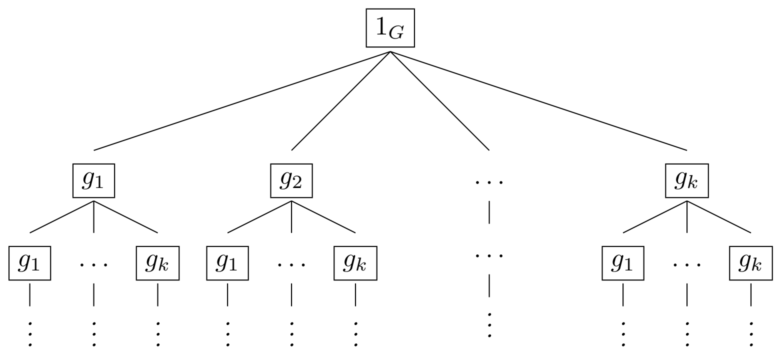
For readers who would like to find more background materials for this thesis, they should see Holt's book [7] for computational group theory, and Michler's book [12] for algorithmic representation theory of finite simple groups.

1. Algorithms

In this section, a finite group G is always assumed to be realized as a permutation group or a matrix group. This allows us to compute the orders of elements and subgroups of G, and check equalities.

Before going into the actual algorithms, it is natural to have definitions of some vocabulary: 'word-tree', 'length of a word', 'short-word', 'lexicographic order of words', and so on. The following definition, as stated in Definition 5.3.9 of [12], is due to M. Kratzer.

Definition 1.1 (Word Tree, Word Length). *Let G be a finite group. Let $\{g_1, g_2, \ldots, g_k\}$ be a fixed set of generators for G. The infinitely deep k-nary tree $\mathcal{C}(G)$ in which the root vertex is marked by the identity $1_G \in G$ and the k successors of each vertex are marked successively by $g_1, g_2, \ldots, g_k$ is called the "complete word tree of G":*



There is a canonical one-to-one correspondence between vertices in $\mathcal{C}(G)$ and words in generators of G: For any vertex v of $\mathcal{C}(G)$ let w_v denote the incremental product of the vertex markers occurring along the direct path from the root vertex to v in $\mathcal{C}(G)$. Conversely, starting from the root vertex and reading a given word $w = w(g_1, g_2, \ldots, g_k)$ in generators of G like a sequence of directions guides one to the unique vertex v_w in $\mathcal{C}(G)$ such that $w_{v_w} = w$.

The "length" of a word w is the depth of the unique vertex v_w corresponding to w in the word tree $\mathcal{C}(G)$, i.e. the number of steps needed to reach the vertex v_w from root vertex. For example, the word $g_1^2 g_3 g_2$ has length 4.

Now, a "short-word" refers to a word of short length, though notion of "short" might not be consistent. To simplify arguments, we may look only at the indices of the generators. That is, $g_1^2 g_3 g_2$ can be identified with $[1, 1, 3, 2]$. Precise definition of this idea is as follows:

Definition 1.2 (Numerical Word). *A "numerical word of k generators" is a finite sequence (can be empty sequence) of integers in $\{1, 2, \ldots, k\}$. For example, $[5, 1, 2, 2, 2, 3, 4, 4, 5]$ is an example of a numerical word of 5 generators, as well as a numerical word of 6 generators, but not a numerical word of 4 generators.*

Let $\{g_1, g_2, \ldots, g_k\}$ be a fixed set of generators of a finite group G. Then, if we identify each g_i with i, there is a natural one-to-one correspondence between the set of all numerical word of k generators and $\mathcal{C}(G)$. For example, the word $g_1 g_4^3 g_2 g_3^2$ corresponds to the numerical word $[1, 4, 4, 4, 2, 3, 3]$.

Now we can define a total ordering on $\mathcal{C}(G)$, i.e. on the set of all finite words in the given (ordered) generators $g_1, g_2, \ldots, g_k$ of G.

Definition 1.3 (Lexicographical Order). *Let $\{g_1, g_2, \ldots, g_k\}$ be a fixed set of generators of a finite group G. Let v_u and v_w be the vertices of $\mathcal{C}(G)$, corresponding to distinct words u and w in $g_1, g_2, \ldots, g_n$, respectively. Let N_u and N_w be the numerical words of k generators corresponding to v_u and v_w, respectively (each g_i is identified with i). Then, $u < w$ if and only if one of the followings hold (this is called "lexicographic order"):*

(1) *length of $u <$ length of w*
(2) *length of $u =$ length of w, and $N_u[n] < N_w[n]$ holds, where n is the smallest number such that $N_u[n] \neq N_w[n]$.*

Remark 1.4. *If u and w are distinct words of G (in given generators), then exactly one of $u < w$ or $w < u$ holds.*

The main algorithm is named `GetShortGens`; for any finite group G with given set of generators, for any subgroup S of G, this algorithm returns a short-word generating set for S in terms of the given generators. It needs three other small programs, named `EnumWords`, `ReduceGensForGroup` and `Word2Elt`. All four programs are described below, with description of the algorithms and their implementation in MAGMA. The first algorithm `EnumWords` enables us to descend one level deeper in the word tree:

Algorithm 1.5 (EnumWords). *Let k be the number of given generators for the finite group of interest. Let W be a sequence of pre-built numerical words in k generators. Let s, e be integers s.t. $1 \leq s \leq e \leq \#W$ (s for "start", e for "end"). Then, we can get a new sequence W_1 of numerical words by adding new words to W, where the new words are obtained by appending $1, 2, \ldots, k$ at the end of the words $W[s], W[s+1], \ldots, W[e]$.*

Implementation 1.6 (EnumWords).

```
function EnumWords(W, st_end, nGens)

local NewW, Newst_end;

NewW := W;
if #NewW eq 0 then
  NewW := [[i] : i in [1..nGens]];
  Newst_end := [1, nGens];
else
  for i:=st_end[1] to st_end[2] do
    for j:=1 to nGens do
      Append(~NewW, Append(W[i],j));
    end for;
  end for;
  Newst_end := [st_end[2]+1, #NewW];
end if;
return NewW, Newst_end;

end function;
```

Example 1.7 (EnumWords). MAGMA *code*

```
> W:=[ [1,3,4], [3,3,2], [2,1], [4] ];
> s:=2; e:=3;
> W:=EnumWords(W, [s,e], 4);
> W;
[ [1,3,4], [3,3,2], [2,1], [4], [3,3,2,1], [3,3,2,2], [3,3,2,3],
 [3,3,2,4], [2,1,1], [2,1,2], [2,1,3], [2,1,4] ]
```

The next algorithm `ReduceGensForGroup` gives a method to reduce the given set of generators as much as possible, while our subgroup of interest still should lie inside the subgroup generated by the reduced set of generators:

Algorithm 1.8 (ReduceGensForGroup)**.** *Let G be a finite group generated by the given generators $g_1, g_2, \ldots, g_k$. For any subgroup S of G, a subset of $\{g_1, g_2, \ldots, g_k\}$ which generates a subgroup of G containing S is obtained by the following method:*

[Reducing original generators] If there is some $i \in \{1, 2, \ldots, k\}$ such that the set $\{g_1, \ldots, g_{i-1}, g_{i+1}, \ldots, g_k\}$ generates a subgroup of G containing S, pick the largest such i. Now, call this function `ReduceGensForGroup` *recursively, with same G and S, with reduced set of generators $g_1, \ldots, g_{i-1}, g_{i+1}, \ldots, g_k$ (with corresponding names). If there is no such i, then return the original set $g_1, g_2, \ldots, g_k$, with their names.*

Implementation 1.9 (ReduceGensForGroup)**.**

```
function ReduceGensForGroup(G, Target : wordgens:=[], gencollection:=[1..#Generators(G)],
exclude:=sub⟨G|⟩,CoverGroup:=G)

local reducedlist;

if #wordgens eq 0 or #Generators(G) ne #wordgens then
   wordgens:=[(“$.” cat Sprint(i)) :i in [1..#Generators(G)]];
end if;
for i:=1 to #Generators(G) do
   reducedlist := Exclude([1..#Generators(G)],#Generators(G)+1-i);
   if (Target meet sub⟨CoverGroup|[G.j : j in reducedlist],exclude⟩) eq Target then
      return ReduceGensForGroup(sub⟨G|[G.j : j in reducedlist]⟩, Target :
wordgens:=[wordgens[j] : j in reducedlist], gencollection:=[gencollection[j]: j in
reducedlist],exclude:=exclude,CoverGroup:=CoverGroup);
   end if;
end for;
return G, wordgens, gencollection;

end function;
```

The following algorithm `Word2Elt` converts a numerical word to the corresponding actual element of the group:

Algorithm 1.10 (Word2Elt)**.** *Let G be a finite group generated by the given generators $g_1, g_2, \ldots, g_k$. For any numerical word w in k generators, return the element in G corresponding to the word w by the following steps:*

Step 1 *Let $e = 1$, the identity element of G.*

Step 2 *If w is an empty word, return e. If not, and if w can be written as $w = [a_1, a_2, \ldots, a_m]$ where $a_i \in \{1, 2, \ldots, k\}$, then let $e := e \cdot g_{a_1}$.*

Step 3 *Let $w := [a_2, a_3, \ldots, a_m]$, and go to* **Step 2**.

Implementation 1.11 (Word2Elt)**.**

```
function Word2Elt(G, word)

local elt;

elt := Id(G);
for i:=1 to #word do
   elt := elt * G.word[i];
end for;
return elt;

end function;
```

Example 1.12 (Word2Elt)**.** MAGMA *code (Note:* MAGMA *composes two permutations from left, not from right)*

```
> G:=sub<Sym(3)| Sym(3)!(1,2), Sym(3)!(1,2,3)>;
> Word2Elt(G, [1,2,2]);
(2,3)
```

Finally, the next algorithm `GetShortGens` enables us to obtain a short-word generating set for a subgroup of a group with given generators:

Algorithm 1.13 (GetShortGens)**.** *Let G be a finite group generated by the given generators $g_1, g_2, \dots, g_k$. For any subgroup S of G, a generating set of S consisting of short-word elements of G in terms of $g_1, g_2, \dots, g_k$ is obtained by the following steps:*

Step 1 *[Reducing original generators] If desired, try to get a subset of $\{g_1, g_2, \dots, g_k\}$ which generates a subgroup of G containing S, using the command* `ReduceGensForGroup`*. For convenience, suppose that the set of generators g_1, g_2, ..., g_k is already a result of this reducing process.*

Step 2 *[Building word list, and finding generators]*

(1) *Set $F = \langle 1 \rangle$ (trivial subgroup).*
(2) *Let $W = [[1], [2], \dots, [k]]$, the set of words, initially set to have only simplest numerical words of k generators of length 1.*
(3) *Take the word w in W of lowest lexicographic order which is not checked yet, and let $x =$ `Word2Elt(G,w)`, the element of G corresponding to the word w. If there is some $m \in \{1, 2, \dots, Order(w) - 1\}$ such that $w^m \in S$ and $w^m \notin F$, then enlarge F by $F := \langle F, w^m \rangle$.*
(4) *If $F = S$, then proceed to* **Step 3***. If not, enlarge W by appending $1, 2, \dots, k$ to all words of W of longest length, using the command* `EnumWords` *(this is same as descending one level deeper in the word tree). Now, go to (3).*

Step 3 *[Printing] Print the words obtained.*

Implementation 1.14 (GetShortGens)**.**

```
function GetShortGens(G, Target : exclude := sub⟨G|⟩, limit:=0, wordgens:=[], Words:=[],
startpoint:=1, powers:=:=[1], Hard:=true, OrderRestriction:=[], CoverGroup:=G,
generatecheck:=true, auto:=true, EltReturn:=false, ReduceMore:=true)

local gens, iter, st_end, tempelt, tempord, temppow, WordsSoFar, WordsNumSoFar,
gencollection, SubgroupSoFar, generatingset;

gens := Generators(G);

function GetGeneratingSet(WordsForGenerators)
  return [Word2Elt(G,WordsForGenerators[i][1])^WordsForGenerators[i][2] :
  i in [1..#WordsForGenerators]];
end function;

if generatecheck and Target meet sub⟨CoverGroup|G,exclude⟩ ne Target then
  print "can't generate subgroup";
  return "";
end if;

if #wordgens eq 0 or #Generators(G) ne #wordgens then
  if #wordgens ne 0 and #Generators(G) ne #wordgens then
    print "the number of generator names you provided is incompatible
    with the number of generators of the group, so re-building names";
  end if;
  wordgens:=[("$." cat Sprint(i)) :i in [1..#Generators(G)]];
```

```
end if;

if Hard then
   print ”Reducing Generators...”;
   time G, wordgens, gencollection := ReduceGensForGroup(G, Target :
wordgens:=wordgens,exclude:=exclude,CoverGroup:=CoverGroup);
   printf “Using only %o generators %o, out of %o ”, #Generators(G),wordgens,#gens;
   gens := Generators(G);
else
   gencollection := [1..#Generators(G)];
end if;

Include(~powers,1);
Sort(~powers);

WordsSoFar:=[];
WordsNumSoFar:=[];
SubgroupSoFar := Target meet exclude;
iter:=0;

while limit eq 0 or iter lt limit do
   if #gens eq 1 and iter ge Order(G.1) then
      print “use other method”;
      return “”,[];
   end if;

   iter:=iter+1;
   printf “%o-th iteration\n”,iter;
   if #Words eq 0 then
      Words, st_end := EnumWords([],[1,1],#gens);
      startpoint:=1;
   end if;

   for j:=startpoint to #Words do
      if #OrderRestriction eq 0 then
         tempelt := Word2Elt(G, Words[j]);
         if auto then powers:=[1..(Order(tempelt)-1)]; end if;
         if exists(temppow){x : x in powers| tempelt^x notin SubgroupSoFar
         and tempelt^x in Target} then
            Append(~WordsSoFar, WordPrint(Words[j], wordgens:power:=temppow));
            Append(~WordsNumSoFar, ⟨[gencollection[Words[j][k]]:k in [1..#Words[j]]],
temppow⟩);
            printf “Got a new elt: %o\n”, WordsSoFar[#WordsSoFar];
            SubgroupSoFar := sub⟨Target|SubgroupSoFar, Word2Elt(G,Words[j])^temppow⟩;
            if SubgroupSoFar eq Target then
               printf “Got a generating set.\n”;
               if ReduceMore then
                  print “Getting a smaller generating set...”;
                  time G, wordgens, gencollection :=
ReduceGensForGroup(sub⟨G|GetGeneratingSet(WordsNumSoFar)⟩, Target :
wordgens:=[],exclude:=exclude,CoverGroup:=CoverGroup);
                  printf “Resulting set has %o generators, out of %o\n”, #wordgens,#WordsSoFar;
                  printf “subcollection indices:%o\n”,gencollection;
                  gens := Generators(G);
               end if;

               WordsSoFar := [WordsSoFar[gencollection[i]] : i in [1..#gencollection]];
               WordsNumSoFar := [WordsNumSoFar[gencollection[i]] : i in [1..#gencollection]];
               if not EltReturn then
                  return WordsSoFar, WordsNumSoFar;
               else
                  return WordsSoFar, WordsNumSoFar, GetGeneratingSet(WordsNumSoFar);
               end if;
```

```
        end if;
      end if;
    else      // if there is some restriction on orders
      tempelt := Word2Elt(G, Words[j]);
      tempord := Order(tempelt);
      for ord in OrderRestriction do
        if tempord mod ord eq 0 then
          temppow := Integers()!(tempord/ord);
          if tempelt^temppow notin SubgroupSoFar and tempelt^temppow in Target then
            Append(~WordsSoFar, WordPrint(Words[j],wordgens:power:=temppow));
            Append(~WordsNumSoFar, ⟨[gencollection[Words[j][k]]:k in
[1..#Words[j]]],temppow⟩);
            printf "Got a new elt of order %o: %o\n",ord,WordsSoFar[#WordsSoFar];
            SubgroupSoFar := sub⟨Target|SubgroupSoFar, Word2Elt(G,Words[j])^temppow⟩;
            if SubgroupSoFar eq Target then
              printf "Done.\n";
              if not EltReturn then
                return WordsSoFar, WordsNumSoFar;
              else
                return WordsSoFar, WordsNumSoFar, Generators(SubgroupSoFar);
              end if;
            end if;
          end if;
        end if;
      end for;
    end if;
  end for;
  Words, st_end := EnumWords(Words, st_end, #gens);
  startpoint := st_end[1];
end while;

print "Couldn't generatate group";
return WordsSoFar;
end function;
```

Example 1.15 (GetShortGens). MAGMA *code*

```
> g1:=Sym(8)!(1,2); g2:=Sym(8)!(1,2,3,4,5,6,7,8);
> G:=sub<Sym(8)|g1,g2>;
> S := sub<Sym(8)| Sym(8)!(1,3,6)(2,4), Sym(8)!(1,7,8)(2,5)>;
> res := GetShortGens(G,S : wordgens:=[''g1'',''g2'']);
> res;
[ (g2*g1*g2^4)^5, (g1*g2*g1*g2^4)^3, (g1*g2^3*g1*g2*g1)^2,
(g1*g2*g1*g2*g1*g2^3*g1)^3 ]
```

For a given element of a finite group with a given generating set, it is often important to obtain a short word for the element in terms of given generating set. This element-version (as opposed to subgroup-version: `GetShortGens`) of short-word program is named `LookupWord`, and it needs a different version of `ReduceGensForGroup` which is called `ReduceGensForElt`. It is designed for finding a word of an element; it reduces the given set of generators as much as possible, while the element of interest still should lie inside the subgroup generated by the reduced set of generators:

Algorithm 1.16 (ReduceGensForElt). *Let G be a finite group generated by the given generators $g_1, g_2, \ldots, g_k$. For any element x of G, a subset of $\{g_1, g_2, \ldots, g_k\}$ which generates a subgroup of G containing S is obtained by the following method:*

[Reducing original generators] If there is some $i \in \{1, 2, \ldots, k\}$ such that the set $\{g_1, \ldots, g_{i-1}, g_{i+1}, \ldots, g_k\}$ generates a subgroup of G containing S, pick the largest such i. Now, call this function `ReduceGensForElt` *recursively, with same G and S,*

with reduced set of generators $g_1, \ldots, g_{i-1}, g_{i+1}, \ldots, g_k$ (with corresponding names). If there is no such i, then return the original set $g_1, g_2, \ldots, g_k$, with their names.

Implementation 1.17 (ReduceGensForElt)**.**

```
function ReduceGensForElt(G, TargetElt : wordgens:=[], gencollection:=[1..#Generators(G)])

local reducedlist;

for i:=1 to #Generators(G) do
   reducedlist := Exclude([1..#Generators(G)],#Generators(G)+1-i);
   if TargetElt in sub⟨G|[G.j : j in reducedlist]⟩ then
      return ReduceGensForElt(sub⟨G|[G.j : j in reducedlist]⟩, TargetElt :
wordgens:=reducedlist], gencollection:=[gencollection[j]:j in reducedlist]);
   end if;
end for;
return G, wordgens, gencollection;
end function;
```

The algorithm `LookupWord` enables us to obtain a short word for an element of a group with given generators:

Algorithm 1.18 (LookupWord)**.** *Let G be a finite group generated by the given generators $g_1, g_2, \ldots, g_k$. For any element x of G, a short word for x in terms of $g_1, g_2, \ldots, g_k$ is obtained by the following steps:*

Step 1 *[Reducing original generators] If desired, try to get a subset of $\{g_1, g_2, \ldots, g_k\}$ which generates a subgroup of G containing x, using the command* `ReduceGensForElt`*. For convenience, suppose that the set of generators $g_1, g_2, \ldots, g_k$ is already a result of this reducing process.*

Step 2 *[Building word list, and finding generators]*

(1) *Let $W = [[1], [2], \ldots, [k]]$, the set of words, initially set to have only simplest numerical words of k generators of length 1.*
(2) *Take the word w in W of lowest lexicographic order which is not checked yet, and let $y =$* `Word2Elt(G,w)`*, the element of G corresponding to the word w.*
(3) *Let $r = Order(y)$ and $s = Order(x)$. If $s \nmid r$, go to (5).*
(4) *If there is some $m \in \{t \in \mathbb{Z} | 1 \leq t < s, gcd(t, Order(x)) = 1\}$ such that $w^{rm/s} = x$, then proceed to* **Step 3***.*
(5) *Enlarge W by appending $1, 2, \ldots, k$ to all words of W of longest length, using the command* `EnumWords` *(this is same as descending one level deeper in the word tree). Now, go to (2).*

Step 3 *[Printing] Print the words obtained.*

Alternative option : *The above algorithm finds a short word which is equal to the given element. A similar algorithm can be used to find a short word which is conjugate to the given element: in* **Step 2**(4)*, we look for m such that $w^{rm/s}$ is conjugate to x (the equality test is replaced by the conjugacy test). This option is incorporated in the following implementation, as the hidden parameter* `ConjugateCheck`*; if we set* `ConjugateCheck:=true`*, then the following program finds a short word which is conjugate to the given element.*

Implementation 1.19 (LookupWord)**.**

```
function LookupWord(G, TargetElt : limit:=0, wordgens:=[], Words:=[], st_end:=[],
startpoint:=1, Hard:=true, ConjugateCheck:=false, CoverGroup:=G, containcheck:=true,
InfoLevel:=2)

local Ord, gens, iter;
```

```
local tempelt, tempord, temppow;

Ord := Order(TargetElt);
gens := Generators(G);

if containcheck and TargetElt notin G then
   print "Coudln't find a word for it";
   return "";
end if;

if TargetElt eq Id(G) then
   if InfoLevel gt 1 then print "it is the identity!"; end if;
   return "Id($)", ⟨[],1⟩;
end if;

if #wordgens eq 0 or #gens ne #wordgens then
   for i:=1 to #gens do
      Append(~wordgens, "$." cat IntegerToString(i));
   end for;
end if;

if Hard and not ConjugateCheck then
   print "Reducing Generators...";
   time G, wordgens, gencollection := ReduceGensForElt(G, TargetElt : wordgens:=wordgens);
   printf "Using only %o generators %o, out of %o ", #Generators(G),wordgens,#gens;
   gens := Generators(G);
else
   gencollection := [1..#Generators(G)];
   if #wordgens eq 0 or #Generators(G) ne #wordgens then
      wordgens:=[("$." cat Sprint(i)) :i in [1..#Generators(G)]];
   end if;
end if;

iter:=0;
while limit eq 0 or iter lt limit do
   iter:=iter+1;
   if InfoLevel gt 1 then
      printf "%o-th iteration\n",iter;
   end if;
   if #Words eq 0 or #st_end eq 0 then
      Words, st_end := EnumWords([],[1,1],#gens);
      startpoint:=1;
   end if;

   for j:=startpoint to #Words do
      tempelt := Word2Elt(G, Words[j]);
      tempord := Order(tempelt);
      if tempord mod Ord eq 0 then
         temppow := Integers()!(tempord/Ord);
         for addi in [a:a in [1..Ord]|GCD(a,Ord) eq 1] do
            if tempelt^(temppow*addi) eq TargetElt then
               if InfoLevel gt 1 then print "Got a word (exact)"; end if;
               return WordPrint(Words[j], wordgens:power:=(temppow*addi)),
⟨[gencollection[Words[j][k]]:k in [1..#Words[j]]], temppow*addi⟩;
            elif ConjugateCheck and IsConjugate(CoverGroup,tempelt^(temppow*addi),TargetElt)
then
               if InfoLevel gt 1 then print "Got a conjugate word (new ver)"; end if;
               return WordPrint(Words[j], wordgens:power:=(temppow*addi)), ⟨[Words[j][k]:k in
[1..#Words[j]]], temppow*addi⟩;
            end if;
         end for;
      end if;
   end for;
```

```
  Words, st_end := EnumWords(Words, st_end, #gens);
  startpoint := st_end[1];
end while;

print "Couldn't find a word";
return "";

end function;
```

Example 1.20 (LookupWord)**.** MAGMA *code*

```
> g1:=Sym(8)!(1,2); g2:=Sym(8)!(1,2,3,4,5,6,7,8);
> G:=sub<Sym(8)|g1,g2>;
> x := Sym(8)!(2, 8, 7, 6, 4, 3);
> res := LookupWord(G,x : wordgens:=[``g1'',``g2'']);
> res;
g1*g2^4*g1*g2^3
```

Sometimes, the subgroup or the element that we want to get short words of lies too deep inside the group (whose generators are given), so `GetShortGens` or `LookupWord` doesn't work well. For example, if an element x can't be represented by a word of length≤ 24 in terms of given generators, then `LookupWord` either takes too long a time to get the word for x, or causes memory overflow due to too much required space for all the words built up so far. The author found a strategy to overcome this issue, described as follows.

The strategy needs a proper subgroup T of G, which contains the target subgroup S or the target element x. A standard trick to get such T is to take the normalizer of S in G, the centralizer of x in G, or the normalizer of $\langle x\rangle$ of G.

Strategy 1.21 (two-step GetShortGens)**.** *Let G be a finite group generated by the given generators $g_1, g_2, \ldots, g_k$. For any subgroups S and T of G such that $S \gneqq T \gneqq G$, a short-word generating set of S in terms of $g_1, g_2, \ldots, g_k$ is obtained by the following steps:*

Step 1 *[Generators for T] Get short-word generators $t_1, t_2, \ldots, t_n$ for T in terms of $g_1, g_2, \ldots, g_k$ using* `GetShortGens`*.*

Step 2 *[Generators for S] Get short-word generators for S in terms of t_1, t_2, $\ldots$, t_n using* `GetShortGens`*.*

Strategy 1.22 (two-step LookupWord)**.** *Let G be a finite group generated by the given generators $g_1, g_2, \ldots, g_k$. For any element $x \in G$ and a subgroup T of G such that $x \in T \gneqq G$, a short word for x in terms of $g_1, g_2, \ldots, g_k$ is obtained by the following steps:*

Step 1 *[Generators for T] Get short-word generators $t_1, t_2, \ldots, t_n$ for T in terms of $g_1, g_2, \ldots, g_k$ using* `GetShortGens`*.*

Step 2 *[Short Words for x] Get short word for x in terms of $t_1, t_2, \ldots, t_n$ using* `LookupWord`*.*

The above two strategies are used throughout this thesis and therefore constantly referred to. Note that sometimes it can be helpful to iterate the strategies many times. For example, for a subgroup S of G, it can be a good idea to try to find subgroups T_1 and T_2 such that $S \gneqq T_1 \gneqq T_2 \gneqq G$ and then find short-word generators for T_2, T_1, and finally S in turn.

2. Michler's Algorithm

In this section, G. Michler's Algorithm 2.5 of [11] is presented. This algorithm gives a uniform method to construct all finite simple groups (not having Sylow 2-subgroups which are cyclic, dihedral or semi-dihedral), from indecomposable subgroups of $\mathrm{GL}_n(2)$. The author's algorithms described in the previous section are used to implement this algorithm of Michler. In particular, in this thesis, Fi_{23} is constructed by the following algorithm.

Algorithm 2.1. *Let T be an indecomposable subgroup of $\mathrm{GL}_n(2)$ acting on $V = F^n$ by matrix multiplication.*

- Step 1: *Calculate a faithful permutation representation PT of T and a finite presentation $T = \langle t_i \mid 1 \leq i \leq r \rangle$ with set $\mathcal{R}(T)$ of defining relations.*
- Step 2: *Compute all extension groups E of T by V by means of Holt's Algorithm* [7]*. Determine a complete set $\mathfrak{S}$ of non isomorphic extension groups E by means of the Cannon-Holt Algorithm* [14]*.*
- Step 3: *Let $E \in \mathfrak{S}$. From the given presentation of E determine a faithful permutation representation PE of E. Using it and Kratzer's Algorithm 5.3.18 of* [12] *calculate a complete system of representatives of all the conjugacy classes of E.*
- Step 4: *Let $z \neq 1$ be a 2-central involution of E. Calculate $D = C_E(z)$ and fix a Sylow 2-subgroup S of D. Check that the elementary abelian normal subgroup V of E is a maximal elementary abelian normal subgroup of S. If it is not maximal, then the algorithm terminates.*
- Step 5: *Construct a group $H > D$ with the following properties:*
  (a) *z belongs to the center $Z(H)$ of H.*
  (b) *The index $|H : D|$ is odd.*
  (c) *The normalizer $N_H(V) = D = C_E(z)$.*
  If no such H exists the algorithm terminates.

  Otherwise, apply for each constructed group H the following steps of Algorithm 7.4.8 of [12]*. By step* 5(*c*) *it may be assumed from now on that $D = H \cap E$.*

After the Step 5, the remaining steps are identical to Algorithm 7.4.8 of [12], so omitted here.

3. Extensions of Mathieu group $\mathcal{M}_{23}$

The Mathieu group M_{23} is defined in Definition 8.2.1 of [12] by means of generators and relations. This beautiful presentation is due to J.A. Todd. The irreducible 2-modular representations of the Mathieu group M_{23} were determined by G. James [8]. Here only the 2 non isomorphic simple modules V_i, $i = 1, 2$, of dimension 11 over $F = \mathrm{GF}(2)$ will be considered. Todd's permutation representations of the Mathieu groups are stated in Lemma 8.2.2 of [12]. Therefore all conditions of Holt's Algorithm [7] implemented in Magma are satisfied. It is applied here. Thus it is shown in this section that for the simple module V_1 there are exactly two extensions of M_{23} by V_1, the split extension E_1 and the non-split extension E and that $\mathcal{M}_{23}$ has only the split extension E_2 by V_2. It will be shown that the applications of Algorithm 2.1 to E_1 and E_2 fail. Therefore only the constructed presentation of E is given in Lemma 3.1.

Lemma 3.1. *Let $\mathcal{M}_{23} = \langle a, b, c, d, t, g, h, i, j \rangle$ be the finitely presented group with set of defining relations $\mathcal{R}(\mathcal{M}_{23})$ given in Definition 8.2.1 of* [12]. *Then the following statements hold:*

(a) *A faithful permutation representation of degree* 23 *of $\mathcal{M}_{23}$ is stated in Lemma 8.2.2 of* [12].
(b) *The first irreducible representation (ρ_1, V_1) of $\mathcal{M}_{23}$ is described by the following matrices:*

$$\rho_1(a) = \begin{pmatrix} 01001111100 \\ 01100011111 \\ 00000010000 \\ 00110111010 \\ 11000110011 \\ 00000100000 \\ 00100000000 \\ 00000001000 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix}, \quad \rho_1(b) = \begin{pmatrix} 01000000000 \\ 10000000000 \\ 00001000000 \\ 11010101110 \\ 00100000000 \\ 00000100000 \\ 11000110011 \\ 00000001000 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix},$$

$$\rho_1(c) = \begin{pmatrix} 00001000000 \\ 00100000000 \\ 01000000000 \\ 10011101001 \\ 10000000000 \\ 00000100000 \\ 01100011111 \\ 00000001000 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix}, \quad \rho_1(d) = \begin{pmatrix} 01111010001 \\ 10111011100 \\ 00110111010 \\ 00000010000 \\ 00011110111 \\ 00000100000 \\ 00010000000 \\ 00000001000 \\ 00000000100 \\ 00000000010 \\ 00000000001 \end{pmatrix},$$

$$\rho_1(t) = \begin{pmatrix} 01000000100 \\ 10011101101 \\ 01001111000 \\ 10111011000 \\ 01111010101 \\ 00000000100 \\ 11010101010 \\ 00000001100 \\ 00000100100 \\ 00000000110 \\ 00000000101 \end{pmatrix}, \quad \rho_1(g) = \begin{pmatrix} 01010000000 \\ 10010000000 \\ 01011111100 \\ 00010000000 \\ 11010110011 \\ 10101011100 \\ 01110011111 \\ 10100000111 \\ 00001110111 \\ 00010000010 \\ 00010000001 \end{pmatrix},$$

$$\rho_1(h) = \begin{pmatrix} 10000100000 \\ 10011001001 \\ 00110011010 \\ 00000110000 \\ 11000010011 \\ 00000100000 \\ 00010100000 \\ 00000100010 \\ 00000100100 \\ 00000101000 \\ 00000100001 \end{pmatrix}, \quad \rho_1(i) = \begin{pmatrix} 10011001001 \\ 01000100000 \\ 11010001110 \\ 00010100000 \\ 00001100000 \\ 00000100000 \\ 01001011100 \\ 00000101000 \\ 00000100100 \\ 11101000101 \\ 00000100001 \end{pmatrix},$$

$$\rho_1(j) = \begin{pmatrix} 01000000000 \\ 10000000000 \\ 00001000000 \\ 00010000000 \\ 00100000000 \\ 00000000100 \\ 01100011111 \\ 00000001000 \\ 00000100000 \\ 00000000010 \\ 11101100101 \end{pmatrix}.$$

(c) *The second irreducible representation* (ρ_2, V_2) *of* $\mathcal{M}_{23}$ *is described by the transpose inverse matrices of the generating matrices of* $\mathcal{M}_{23}$ *defining* (ρ_1, V_1)*:*

$$\begin{gathered}\rho_2(a) = [\rho_1(a)^{-1}]^T,\ \rho_2(b) = [\rho_1(b)^{-1}]^T,\ \rho_2(c) = [\rho_1(c)^{-1}]^T,\\ \rho_2(d) = [\rho_1(d)^{-1}]^T,\ \rho_2(t) = [\rho_1(t)^{-1}]^T,\ \rho_2(g) = [\rho_1(g)^{-1}]^T,\\ \rho_2(h) = [\rho_1(h)^{-1}]^T,\ \rho_2(i) = [\rho_1(i)^{-1}]^T,\ \text{and } \rho_2(j) = [\rho_1(j)^{-1}]^T.\end{gathered}$$

(d) $dim_F[H^2(\mathcal{M}_{23}, V_1)] = 1$ *and* $dim_F[H^2(\mathcal{M}_{23}, V_2)] = 0$.

(e) *The presentations of the split extensions* E_1 *and* E_2 *of* $\mathcal{M}_{23}$ *by* V_1 *and* V_2*, respectively, can be constructed from the matrices of (b) and (c) using* MAGMA.

(f) *The unique non-split extension* E *of* $\mathcal{M}_{23}$ *by* V_1 *has the presentation*

$$E = \langle a_1, b_1, c_1, d_1, t_1, g_1, h_1, i_1, j_1, v_i \mid 1 \le i \le 11\rangle$$

with set $\mathcal{R}(E)$ *of defining relations consisting of the following set of relations:*

$$\begin{aligned}
&a_1^2 = b_1^2 = c_1^2 = d_1^2 = t_1^3 = g_1^4 = h_1^4 = j_1^2 = 1,\\
&(b_1, a_1) = (c_1, a_1) = (d_1, a_1) = (h_1, a_1) = (c_1, b_1) = (d_1, b_1) = (d_1, c_1) = 1,\\
&(j_1, b_1) = (j_1, c_1) = (j_1, g_1) = 1,\\
&v_r^2 = 1 \quad \text{for} \quad 1 \le r \le 11,\\
&(v_r, v_s) = 1 \quad \text{for} \quad 1 \le r, s \le 11,\\
&a_1^{-1}v_1a_1v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1} = a_1^{-1}v_2a_1v_2^{-1}v_3^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1,\\
&a_1^{-1}v_3a_1v_7^{-1} = a_1^{-1}v_7a_1v_3^{-1} = a_1^{-1}v_4a_1v_3^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1,\\
&a_1^{-1}v_5a_1v_1^{-1}v_2^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} = 1,\\
&(a_1, v_6) = (a_1, v_8) = (a_1, v_9) = (a_1, v_{10}) = (a_1, v_{11}) = 1,\\
&b_1^{-1}v_1b_1v_2^{-1} = b_1^{-1}v_2b_1v_1^{-1} = b_1^{-1}v_3b_1v_5^{-1} = b_1^{-1}v_5b_1v_3^{-1} = 1,\\
&b_1^{-1}v_4b_1v_1^{-1}v_2^{-1}v_4^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1} = 1,\\
&(b_1, v_6^{-1}) = (b_1, v_8^{-1}) = (b_1, v_9^{-1}) = (b_1, v_{10}^{-1}) = (b_1, v_{11}^{-1}) = 1,\\
&b_1^{-1}v_7b_1v_1^{-1}v_2^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1} = c_1^{-1}v_1c_1v_5^{-1} = c_1^{-1}v_2c_1v_3^{-1} = 1,\\
&c_1^{-1}v_3c_1v_2^{-1} = c_1^{-1}v_5c_1v_1^{-1} = c_1^{-1}v_4c_1v_1^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_{11}^{-1} = 1,\\
&(c_1, v_6^{-1}) = (c_1, v_8^{-1}) = (c_1, v_9^{-1}) = (c_1, v_{10}^{-1}) = (c_1, v_{11}^{-1}) = 1,\\
&c_1^{-1}v_7c_1v_2^{-1}v_3^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = d_1^{-1}v_1d_1v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_{11}^{-1} = 1,\\
&d_1^{-1}v_2d_1v_1^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1} = d_1^{-1}v_3d_1v_3^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1} = 1,\\
&d_1^{-1}v_4d_1v_7^{-1} = d_1^{-1}v_7d_1v_4^{-1} = d_1^{-1}v_5d_1v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1} = 1,\\
&(d_1, v_6^{-1}) = (d_1, v_8^{-1}) = (d_1, v_9^{-1}) = (d_1, v_{10}^{-1}) = (d_1, v_{11}^{-1}) = 1,\\
&t_1^{-1}v_1t_1v_2^{-1}v_9^{-1} = t_1^{-1}v_6t_1v_9^{-1} = t_1^{-1}v_2t_1v_1^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_9^{-1}v_{11}^{-1} = 1,\\
&t_1^{-1}v_3t_1v_2^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1} = t_1^{-1}v_4t_1v_1^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_8^{-1} = 1,\\
&t_1^{-1}v_5t_1v_2^{-1}v_3^{-1}v_4^{-1}v_5^{-1}v_7^{-1}v_9^{-1}v_{11}^{-1} = t_1^{-1}v_7t_1v_1^{-1}v_2^{-1}v_4^{-1}v_6^{-1}v_8^{-1}v_{10}^{-1} = 1,\\
&t_1^{-1}v_8t_1v_8^{-1}v_9^{-1} = t_1^{-1}v_9t_1v_6^{-1}v_9^{-1} = t_1^{-1}v_{10}t_1v_9^{-1}v_{10}^{-1} = t_1^{-1}v_{11}t_1v_9^{-1}v_{11}^{-1} = 1,\\
&g_1^{-1}v_1g_1v_2^{-1}v_4^{-1} = g_1^{-1}v_2g_1v_1^{-1}v_4^{-1} = g_1^{-1}v_3g_1v_2^{-1}v_4^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_8^{-1}v_9^{-1} = 1,
\end{aligned}$$

$$(g_1,v_4)=g_1^{-1}v_5g_1v_1^{-1}v_2^{-1}v_4^{-1}v_6^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$g_1^{-1}v_6g_1v_1^{-1}v_3^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}=g_1^{-1}v_7g_1v_2^{-1}v_3^{-1}v_4^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$g_1^{-1}v_8g_1v_1^{-1}v_3^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=g_1^{-1}v_9g_1v_5^{-1}v_6^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$g_1^{-1}v_{10}g_1v_4^{-1}v_{10}^{-1}=g_1^{-1}v_{11}g_1v_4^{-1}v_{11}^{-1}=h_1^{-1}v_1h_1v_1^{-1}v_6^{-1}=1,$$
$$h_1^{-1}v_4h_1v_6^{-1}v_7^{-1}=(h_1,v_6)=h_1^{-1}v_2h_1v_1^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_{11}^{-1}=1,$$
$$h_1^{-1}v_3h_1v_3^{-1}v_4^{-1}v_7^{-1}v_8^{-1}v_{10}^{-1}=h_1^{-1}v_5h_1v_1^{-1}v_2^{-1}v_7^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$h_1^{-1}v_7h_1v_4^{-1}v_6^{-1}=h_1^{-1}v_8h_1v_6^{-1}v_{10}^{-1}=h_1^{-1}v_9h_1v_6^{-1}v_9^{-1}=1,$$
$$h_1^{-1}v_{10}h_1v_6^{-1}v_8^{-1}=h_1^{-1}v_{11}h_1v_6^{-1}v_{11}^{-1}=i_1^{-1}v_1i_1v_1^{-1}v_4^{-1}v_5^{-1}v_8^{-1}v_{11}^{-1}=1,$$
$$i_1^{-1}v_2i_1v_2^{-1}v_6^{-1}=i_1^{-1}v_4i_1v_4^{-1}v_6^{-1}=i_1^{-1}v_5i_1v_5^{-1}v_6^{-1}=1,$$
$$i_1^{-1}v_3i_1v_1^{-1}v_2^{-1}v_4^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}=1,\quad (i_1,v_6)=1,$$
$$i_1^{-1}v_7i_1v_2^{-1}v_5^{-1}v_7^{-1}v_8^{-1}v_9^{-1}=i_1^{-1}v_{11}i_1v_6^{-1}v_{11}^{-1}=i_1^{-1}v_8i_1v_6^{-1}v_8^{-1}=1,$$
$$i_1^{-1}v_9i_1v_6^{-1}v_9^{-1}=i_1^{-1}v_{10}i_1v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_9^{-1}v_{11}^{-1}=1,$$
$$j_1^{-1}v_1j_1v_2^{-1}=j_1^{-1}v_2j_1v_1^{-1}=j_1^{-1}v_3j_1v_5^{-1}=(j_1,v_4)=(j_1,v_8)=1,$$
$$(j_1,v_{10})=j_1^{-1}v_5j_1v_3^{-1}=j_1^{-1}v_6j_1v_9^{-1}=j_1^{-1}v_9j_1v_6^{-1}=1,$$
$$j_1^{-1}v_7j_1v_2^{-1}v_3^{-1}v_7^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=j_1^{-1}v_{11}j_1v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{11}^{-1}=1,$$
$$t_1^{-1}a_1t_1d_1^{-1}c_1^{-1}v_6^{-1}v_9^{-1}=t_1^{-1}b_1t_1d_1^{-1}a_1^{-1}v_6^{-1}v_9^{-1}=1,$$
$$t_1^{-1}c_1t_1d_1^{-1}b_1^{-1}v_6^{-1}v_9^{-1}=t_1^{-1}d_1t_1c_1^{-1}b_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_9^{-1}=1,$$
$$g_1^2v_1^{-1}v_3^{-1}v_8^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=(g_1a_1)^3v_{10}^{-1}v_{11}^{-1}=1,$$
$$(g_1b_1)^3v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}=(g_1c_1)^3=1,\quad (i_1j_1)^3=1$$
$$(g_1t_1)^2v_1^{-1}v_3^{-1}v_8^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$h_1^2v_6^{-1}=1,\quad h_1^{-1}b_1h_1d_1^{-1}b_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}=1,$$
$$h_1^{-1}c_1h_1c_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}=h_1^{-1}d_1h_1d_1^{-1}v_6^{-1}=1,$$
$$h_1^{-1}t_1h_1t_1v_9^{-1}=(g_1h_1)^3v_3^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$i_1^{-1}a_1i_1d_1^{-1}c_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$i_1^{-1}b_1i_1d_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$i_1^{-1}c_1i_1d_1^{-1}c_1^{-1}b_1^{-1}a_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$i_1^{-1}d_1i_1d_1^{-1}c_1^{-1}b_1^{-1}=i_1^{-1}t_1i_1t_1=i_1^{-1}g_1i_1g_1^{-1}t_1^{-1}v_5^{-1}v_6^{-1}v_7^{-1}v_9^{-1}v_{10}^{-1}v_{11}^{-1}=1,$$
$$(h_1i_1)^3v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_8^{-1}v_{10}^{-1}=j_1^{-1}a_1j_1c_1^{-1}b_1^{-1}a_1^{-1}=1,$$
$$j_1^{-1}d_1j_1d_1^{-1}c_1^{-1}v_1^{-1}v_2^{-1}v_3^{-1}v_5^{-1}v_6^{-1}v_9^{-1}=j_1^{-1}t_1j_1t_1=1,\quad j_1^{-1}h_1j_1h_1^{-1}t_1^{-1}=1.$$

Proof. The 2 irreducible $F\mathcal{M}_{23}$-modules V_i, $i=1,2$, occur as composition factors with multiplicity 1 in the permutation module $(1_{\mathcal{M}_{22}})^{\mathcal{M}_{23}}$ of degree 23 where $\mathcal{M}_{22}=\langle a_1,b_1,c_1,d_1,t_1,g_1,h_1,i_1\rangle$. They are dual to each other. Using the faithful permutation representation of $\mathcal{M}_{23}$ stated in (a) and the Meat-axe Algorithm implemented in Magma one obtains the generating matrices of $\mathcal{M}_{23}$ stated in (b) defining V_1. Their dual matrices define V_2. They are stated in (c).

(d) The cohomological dimensions $dim_F[H^2(\mathcal{M}_{23},V_i)]$, $i=1,2$, have been calculated by means of Magma using Holt's Algorithm 7.4.5 of [12]. Its hypothesis is satisfied by the presentation of $\mathcal{M}_{23}$ stated in Definition 8.2.1 of [12] and all the data of (a), (b) and (c). It follows that $dim_F[H^2(\mathcal{M}_{23},V_1)]=1$ and $dim_F[H^2(\mathcal{M}_{23},V_2)]=0$.

(e) This statement is checked easily with Magma.

(g) The presentation of the non-split extension E has been obtained by means of the commands `ExtensionProcess` and `Extension` of Holt's Algorithm 7.4.5 of [12] implemented in Magma [7]. This completes the proof.

□

The first statement of the following subsidiary lemma is mainly due to Paul Young.

Lemma 3.2. *With the notations of Lemma 3.1 the following statements hold:*

(a) *The non-split extension E of $\mathcal{M}_{23}$ by its simple module V_1 has a faithful permutation representation PE of degree 1012 with stabilizer U generated by the two elements $(i_1^{-1}j_1^{-1}b_1g_1)^2$ and $(i_1^{-1}b_1t_1h_1^{-1})^4$.*

(b) $E = \langle a_1, b_1, c_1, d_1, t_1, g_1, h_1, i_1, j_1 \rangle$.

(c) *E has 3 conjugacy classes of 2-central involutions. They are represented by $z_1 = (d_1g_1)^5$, $z_2 = h_1^2$ and $z_3 = g_1^2$. Their centralizers have orders $|C_E(z_1)| = 2^{18}\cdot 3^2\cdot 5\cdot 7\cdot 11$, $|C_E(z_2)| = 2^{18}\cdot 3^2\cdot 5\cdot 7$ and $|C_E(z_3)| = 2^{18}\cdot 3^2\cdot 5$.*

(d) *$D = C_E(z_1) = \langle x_1, y_1\rangle$ and $E = \langle D, e_1\rangle$ where $x_1 = a_1$, $y_1 = b_1g_1h_1i_1$ and $e_1 = j_1$ have respective orders 2, 14 and 2.*

(e) *$E = \langle x_1, y_1, e_1\rangle$ has 56 conjugacy classes. A system of representatives is given in Table A.1.*

(f) *$D = \langle x_1, y_1\rangle$ has 69 conjugacy classes. A system of representatives is given in Table A.2.*

(g) *The character tables of E and D are given in Tables B.1 and B.2, respectively.*

Proof. (a) The two generators of the stabilizer U have been found by means of a program due to P. Young. Using the Magma command `CosetAction(E,U)` one obtains a faithful permutation representation PE of E with stabilizer U and degree 1012.

(b), (c) and (d) These statements are easily checked using the permutation representation PE and Magma.

(e) and (f) The faithful permutation representation PE of E, Magma and Kratzer's Algorithm 5.3.18 of [12] are employed to calculate a system of representatives of all conjugacy classes of D and E in terms of their generators given in (d). The results are stated in Tables A.1 and A.2.

The results of (g) were computed by means of Magma.

□

4. Construction of the 2-central involution centralizer of Fi_{23}

Let z_1 denote the central involution of the extension E of $\mathcal{M}_{23}$ defined in Lemma 3.1 and let $D = C_E(z_1)$. Then in the following subsidiary result it is shown that $D_1 = D/\langle z_1\rangle$ is isomorphic to the extension $E_2 = E(\mathrm{Fi}_{22})$ of Lemma 3.1. As proved in [9], the Fischer's simple group $G_1 \cong \mathrm{Fi}_{22}$ can be contructed from the group D_1 by Algorithm 7.4.8 of [12]. Unfortunately, (in 2007) Magma was not able to perform all steps of Holt's Algorithm to establish the 2-fold cover H of G_1 from the presentation of the given group G_1. Therefore the author constructs first all the central extensions H_2 of the centralizer $H_1 = C_{G_1}(t)$ of a 2-central involution t of G_1 by a cyclic group of order 2. It will be shown that one of them has a Sylow 2-subgroup which is isomorphic to the ones of D. Thus H_2 which will be a subgroup of H is uniquely determined up to isomorphism.

The group H, with center $Z(H) = \langle z_1\rangle$ such that $H/Z(H) \cong \mathrm{Fi}_{22}$, isomorphic to the centralizer of a 2-central involution of the target group $\mathfrak{G}$, is then constructed by means of Algorithm 2.1 as a matrix subgroup $\mathfrak{H}$ of $\mathrm{GL}_{352}(17)$. In this way a faithful permutation representation representation of degree 28160 and a presentation of H are built. All these results are described in this section.

Proposition 4.1. *Keep the notations of Lemma 3.1 and 3.2, and let* $E = \langle a_1, b_1, c_1, d_1, t_1, g_1, h_1, i_1, j_1, v_i | 1 \le i \le 11\rangle$ *be the nonsplit extension of* $\mathcal{M}_{23}$ *by its simple module* V_1 *of dimension* 11 *over* $F = \mathrm{GF}(2)$*, and* $D = C_E(z_1)$*, where* $z_1 = (d_1g_1)^5$*. Then the following statements hold:*

(a) $z_1 = (d_1g_1)^5$ *is a* 2*-central involution of* E *with centralizer* $D = C_E(z_1)$ *of order* $2^{18}\cdot 3^2\cdot 5\cdot 7\cdot 11$.

(b) $D = \langle x_1, y_1\rangle$, $z_1 = (x_1y_1^2)^7$ *and* $E = \langle D, e_1\rangle$*, where* $x_1 = a_1$, $y_1 = b_1g_1h_1i_1$ *and* $e_1 = j_1$ *have respective orders* 2, 14 *and* 2. D *has a center of order* 2*, generated by* z_1.

(c) V *is a unique normal subgroup of* D *of order* 2^{11}. V *is elementary abelian, and has a basis* $\mathcal{B} = \{z_1, v_i \mid 1 \le i \le 10\}$*, where*

$$v_1 = y_1^7,\quad v_2 = (x_1y_1x_1)^7,\quad v_3 = (x_1y_1x_1y_1^2)^8,\quad v_4 = (x_1y_1^2x_1y_1)^8,$$
$$v_5 = (y_1x_1y_1^2x_1)^8,\quad v_6 = (x_1y_1x_1y_1^3)^6,\quad v_7 = (x_1y_1^3x_1y_1)^6,$$
$$v_8 = (x_1y_1^5)^7,\quad v_9 = (y_1x_1y_1x_1y_1^2)^6,\quad v_{10} = (y_1^2x_1y_1^3)^7.$$

(d) $V_1 = V/\langle z_1\rangle$ *has a complement* W_1 *in* $D_1 = D/\langle z_1\rangle$ *such that* $W_1 \cong \mathcal{M}_{22}$.
In particular, $D_1 \cong E(\mathrm{Fi}_{22})$ *defined in Proposition 3.3 in* [9] *(in* [9]*,* $E(\mathrm{Fi}_{22})$ *appears as* E_2*).*

(e) *Let* $z_1 = (x_1y_1^2)^7$ *and let* v_i *be the* 10 *basis elements of* $\mathcal{B}$ *given in (c). Then* $D = \langle x_1, y_1\rangle$ *has the following set* $\mathcal{R}(D)$ *of defining relations:*

$$z_1^2 = 1, (x_1, z_1) = (y_1, z_1) = 1,$$
$$v_i^2 = 1, \quad \textit{for all} \quad 1 \le i \le 10,$$
$$(v_j, v_k) = 1 \quad \textit{for all} \quad 1 \le j < k \le 10,$$
$$x_1^2 = y_1^{-7}v_1 = (x_1y_1^{-1})^7 = (y_1^{-1}x_1y_1x_1)^5 = (y_1^{-2}x_1)^7z_1 = 1,$$
$$(y_1^{-2}x_1y_1^2x_1y_1^{-1}x_1)^3v_2 = (x_1y_1^{-2}x_1y_1)^5v_1v_2v_3v_4v_7v_8v_{10} = 1,$$
$$(y_1x_1y_1x_1y_1^2x_1y_1x_1y_1^{-1}x_1y_1^2)^2v_1v_3v_5v_6v_9z_1 = 1,$$
$$x_1y_1^{-1}x_1y_1x_1y_1^2x_1y_1^{-1}x_1y_1^{-1}x_1y_1^2x_1y_1x_1y_1^{-1}x_1y_1^2x_1y_1^2x_1y_1v_1v_3v_6v_8z_1,$$
$$y_1x_1y_1^3x_1y_1x_1y_1^{-2}x_1y_1^{-3}x_1y_1x_1y_1^3x_1y_1^{-1}x_1y_1^{-3}x_1v_4v_8v_{10} = 1,$$

$$
\begin{aligned}
&x_1v_1x_1^{-1}v_2 = x_1v_2x_1^{-1}v_1 = x_1v_3x_1^{-1}v_5 = x_1v_4x_1^{-1}v_3v_4v_5 = x_1v_5x_1^{-1}v_3 = 1,\\
&x_1v_6x_1^{-1}v_1v_2v_6 = x_1v_7x_1^{-1}v_1v_2v_7 = x_1v_8x_1^{-1}v_4v_6v_9 = 1,\\
&x_1v_9x_1^{-1}v_1v_2v_3v_4v_5v_6v_8 = 1,\\
&x_1v_{10}x_1^{-1}v_1v_3v_8v_9v_{10}z_1 = y_1v_1y^{-1}v_1 = y_1v_2y^{-1}v_3v_4v_5v_{10} = 1,\\
&y_1v_3y^{-1}v_1v_3v_4z_1 = y_1v_4y^{-1}v_5 = y_1v_5y^{-1}v_2v_6v_8v_9z_1 = y_1v_6y^{-1}v_9 = 1,\\
&y_1v_7y^{-1}v_1v_2v_6 = y_1v_8y^{-1}v_1v_2v_3v_4z_1 = y_1v_9y^{-1}v_1v_6v_7v_9z_1 = 1,\\
&y_1v_{10}y^{-1}v_2v_4v_5v_7z_1 = 1.
\end{aligned}
$$

(f) *The involution $t = (x_1y_1x_1y_1^3)^6$ of D has a centralizer $T = C_D(t)$ of order $2^{18} \cdot 3 \cdot 5$.*

Proof. (a) and (b) are restatements of Lemma 3.2.

(c) It can be checked by means of the MAGMA command

```
Subgroups(D : Al:=‘‘Normal’’)
```

that D has a unique normal subgroup V of order 2^{11}, and that V is elementary abelian. Now, by means of

```
GetShortGens(sub<D|x_1,y_1>, V : exclude:=sub<D|z_1>)
```

we get the 10 generators v_i which together with z_1 generate V. Since V is elementary elementary abelian, we can regard the generators as a basis.

(d) As $z_1 \in V$ it follows that $V_1 = V/\langle z_1 \rangle$ is the unique normal subgroup of $D_1 = D/\langle z_1 \rangle$ of order 2^{10}. Clearly, V_1 is elementary abelian.

Applying the MAGMA command `CompositionFactors(D)` one sees that $D_1/V_1 \cong D/V \cong \mathcal{M}_{22}$. Thus Lemma 2.4(d) of [9] asserts that D_1 splits over V_1.

Let Mx and My be the matrices of the generators x_1 and y_1 of D w.r.t. the basis $\mathcal{B}$ of V. Then

$$
Mx = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&1&1&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0\\
0&1&1&0&0&0&1&0&0&0&0\\
0&1&1&0&0&0&0&1&0&0&0\\
0&0&0&0&1&0&1&0&0&1&0\\
0&1&1&1&1&1&1&0&1&0&0\\
1&1&0&1&0&0&0&0&1&1&1
\end{pmatrix}, \quad
My = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&1&0&0\\
1&1&0&1&1&0&1&1&0&1&1\\
0&0&0&0&1&0&1&1&0&1&1\\
0&0&0&0&1&0&0&0&0&0&0\\
0&1&0&1&0&0&0&1&1&0&0\\
1&0&0&1&0&0&1&1&1&1&0\\
1&1&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0\\
1&1&1&1&1&0&0&0&0&0&0
\end{pmatrix},
$$

Both matrices are blocked lower triangular matrices with upper left diagonal blocks equal to 1 and lower diagonal 10×10 blocks Mx_1 and My_1 in $\mathrm{GL}_{10}(2)$. Hence $D_1/V_1 \cong W_1$, where $W_1 = \langle Mx_1, My_1 \rangle \le \mathrm{GL}_{10}(2)$. Applying the MAGMA command `FPGroup(sub<GL(10,2)|Mx1,My1>)` to W_1 one obtains the following set $\mathcal{R}(W_1)$ of defining relations of $W_1 = \langle x_1, y_1 \rangle$:

$$
\begin{aligned}
&x_1^2 = 1, \quad y_1^7 = 1,\\
&(x_1y_1^{-1})^7 = 1, \quad (y_1^{-1}x_1y_1x_1)^5 = 1, \quad (y_1^{-2}x_1)^7 = 1,\\
&(y_1^{-2}x_1y_1^2x_1y_1^{-1}x_1) = 1, \quad (x_1y_1^{-2}x_1y_1)^5 = 1,\\
&(y_1x_1y_1x_1y_1^2x_1y_1x_1y_1^{-1}x_1y_1^2)^2 = 1,\\
&x_1y_1^{-1}x_1y_1x_1y_1^2x_1y_1^{-1}x_1y_1^{-1}x_1y_1^2x_1y_1x_1y_1^{-1}x_1y_1^2x_1y_1^2x_1y_1 = 1,\\
&y_1x_1y_1^3x_1y_1x_1y_1^{-2}x_1y_1^{-3}x_1y_1x_1y_1^3x_1y_1^{-1}x_1y_1^{-3}x_1 = 1.
\end{aligned}
$$

Let PD_1 be the faithful permutation representation of D_1 of degree 1024 with stabilizer W_1. Let $E_2 = E(\mathrm{Fi}_{22})$ be the finitely presented group of Lemma 2.4(f) of [9] with faithful permutation representation PE_2 defined in Lemma 2.6(a) of [9]. By means of the MAGMA command `IsIsomorphic(PE_2,PD_1)` it is verified that D_1 is isomorphic to E_2.

(e) By means of MAGMA the semidirect product $D_1 = \langle x_1, y_1, v_i \mid 1 \le i \le 10\rangle$ of W_1 by $V_1 = \langle v_i \mid 1 \le i \le 10\rangle$ has a set of defining relations $\mathcal{R}(D_1)$ consisting of $\mathcal{R}(W_1)$ and the following set of relations:

$$\begin{aligned}
&v_i^2 = 1, (v_j, v_k) = 1 \quad \text{for all} \quad 1 \le i, j, k \le 10,\\
&x_1v_1x_1^{-1}v_2 = x_1v_2x_1^{-1}v_1 = x_1v_3x_1^{-1}v_5 = 1,\\
&x_1v_4x_1^{-1}v_3v_4v_5 = x_1v_5x_1^{-1}v_3 = 1,\\
&x_1v_6x_1^{-1}v_1v_2v_6, x_1v_7x_1^{-1}v_1v_2v_7 = 1,\\
&x_1v_8x_1^{-1}v_4v_6v_9, x_1v_9x_1^{-1}v_1v_2v_3v_4v_5v_6v_8 = 1,\\
&x_1v_{10}x_1^{-1}v_1v_3v_8v_9v_{10} = y_1v_1y_1^{-1}v_1 = 1,\\
&y_1v_2y_1^{-1}v_3v_4v_5v_{10} = y_1v_3y_1^{-1}v_1v_3v_4 = 1,\\
&y_1v_4y_1^{-1}v_5 = y_1v_5y_1^{-1}v_2v_6v_8v_9 = 1,\\
&y_1v_6y_1^{-1}v_9 = y_1v_7y_1^{-1}v_1v_2v_6 = 1,\\
&y_1v_8y_1^{-1}v_1v_2v_3v_4 = y_1v_9y_1^{-1}v_1v_6v_7v_9 = y_1v_{10}y_1^{-1}v_2v_4v_5v_7 = 1.
\end{aligned}$$

By (c) each v_i is a word in the generators x_1 and y_1 of D. Evaluating the relations of $\mathcal{R}(D_1)$ in D, we get that each relation has value equal to either 1 or z_1. Appending z_1 to appropriate relations, we get the defining set of relations $\mathcal{R}(D)$, as written in the statement.

□

Proposition 4.2. *Keep the notations of Proposition 4.1. The following statements hold:*

(a) *Let $H_1 = \langle r_i | 1 \le i \le 14\rangle \cong H(\mathrm{Fi}_{22})$ be the finitely presented group constructed in Proposition 3.3(n) of* [9] *(for the notation, h_i's in Proposition 3.3(n) of* [9] *are replaced by r_i's). Then H_1 has a central extension $H_2 = \langle k_j \mid 1 \le j \le 15\rangle$ of order $2^{18} \cdot 3^4 \cdot 5$ having the following set of $\mathcal{R}(H_2)$ of defining relations:*

$$\begin{aligned}
&k_1^2 = k_2^5 = k_3^3 = k_4^3 = k_5^2 = k_6^2 = k_7^2 = 1,\\
&k_8^2 = k_9^2 = k_{10}^4 = k_{11}^2 = k_{12}^2 = k_{13}^2 = k_{14}^2 = k_{15}^2 = 1,\\
&(k_i, k_{15}) = 1 \quad \textit{for} \quad 1 \le i \le 14,\\
&(k_i, k_{14}) = 1 \quad \textit{for} \quad 1 \le i \le 13,\\
&(k_i, k_{13}) = 1 \quad \textit{for} \quad 2 \le i \le 12,\\
&k_1^{-1}k_{13}k_1k_{13}^{-1}k_{14}^{-1} = 1,\\
&(k_3, k_4) = 1, \quad k_3^{-1}k_2k_1k_2^{-1}k_3k_1 = 1,\\
&(k_2^{-1}k_1)^4 = 1, \quad k_3^{-1}k_2k_3^{-1}k_1k_2k_3k_1k_2^{-1} = 1,\\
&(k_2k_3^{-1})^4 = 1, \quad (k_4, k_2^{-1}, k_4) = 1,\\
&k_1k_4^{-1}k_1k_4^{-1}k_1k_4k_1k_4 = 1, \quad k_2^{-1}k_3^{-1}k_1k_2^{-2}k_3k_4k_1k_4^{-1} = 1,\\
&k_2k_3^{-1}k_2k_4k_2^2k_3^{-1}k_2^{-1}k_3k_4 = 1, \quad k_1k_5k_1^{-1}k_5k_6k_{13}^{-1} = 1,\\
&k_1k_6k_1^{-1}k_6k_{14}^{-1} = k_1k_7k_1^{-1}k_7k_8k_{14}^{-1} = k_1k_8k_1^{-1}k_8 = 1,
\end{aligned}$$

$$
\begin{aligned}
&k_1k_9k_1^{-1}k_{11}k_{13}^{-1}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_1k_{10}k_1^{-1}k_6k_9k_{10}k_{11}k_{12}k_{13}^{-1}k_{15}^{-1} = 1,\\
&k_1k_{11}k_1^{-1}k_9k_{13}^{-1}k_{15}^{-1} = k_1k_{12}k_1^{-1}k_{12}k_{14}^{-1} = 1,\\
&k_2k_5k_2^{-1}k_5k_6k_7k_{14}^{-1}k_{15}^{-1} = k_2k_6k_2^{-1}k_7k_8k_{13}^{-1}k_{14}^{-1} = 1,\\
&k_2k_7k_2^{-1}k_6k_7k_{14}^{-1}k_{15}^{-1} = k_2k_8k_2^{-1}k_5k_6k_7k_8 = 1,\\
&k_2k_9k_2^{-1}k_5k_6k_{10}k_{11}k_{13}^{-1}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_2k_{10}k_2^{-1}k_7k_8k_9k_{13}^{-1} = k_2k_{11}k_2^{-1}k_5k_{12}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_2k_{12}k_2^{-1}k_6k_7k_8k_{10}k_{12} = k_3k_5k_3^{-1}k_6k_{14}^{-1} = 1,\\
&k_3k_6k_3^{-1}k_5k_6k_{13}^{-1} = k_3k_7k_3^{-1}k_5k_8k_{15}^{-1} = 1,\\
&k_3k_8k_3^{-1}k_5k_6k_7k_8 = k_3k_9k_3^{-1}k_5k_9k_{10}k_{14}^{-1} = 1,\\
&k_3k_{10}k_3^{-1}k_6k_9k_{15}^{-1} = k_3k_{11}k_3^{-1}k_9k_{12}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_3k_{12}k_3^{-1}k_5k_{10}k_{11}k_{12}k_{13}^{-1}k_{14}^{-1} = 1,\\
&k_4k_5k_4^{-1}k_5k_6k_{13}^{-1}k_{14}^{-1} = 1,\\
&k_4k_6k_4^{-1}k_5k_{14}^{-1} = k_4k_7k_4^{-1}k_6k_{10}k_{11}k_{12}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_4k_8k_4^{-1}k_5k_6k_{10}k_{11}k_{13}^{-1}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_4k_9k_4^{-1}k_5k_6k_{11}k_{12}k_{14}^{-1} = 1,\\
&k_4k_{10}k_4^{-1}k_9k_{10}k_{12}k_{13}^{-1}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_4k_{11}k_4^{-1}k_5k_6k_7k_8k_9k_{11}k_{13}^{-1}k_{15}^{-1} = 1,\\
&k_4k_{12}k_4^{-1}k_5k_7k_{10}k_{11}k_{14}^{-1} = k_{10}^2k_{14}^{-1}k_{15}^{-1} = 1,\\
&(k_5,k_6) = (k_5,k_7) = (k_5,k_8) = (k_5,k_9) = 1,\\
&k_5^{-1}k_{10}^{-1}k_5k_{10}k_{14}^{-1}k_{15}^{-1} = k_5^{-1}k_{11}^{-1}k_5k_{11}k_{14}^{-1}k_{15}^{-1} = 1,\\
&(k_5,k_{12}) = (k_6,k_7) = (k_6,k_8) = 1,\\
&k_6^{-1}k_9^{-1}k_6k_9k_{14}^{-1}k_{15}^{-1} = k_6^{-1}k_{10}^{-1}k_6k_{10}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_6^{-1}k_{11}^{-1}k_6k_{11}k_{14}^{-1}k_{15}^{-1} = 1,\\
&(k_6,k_{12}) = (k_7,k_8) = (k_7,k_9) = (k_8,k_{10}) = (k_8,k_{12}) = 1,\\
&k_7^{-1}k_{10}^{-1}k_7k_{10}k_{14}^{-1}k_{15}^{-1} = k_7^{-1}k_{11}^{-1}k_7k_{11}k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_7^{-1}k_{12}^{-1}k_7k_{12}k_{14}^{-1}k_{15}^{-1} = k_8^{-1}k_9^{-1}k_8k_9k_{14}^{-1}k_{15}^{-1} = 1,\\
&k_8^{-1}k_{11}^{-1}k_8k_{11}k_{14}^{-1}k_{15}^{-1} = 1,\\
&(k_9,k_{10}) = (k_9,k_{11}) = (k_9,k_{12}) = (k_{10},k_{12}) = (k_{11},k_{12}) = 1,\\
&k_{10}^{-1}k_{11}^{-1}k_{10}k_{11}k_{14}^{-1}k_{15}^{-1} = 1.
\end{aligned}
$$

(b) *H_2 has a faithful permutation representation of degree* 2048 *with stabilizer* $U_2 = \langle k_1, k_2, k_3, k_4\rangle$.

(c) *H_2 has a Sylow* 2*-subgroup S_2 generated by the four involutions* $m_1 = k_1$, $m_2 = (k_2k_1k_2k_4k_5)^6$, $m_3 = (k_2^3k_1k_2)^2$ *and* $m_4 = (k_4^2k_2k_4k_2k_4)^6$.

(d) *A_2 is the unique maximal elementary abelian normal subgroup of S_2 of order* 2^{11}*, and is generated by the following* 11 *elements:*

$$
\begin{gathered}
m_4, (m_2m_4)^2, (m_3m_4)^2, (m_1m_3m_4)^4, (m_2m_3m_4)^4,\\
(m_1m_2m_3m_4)^8, (m_1m_3m_1m_4)^2, (m_2m_3m_2m_4)^2,\\
(m_2m_3m_4m_2)^2, (m_1m_2m_3m_1m_4)^4, \text{ and } (m_1m_2m_3m_2m_4)^4.
\end{gathered}
$$

(e) $D_2 = N_{H_2}(A_2) = \langle p_2, q_2\rangle$ *where* $p_2 = k_1k_2$ *and* $q_2 = k_1(k_1k_5k_4k_2)^5$*. Here* p_2 *and* q_2 *have orders* 4 *and* 6*, respectively.*

(f) *$H_2 = \langle D_2, h_2\rangle = \langle p_2, q_2, h\rangle$ where $h_2 = k_4$ has order 3.*

(g) *There is an isomorphism $\alpha : D_2 \to T$ s.t. $\alpha(p_2) = p_1 = (u_1u_2u_4u_6u_7)^3$ and $\alpha(q_2) = q_1 = (w_1w_2w_3w_4w_2w_3w_4)^5$, where*

$$u_1 = (x_1y_1^2)^7, \quad u_2 = (x_1y_1x_1y_1^3)^6, \quad u_3 = (y_1^2x_1y_1^3)^7,$$
$$u_4 = (y_1^3x_1y_1^2)^7, \quad u_5 = x_1y_1x_1y_1x_1y_1x_1y_1^2x_1y_1x_1y_1,$$
$$u_6 = (y_1^3x_1y_1^2x_1y_1^5x_1y_1^3x_1y_1)^2, \quad u_7 = (y_1^5x_1y_1^2x_1y_1x_1y_1x_1y_1x_1y_1^2x_1)^4.$$

and

$$w_1 = (s_1)^7, \quad w_2 = (s_1^4s_2^2s_1^2s_2^2s_1)^2,$$
$$w_3 = s_1^2s_2s_1^3s_2^2s_1^3, \quad w_4 = (s_1^4s_2^2s_1^2s_2s_1^3s_2s_1^3)^2,$$

where $s_1 = x_1y_1x_1$ and $s_2 = (x_1y_1x_1y_1^3)^2$.

(h) *So, $D = \langle p_1, q_1, x_1, y_1\rangle$, $T = C_D(t) = \langle p_1, q_1\rangle$, $H_2 = \langle p_2, q_2, h_2\rangle$, and $D_2 = N_{H_2}(A) = \langle p_2, q_2\rangle$.*

(i) *A system of representatives r_i of the 189 conjugacy classes of H_2 and the corresponding centralizers orders $|C_{H_2}(r_i)|$ are given in Table A.3.*

(j) *A system of representatives d_i of the 151 conjugacy classes of D_2 and the corresponding centralizers orders $|C_{D_2}(d_i)|$ are given in Table A.4.*

(k) *The character tables of H_2 and D_2 are given in Tables B.4 and B.5, respectively.*

Proof. (a) By Table A.8 of [9] the simple group Fi_{22} has a unique conjugacy class of 2-central involutions, represented by an element u. Theorem 5.1 of [9] asserts that $H_1 = C_{\mathrm{Fi}_{22}}(u)$ is isomorphic to the finitely presented group $H_1 = \langle r_i \mid 1 \le i \le 14\rangle$ constructed in Proposition 3.3(n) of [9] (with notation changed from h_i in [9] to r_i here). H_1 has a faithful permutation representation PH_1 of degree 1024 with stabilizer $\langle r_1, r_2, r_3, r_4\rangle$ by Lemma 3.4 of [9].

Let $F = \mathrm{GF}(2)$, and let M be the trivial FH_1-module, which can be constructed in Magma by the commands

```
FEalg := MatrixAlgebra< FiniteField(2), 1 | [[1] : i in [1..14]]>
```

and

```
M := GModule(PH_1, FEalg).
```

Using the faithful permutation representation PH_1 and the Magma command

```
CohomologicalDimension(PH_1, M, 2)
```

we get that this cohomological dimension is 3. For each of the eight 2-cocycles (a, b, c) with $a, b, c \in F$, Magma constructs a finitely presented group $E_{(a,b,c)}$ by means of its commands

```
P := ExtensionProcess(PH_1,M,H_1),
E_{a,b,c} := Extension(P, [a,b,c]).
```

Since D is a non-split extension of D_1 by $\langle z_1\rangle$ the split extension $E_{(0,0,0)}$ can be neglected.

It is checked by means of Magma using the faithful permutation representation PD of D of degree 1012 that $E_{(1,1,0)}$ is a non-split extension of order $2^{18} \cdot 3^4 \cdot 5$ of H_1 having a Sylow 2-subgroup which is isomorphic to any of the Sylow 2-subgroups of D. The presentation of $H_2 = E_{(1,1,0)}$ is given in the statement.

(b) The Magma command `CosetAction(H_2,U_2)` for the subgroup $U_2 = \langle k_1, k_2, k_3, k_4\rangle$ of the finitely presented group H_2 gives the faithful permutation representation of H_2 of degree 2048. We know it is faithful because the order of the resulting permutation group has the correct order $2^{18} \cdot 3^4 \cdot 5$.

(c) A Sylow 2-subgroup S_2 of H_2 can be obtained by the MAGMA command

```
S_2 := SylowSubgroup(H_2,2)
```

and its generators can be obtained by the command

```
GetShortGens(H_2,S_2),
```

which gave the four generators $m_1 = k_1$, $m_2 = (k_2k_1k_2k_4k_5)^6$, $m_3 = (k_2^3k_1k_2)^2$ and $m_4 = (k_4^2k_2k_4k_2k_4)^6$, as asserted. The four generators of S_2 all have order 2.

(d) It can be verified by the MAGMA command

```
Subgroups(S_2 : Al:=''Normal'', IsElementaryAbelian:=true)
```

that S_2 has a unique maximal elementary abelian subgroup A_2 of order 2^{11}. Now the generators for A_2 can be obtained by the command

```
GetShortGens(sub<H_2|m_1,m_2,m_3,m_4>,A_2),
```

which gave the 11 generators as written in the statement.

(e) Letting

```
D_2 := Normalizer(H_2,A_2)
```

yields a subgroup D_2 of H_2 of order $2^{18} \cdot 3 \cdot 5$, and its generators can be obtained by the command

```
GetShortGens(H_2,D_2),
```

which gave the two generators $p_2 = k_1k_2$ and $q_2 = k_1(k_1k_5k_4k_2)^5$, having orders 4 and 6, respectively.

(f) It can be verified with MAGMA that $H_2 = \langle D_2, k_4\rangle = \langle p_2, q_2, k_4\rangle$. We let $h_2 = k_4$, and it is easy to check with MAGMA that h_2 has order 3.

(g) The isomorphism $\alpha : D_2 \to T$ can be obtained by the MAGMA command

```
IsIsomorphic(D_2,T).
```

Recall that T is a subgroup of $D = \langle x_1, y_1\rangle$. For documentation of this isomorphism α we need to get words for $\alpha(p_2)$ and $\alpha(q_2)$ in terms of x_1 and y_1. Now, for convenience, let $p_1 = \alpha(p_2)$ and $q_1 = \alpha(q_2)$.

In order to obtain short words for p_1 and q_1, the author employed Strategy 1.22. First, it is checked with MAGMA that $N_D(\langle p_1\rangle)$ has order 2^8, and it is obtained by `GetShortGens` that $N_D(\langle p_1\rangle) = \langle u_1, u_2, u_3, u_4u_5, u_6, u_7\rangle$, where the words u_1 through u_7 are as written in the statement. Then, the command `LookupWord(sub<D|u_1,u_2,u_3,u_4,u_5,u_6,u_7>, p_1)` gave the answer $p_1 = (u_1u_2u_4u_6u_7)^3$.

Note that q_1 has order 6, since so does q_2. It can be checked with MAGMA that the $N_D(\langle q_1\rangle)$ has order $2^6 \cdot 3^2$ and that $C_D(q_1^3)$ has order $2^{17} \cdot 3^2 \cdot 5 \cdot 7$. It is easy to observe that $N_D(\langle q_1\rangle) \leq C_D(q_1^3)$. Employing Strategy 1.21, we can find out that $C_D(q_1^3) = \langle s_1, s_2\rangle$ and $N_D(\langle q_1\rangle) = \langle w_1, w_2, w_3, w_4\rangle$, where $s_1 = x_1y_1x_1$, $s_2 = (x_1y_1x_1y_1^3)^2$, $w_1 = (s_1)^7$, $w_2 = (s_1^4s_2^2s_1^2s_2^2s_1)^2$, $w_3 = s_1^2s_2s_1^3s_2^2s_1^3$, and $w_4 = (s_1^4s_2^2s_1^2s_2s_1^3s_2s_1^3)^2$. Finally, the command `LookupWord(sub<D|w_1,w_2,w_3,w_4>, q_1)` successfully gave the word for q_1, namely $q_1 = (w_1w_2w_3w_4w_2w_3w_4)^5$.

(h) From (g) we know that $D_2 = \langle p_2, q_2\rangle$ and $\alpha : D_2 \to T$ is an isomorphism, and also that $p_1 = \alpha(p_2)$ and $q_1 = \alpha(q_2)$. Therefore we have $T = \langle p_1, q_1\rangle$. Since $D = \langle x_1, y_1\rangle$, it is clear that $D = \langle p_1, q_1, x_1, y_1\rangle$. It is verified in (f) that $H_2 = \langle D_2, k_4\rangle = \langle p_2, q_2, k_4\rangle$.

The results of (i) and (j) can be obtained by applying Kratzer's Algorithm 5.3.18 of [12] in Magma to the relevant groups, with generators as stated in (h). The results of (k) were computed by means of Magma.

□

It will be shown in the rest of this section that the free product $H_2 *_{D_2} D$ of H_2 and D with amalgamated subgroup D_2 has an irreducible 352-dimensional faithful representation over GF(17) which gives rise to the group H such that $|Z(H)| = 2$ and $H/Z(H) \cong \mathrm{Fi}_{22}$.

Theorem 4.3. *Keep the notations in Proposition 4.2. Let $K = \mathrm{GF}(17)$. Using the notations of the character tables of B.4, B.5, and B.2 of $H_2 = \langle p_2, q_2, h_2\rangle$, $D_2 = \langle p_2, q_2\rangle$, $D = \langle p_1, q_1, x_1, y_1\rangle$, the following statements hold:*

(a) *The smallest degree of a non-trivial compatible pair $(\chi, \tau) \in mf\,char_{\mathbb{C}}(H_2)\times mf\,char_{\mathbb{C}}(D)$ is 352.*

(b) *There are exactly two compatible pairs $(\chi,\tau), (\chi',\tau') \in mf\,char_{\mathbb{C}}(H_2) \times mf\,char_{\mathbb{C}}(D)$ of degree 352 of $H_2 = \langle D_2, h\rangle$ and $D = \langle \alpha(D_2), x, y\rangle$:*

$$(\chi,\tau) = (\chi_{57} + \chi_{40} + \chi_{41}, \tau_9 + \tau_{10})$$

and

$$(\chi',\tau') = (\chi_{31} + \chi_{34} + \chi_{40} + \chi_{41}, \tau_9 + \tau_{10})$$

with common restriction

$$\chi_{|D_2} = \tau_{|\alpha(D_2)} = \chi'_{|D_2} = \tau'_{|\alpha(D_2)} = \psi_{26} + \psi_{27} + \psi_{59} + \psi_{63} + \psi_{70} + \psi_{73},$$

where irreducible characters with bold face indices denote faithful irreducible characters.

(c) *Let $\mathfrak{V}$ and $\mathfrak{W}$ be the up to isomorphism uniquely determined faithful semi-simple multiplicity-free 352-dimensional modules of H_2 and D over $F = \mathrm{GF}(17)$ corresponding to the compatible pair χ, τ, respectively.*

Let $\kappa_{\mathfrak{V}} : H_2 \to \mathrm{GL}_{352}(17)$ and $\kappa_{\mathfrak{W}} : D \to \mathrm{GL}_{352}(17)$ be the representations of H_2 and D afforded by the modules $\mathfrak{V}$ and $\mathfrak{W}$, respectively.

Let $\mathfrak{p}_2 = \kappa_{\mathfrak{V}}(p_2)$, $\mathfrak{q}_2 = \kappa_{\mathfrak{V}}(q_2)$, $\mathfrak{h}_2 = \kappa_{\mathfrak{V}}(h_2)$ in $\kappa_{\mathfrak{V}}(H_2) \le \mathrm{GL}_{352}(17)$. Then the following assertions hold:

(1) *$\mathfrak{V}_{|D_2} \cong \mathfrak{W}_{|\alpha(D_2)}$, and there is a transformation matrix $\mathcal{T}_1 \in \mathrm{GL}_{352}(17)$ such that*

$$\mathfrak{p}_2 = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}}(p_1)\mathcal{T}_1, \quad \mathfrak{q}_2 = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}}(q_1)\mathcal{T}_1.$$

(2) *There is a transformation matrix $\mathcal{S}_1 \in \mathrm{GL}_{352}(17)$ such that*

$$\mathfrak{p}_2 = \mathcal{S}_1^{-1}\mathfrak{p}_2\mathcal{S}_1, \quad \mathfrak{q}_2 = \mathcal{S}_1^{-1}\mathfrak{q}_2\mathcal{S}_1,$$

and that if we let

$$\mathfrak{x}_2 = (\mathcal{T}_1\mathcal{S}_1)^{-1}\kappa_{\mathfrak{W}}(x_1)\mathcal{T}_1\mathcal{S}_1 \in \mathrm{GL}_{352}(17), \quad \text{and}$$
$$\mathfrak{y}_2 = (\mathcal{T}_1\mathcal{S}_1)^{-1}\kappa_{\mathfrak{W}}(y_1)\mathcal{T}_1\mathcal{S}_1 \in \mathrm{GL}_{352}(17),$$

then $\mathfrak{H} = \langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2\rangle = \langle \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2\rangle$ satisfies the Sylow 2-subgroup test of Algorithm 7.4.8 Step 5(c) of [12]. The proof showing that $\mathfrak{H}$ satisfies the Sylow 2-subgroup test is split into two parts; first half is shown in the proof of this theorem (namely, the order of $\mathfrak{h}_2\mathfrak{x}_2$ has to be the order of an element in $2\,\mathrm{Fi}_{22}$; it turned out to be 12 in this case), and the other half in Theorem 4.5.

(3) *The three generating matrices of $\mathfrak{H}$ are stated in [10].*

(d) *The construction shown above in* (c) *can be applied to the compatible pair* χ', τ'. *However, there is no solution in this case which satisfies the Sylow 2-subgroup test of Algorithm 7.4.8 Step 5(c) of* [12].

Proof. (a) The character tables of the groups H_2, D_2, and D are stated in the appendix. In the following we use their notations. Using MAGMA and the character tables of H_2, D_2, and D and the fusion of the classes of D_2 in H_2 and $T = C_D(t)(\cong D_2)$ in D, an application of Kratzer's Algorithm 7.3.10 of [12] yields the compatible pair stated in assertion (a).

(b) The application of Kratzer's Algorithm 7.3.10 of [12] also shows that the pairs (χ, τ) and (χ', τ') of (b) are all the compatibles pairs of degree 352 with respect to the fusion of the D_2-classes into the H_2- and into the D-classes.

(c) In order to construct the faithful irreducible representation $\mathfrak{V}$ corresponding to the character $\chi = \chi_{57} + \chi_{40} + \chi_{41}$ of degree 352, the MAGMA command `LowIndexSubgroups(PH_2, 1500)` is applied, using the faithful permutation presentation PH_2 of H_2 of degree 2048. MAGMA found subgroups U_1, U_2, and U_3 such that the followings hold.

U_1 is of index 480 in H_2, and χ_{57} (dimension 160) is a constituent of the permutation character $(1_{U_1})^{H_2}$. The program `GetShortGens(PH_2, U_1)` gives a generating set of U_1, so we have $U_1 = \langle p_2h_2p_2, (p_2q_2h_2q_2)^3, (q_2h_2^2p_2)^4, (h_2p_2q_2h_2q_2)^6\rangle$. Using Meat-axe Algorithm to the permutation module $(1_{U_1})^{H_2}$, the author obtained the irreducible KH_2-module $\mathfrak{V}_{57}$ over K corresponding to χ_{57}. Here, the permutation module $(1_{U_1})^{H_2}$ is obtained by applying the MAGMA command `PermutationMatrix` to the generators of the permutation group obtained by `CosetAction(PH_2, U_1)`.

U_2 is of index 512 in H_2, and χ_{40} (dimension 96) is a constituent of the permutation character $(1_{U_2})^{H_2}$. By `GetShortGens` we get $U_2 = \langle (q_2p_2h_2)^5, (q_2h_2^2)^4, (p_2h_2q_2p_2)^4, (p_2^2q_2^2h_2)^5\rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{U_2})^{H_2}$, the irreducible KH_2-module $\mathfrak{V}_{40}$ corresponding to χ_{40} is obtained.

U_3 is of index 512 in H_2, and χ_{41} (dimension 96) is a constituent of the permutation character $(1_{U_3})^{H_2}$. By `GetShortGens` we get $U_3 = \langle q_2^3, (h_2q_2)^3, (q_2p_2h_2^2)^6, (q_2p_2h_2q_2^2h_2)^5\rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{U_3})^{H_2}$, the irreducible KH_2-module $\mathfrak{V}_{41}$ corresponding to χ_{41} is obtained.

In order to construct the faithful irreducible representation $\mathfrak{W}$ corresponding to the character $\tau = \tau_9 + \tau_{10}$ of degree 352, the MAGMA command `LowIndexSubgroups(PD, 1500)` is applied, using the faithful permutation presentation PD of D of degree 1012. MAGMA found subgroups S_1 and S_2 such that the followings hold.

S_1 is of index 352 in D, and τ_9 (dimension 176) is a constituent of the permutation character $(1_{S_1})^D$. By `GetShortGens` we get $S_1 = \langle (q_1p_1^3)^3, (x_1q_1p_1q_1x_1)^3, (x_1q_1x_1p_1q_1)^2\rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{S_1})^D$, the irreducible KD-module $\mathfrak{W}_9$ corresponding to τ_9 is obtained.

S_2 is of index 352 in D, and τ_{10} (dimension 176) is a constituent of the permutation character $(1_{S_2})^D$. By `GetShortGens` we get $S_2 = \langle (p_1x_1q_1)^4, x_1q_1x_1, (p_1q_1x_1p_1)^2\rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{S_2})^D$, the irreducible KD-module $\mathfrak{W}_{10}$ corresponding to τ_{10} is obtained.

Thus, we can obtain $\mathfrak{V}$ and $\mathfrak{W}$ by letting $\mathfrak{V} = \mathfrak{V}_{57} \oplus \mathfrak{V}_{40} \oplus \mathfrak{V}_{41}$ and $\mathfrak{W} = \mathfrak{W}_9 \oplus \mathfrak{W}_{10}$, corresponding to the characters $\chi = \chi_{57} + \chi_{40} + \chi_{41}$ and $\tau = \tau_9 + \tau_{10}$, respectively. Direct summation of modules here means block diagonal joining of matrices, in matrix sense. For example, $\mathfrak{p}_2 = \kappa_{\mathfrak{V}}(p_2)$ is obtained by

$$\kappa_{\mathfrak{V}}(p_2) = \mathrm{diag}(\kappa_{\mathfrak{V}_{57}}(p_2), \kappa_{\mathfrak{V}_{40}}(p_2), \kappa_{\mathfrak{V}_{41}}(p_2)),$$

where $\kappa_{\mathfrak{V}_{57}} : H_2 \to \mathrm{GL}_{160}(17)$, $\kappa_{\mathfrak{V}_{57}} : H_2 \to \mathrm{GL}_{96}(17)$, and $\kappa_{\mathfrak{V}_{57}} : H_2 \to \mathrm{GL}_{96}(17)$ are the representations of H_2 afforded by the modules $\mathfrak{V}_{57}$, $\mathfrak{V}_{40}$, and $\mathfrak{V}_{41}$, respectively. The other matrices $\mathfrak{q}_2 = \kappa_{\mathfrak{V}}(q_2)$ and $\mathfrak{h}_2 = \kappa_{\mathfrak{V}}(h_2)$ are obtained by a similar manner, by block diagonal joining. Similar idea applies to the other side $\mathfrak{W}$, too.

$\chi_{|D_2} = \tau_{|\alpha(D_2)}$ means $\mathfrak{V}_{|D_2} \cong \mathfrak{W}_{|\alpha(D_2)} = \mathfrak{W}_{|T}$. Recall that two representations of a group being isomorphic means that there is some matrix $\mathcal{T}$ such that for every element of the group, the matrix for the element corresponding to one of the two representations is conjugate to the matrix for the same element corresponding to the other representation by $\mathcal{T}$. However, here we have two isomorphic groups D_2 and T instead of identical groups. So, employing the isomorphism $\alpha : D_2 \to T$, now we can see that $\mathfrak{V}_{|D_2} \cong \mathfrak{W}_{|T}$ means that there is some $\mathcal{T}_1 \in \mathrm{GL}_{352}(17)$ such that $\kappa_{\mathfrak{V}|D_2}(g) = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}|T}(\alpha(g))\mathcal{T}_1$ for all $g \in D_2$.

Assuming we have such $\mathcal{T}_1$, we get

$$\begin{aligned}\mathfrak{p}_2 &= \kappa_{\mathfrak{V}|D_2}(p_2) = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}|T}(\alpha(p_2))\mathcal{T}_1 = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}}(p_1)\mathcal{T}_1, \quad \text{and}\\ \mathfrak{q}_2 &= \kappa_{\mathfrak{V}|D_2}(q_2) = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}|T}(\alpha(q_2))\mathcal{T}_1 = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}}(q_1)\mathcal{T}_1,\end{aligned}$$

thus $\mathfrak{p}_2 = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}}(p_1)\mathcal{T}_1$ and $\mathfrak{q}_2 = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}}(q_1)\mathcal{T}_1$, as desired in the statement (1). Knowing that such $\mathcal{T}_1$ should exist, we can apply the Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

```
IsIsomorphic(GModule(sub<Y|W(p1),W(q1)>),GModule(sub<Y|V(p2),V(q2)>)),
```

which gives the boolean value, which is `true` in this case, and the desired transformation matrix $\mathcal{T}_1$. So (1) is done.

By assertion (a) and Corollary 7.2.4 of [12] this transformation matrix $\mathcal{T}_1$ has to be multiplied by a diagonal matrix $\mathcal{S}_1$ of $\mathrm{GL}_{352}(17)$. In order to calculate its entries one has to get the composition factors of the restrictions $\chi_{i|D_2}$ and $\tau_{j|T}$ to D_2 and T, respectively. From the fusion and the 3 character tables B.4, B.5 and B.2 follows that:

$$\begin{aligned}&\chi_{40}|_{D_2} = \psi_{27} + \psi_{63}, \quad \chi_{41}|_{D_2} = \psi_{26} + \psi_{59},\\ &\chi_{57}|_{D_2} = \psi_{70} + \psi_{73}, \quad \text{and}\\ &\tau_9|_T = \psi_{26} + \psi_{63} + \psi_{73}, \quad \tau_{10}|_T = \psi_{27} + \psi_{59} + \psi_{70}.\end{aligned}$$

Thus, by applying the Meat-axe Algorithm to $\chi_{40}|_{D_2}, \chi_{41}|_{D_2}, \chi_{57}|_{D_2}$, we get the KD_2-modules $\mathfrak{U}_{26}, \mathfrak{U}_{27}, \mathfrak{U}_{59}, \mathfrak{U}_{63}, \mathfrak{U}_{70}$, and $\mathfrak{U}_{73}$ of $D_2 \cong T$, such that $\mathfrak{V}_{40|D_2} \cong \mathfrak{U}_{27} \oplus \mathfrak{U}_{63}$, $\mathfrak{V}_{41|D_2} \cong \mathfrak{U}_{26} \oplus \mathfrak{U}_{59}$, $\mathfrak{V}_{57|D_2} \cong \mathfrak{U}_{70} \oplus \mathfrak{U}_{73}$, $\mathfrak{W}_{9|T} \cong \mathfrak{U}_{26} \oplus \mathfrak{U}_{63} \oplus \mathfrak{U}_{73}$, and $\mathfrak{W}_{9|T} \cong \mathfrak{U}_{26} \oplus \mathfrak{U}_{63} \oplus \mathfrak{U}_{73}$. Then, we know that

$$\mathfrak{V}_{|D_2} \cong \mathfrak{U}_{26} \oplus \mathfrak{U}_{27} \oplus \mathfrak{U}_{59} \oplus \mathfrak{U}_{63} \oplus \mathfrak{U}_{70} \oplus \mathfrak{U}_{73} \cong \mathfrak{W}_{|T}.$$

Therefore, there is a transformation matrix $\mathcal{S}_0 \in \mathrm{GL}_{352}(17)$ such that

$$\begin{aligned}&\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(p_2)\mathcal{S}_0 = \mathrm{diag}(\kappa_{\mathfrak{U}_{26}}(p_2), \kappa_{\mathfrak{U}_{27}}(p_2), \kappa_{\mathfrak{U}_{59}}(p_2), \kappa_{\mathfrak{U}_{63}}(p_2), \kappa_{\mathfrak{U}_{70}}(p_2), \kappa_{\mathfrak{U}_{73}}(p_2)), \text{ and}\\ &\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(q_2)\mathcal{S}_0 = \mathrm{diag}(\kappa_{\mathfrak{U}_{26}}(q_2), \kappa_{\mathfrak{U}_{27}}(q_2), \kappa_{\mathfrak{U}_{59}}(q_2), \kappa_{\mathfrak{U}_{63}}(q_2), \kappa_{\mathfrak{U}_{70}}(q_2), \kappa_{\mathfrak{U}_{73}}(q_2)),\end{aligned}$$

where $\kappa_{\mathfrak{U}_{26}} : D_2 \to \mathrm{GL}_{16}(17)$, $\kappa_{\mathfrak{U}_{27}} : D_2 \to \mathrm{GL}_{16}(17)$, $\kappa_{\mathfrak{U}_{59}} : D_2 \to \mathrm{GL}_{80}(17)$, $\kappa_{\mathfrak{U}_{63}} : D_2 \to \mathrm{GL}_{80}(17)$, $\kappa_{\mathfrak{U}_{70}} : D_2 \to \mathrm{GL}_{80}(17)$, and $\kappa_{\mathfrak{U}_{73}} : D_2 \to \mathrm{GL}_{80}(17)$ are the representations of D_2 afforded by the modules $\mathfrak{U}_{26}$, $\mathfrak{U}_{27}$, $\mathfrak{U}_{59}$, $\mathfrak{U}_{63}$, $\mathfrak{U}_{70}$, and $\mathfrak{U}_{73}$, respectively. This matrix $\mathcal{S}_0$ can be obtained by applying Parker's isomorphism test to the two modules $\mathfrak{V}$ and $\mathfrak{U}_{26} \oplus \mathfrak{U}_{27} \oplus \mathfrak{U}_{59} \oplus \mathfrak{U}_{63} \oplus \mathfrak{U}_{70} \oplus \mathfrak{U}_{73}$. We can assume that we started with $\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(p_2)\mathcal{S}_0$, $\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(q_2)\mathcal{S}_0$, and $\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(h_2)\mathcal{S}_0$ as $\mathfrak{p}_2$, $\mathfrak{q}_2$, and $\mathfrak{h}_2$, at the beginning of the statement (c). Hence, $\mathfrak{p}_2$ and $\mathfrak{q}_2$ are now assumed to be in the block diagonal form as follows, from the beginning (then $\mathfrak{h}_2$ would be in a certain block form; although not block diagonal, it still carries the structure of $\mathfrak{V}_{57} \oplus \mathfrak{V}_{40} \oplus \mathfrak{V}_{41}$):

$$\mathfrak{p}_2 = \mathrm{diag}(\kappa_{\mathfrak{U}_{26}}(p_2), \kappa_{\mathfrak{U}_{27}}(p_2), \kappa_{\mathfrak{U}_{59}}(p_2), \kappa_{\mathfrak{U}_{63}}(p_2), \kappa_{\mathfrak{U}_{70}}(p_2), \kappa_{\mathfrak{U}_{73}}(p_2)), \text{ and}$$
$$\mathfrak{q}_2 = \mathrm{diag}(\kappa_{\mathfrak{U}_{26}}(q_2), \kappa_{\mathfrak{U}_{27}}(q_2), \kappa_{\mathfrak{U}_{59}}(q_2), \kappa_{\mathfrak{U}_{63}}(q_2), \kappa_{\mathfrak{U}_{70}}(q_2), \kappa_{\mathfrak{U}_{73}}(q_2)).$$

Now, by Schur's Lemma (Lemma 2.1.8 of [12]) and the degrees of these characters appearing in the restriction pattern, the following linear system in the variables $\sigma_a, \sigma_b, \sigma_c, \sigma_d, \sigma_e, \sigma_f \in K$ holds, where the variables $\sigma_a, \sigma_b, \sigma_c, \sigma_d, \sigma_e, \sigma_f$ correspond to $\psi_{26}, \psi_{27}, \psi_{59}, \psi_{63}, \psi_{70}, \psi_{73}$, respectively :

$$96 = 16\sigma_b + 80\sigma_d, \quad 96 = 16\sigma_a + 80\sigma_c, \quad 160 = 80\sigma_e + 80\sigma_f \quad \text{and}$$
$$176 = 16\sigma_a + 80\sigma_d + 80\sigma_f, \quad 176 = 16\sigma_b + 80\sigma_c + 80\sigma_e.$$

This system of equations in K has the solution: $\sigma_c = 8 - 7\sigma_a$, $\sigma_d = 8 - 7\sigma_b$, $\sigma_e = 7\sigma_a - 7\sigma_b + 1$, $\sigma_f = -7\sigma_a + 7\sigma_b + 1$, where σ_a and σ_b run through all nonzero elements of K. Hence the diagonal matrix $\mathcal{S}_1$ has the form

$$\mathcal{S}_{(\sigma_a,\sigma_b)} = diag(\sigma_a^{16}, \sigma_b^{16}, [8-7\sigma_a]^{80}, [8-7\sigma_b]^{80}, [7\sigma_a - 7\sigma_b + 1]^{80}, [-7\sigma_a + 7\sigma_b + 1]^{80})$$

for suitable elements $\sigma_a, \sigma_b \in K$. Recall that $\mathfrak{p}_2$ and $\mathfrak{q}_2$ are in the block diagonal form as described above, where the sizes of the blocks are $16, 16, 80, 80, 80$, and 80, in this order. Therefore, for any σ_a and σ_b in K, the diagonal matrix $\mathcal{S}_{(\sigma_a,\sigma_b)}$ centralizes the two matrices $\mathfrak{p}_2$ and $\mathfrak{q}_2$.

Let

$$\mathfrak{x}_{(\sigma_b,\sigma_d)} = \left(\mathcal{T}_1\mathcal{S}_{(\sigma_b,\sigma_d)}\right)^{-1}\kappa_{\mathfrak{W}}(x_1)\mathcal{T}_1\mathcal{S}_{(\sigma_b,\sigma_d)}, \quad \text{and}$$
$$\mathfrak{y}_{(\sigma_b,\sigma_d)} = \left(\mathcal{T}_1\mathcal{S}_{(\sigma_b,\sigma_d)}\right)^{-1}\kappa_{\mathfrak{W}}(y_1)\mathcal{T}_1\mathcal{S}_{(\sigma_b,\sigma_d)}.$$

Then the field elements σ_a, σ_b have to be chosen so that all the entries on the main diagonal of the matrix $\mathcal{S}_{(\sigma_a,\sigma_b)}$ are nonzero and that the matrix group

$$\mathfrak{H}_{\sigma_a,\sigma_b} = \langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{x}_{\sigma_a,\sigma_b}, \mathfrak{y}_{\sigma_a,\sigma_b}, \mathfrak{h}_2 \rangle$$

satisfies the Sylow 2-subgroup test of Algorithm 7.4.8 Step 5(c) of [12]. The test used here is the order of $\mathfrak{x}_{\sigma_a,\sigma_b}\mathfrak{h}_2$; the order has to be the order of an element in $2\,\mathrm{Fi}_{22}$. Running through all possible pairs $(\sigma_a, \sigma_b) \in F^2$ it follows that this test is only successful for the pair $(\sigma_a, \sigma_b) = (15, 9)$, giving the order of $\mathfrak{x}_{\sigma_a,\sigma_b}\mathfrak{h}_2$ equal to 12.

Now, let $\mathcal{S}_1 = \mathcal{S}_{(15,9)}$, $\mathfrak{x}_2 = (\mathcal{T}_1\mathcal{S}_1)^{-1}\kappa_{\mathfrak{W}}(x_1)\mathcal{T}_1\mathcal{S}_1$, $\mathfrak{y}_2 = (\mathcal{T}_1\mathcal{S}_1)^{-1}\kappa_{\mathfrak{W}}(y_1)\mathcal{T}_1\mathcal{S}_1$, and $\mathfrak{H} = \langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle$. Then we have $\mathfrak{p}_2 = \mathcal{S}_1^{-1}\mathfrak{p}_2\mathcal{S}_1$, $\mathfrak{q}_2 = \mathcal{S}_1^{-1}\mathfrak{q}_2\mathcal{S}_1$, since any $\mathcal{S}_{(\sigma_a,\sigma_b)}$ centralizes the two matrices $\mathfrak{p}_2$ and $\mathfrak{q}_2$. This matrix group $\mathfrak{H}$ is a candidate which can satisfy the Sylow 2-subgroup test. In Theorem 4.5, it is shown that $\mathfrak{H}$ indeed satisfies the Sylow 2-subgroup test. So (2) is done.

Finally, since $\mathfrak{p}_2$ and $\mathfrak{q}_2$ can be expressed as words in terms of $\mathfrak{x}_2$ and $\mathfrak{y}_2$, we have $\mathfrak{H} = \langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle = \langle \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle$.

For (3), the matrices are stated in [10].

(d) Same idea and process as in (c) are applied to the compatible pair χ', τ'. As done in (2) of (c), a suitable transformation matrix has to be found. However, in this case, there is no such transformation matrix which makes the final matrix group to satisfy the Sylow 2-subgroup test of Algorithm 7.4.8 of Step 5(c) of [12]. In other words, there was no $\mathcal{S}_1$ which makes the order of $\mathfrak{x}_2\mathfrak{y}_2$ to be the order of an element in $2\,\mathrm{Fi}_{22}$.

□

Lemma 4.4. *Using a very nice presentation for Fischer's simple group* Fi_{22} *which is stated by Praeger and Soicher in* [13], *the finitely presented group* $G_n = \langle a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n\rangle$ *(here the subscript* n *stands for "nice") with set* $\mathcal{R}(G_n)$ *of defining relations*

$$\begin{aligned}
&a_n^2 = b_n^2 = c_n^2 = d_n^2 = e_n^2 = f_n^2 = g_n^2 = h_n^2 = i_n^2 = 1,\\
&(a_nb_n)^3 = 1, (b_nc_n)^3 = (c_nd_n)^3 = (d_ne_n)^3 = (e_nf_n)^3 = (f_ng_n)^3 = 1,\\
&(a_nc_n)^2 = (a_nd_n)^2 = (a_ne_n)^2 = (a_nf_n)^2 = (a_ng_n)^2 = (a_nh_n)^2 = (a_ni_n)^2 = 1,\\
&(b_nd_n)^2 = (b_ne_n)^2 = (b_nf_n)^2 = (b_ng_n)^2 = (b_nh_n)^2 = (b_ni_n)^2 = 1,\\
&(c_ne_n)^2 = (c_nf_n)^2 = (c_ng_n)^2 = (c_nh_n)^2 = (c_ni_n)^2 = 1,\\
&(d_nf_n)^2 = (d_ng_n)^2 = (e_ng_n)^2 = (e_nh_n)^2 = (e_ni_n)^2 = 1,\\
&(d_nh_n)^3 = (h_ni_n)^3 = (d_ni_n)^2 = (f_nh_n)^2 = (f_ni_n)^2 = (g_nh_n)^2 = (g_ni_n)^2 = 1,\\
&(d_nc_nb_nd_ne_nf_nd_nh_ni_n)^{10} = (a_nb_nc_nd_ne_nf_nh_n)^9 = (b_nc_nd_ne_nf_ng_nh_n)^9 = 1
\end{aligned}$$

satisfies the following properties:

(a) *As in page 110 of* [13], G_n *has a subgroup*
$$U_n = \langle a_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n, (a_nb_nc_nd_ne_nh_n)^5\rangle$$
which is isomorphic to $2.U_6(2)$. G_n *has a faithful permutation representation* PG_n *of degree* 3510 *with permutation stabilizer* U_n.

(b) *As in page 110 of* [13], G_n *has a subgroup*
$$V_n = \langle b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n\rangle$$
which is isomorphic to the (orthogonal) simple group $O_7(3)$. G_n *has a faithful permutation representation* $(PG_n)'$ *of degree* 14080 *with permutation stabilizer* V_n.

(c) *As in page 111 of* [13], G_n *has a subgroup*
$$E_n = \langle a_n, c_n, e_n, g_n, h_n, u_n, v_n, w_n, x_n, y_n, t_n\rangle$$
which is isomorphic to $2^{10} : \mathcal{M}_{22}$, *where*
$$u_n = b_na_nc_nb_n, \quad v_n = d_nc_ne_nd_n, \quad w_n = d_ne_nh_nd_n,$$
$$x_n = f_ne_ng_nf_n, \quad y_n = d_nc_nh_nd_n, \quad t_n = (c_nd_ne_nh_ni_n)^4.$$
In particular, E_n *has order* $2^{17}\cdot 3^2\cdot 5\cdot 7\cdot 11$.

(d) E_n *has exactly one conjugacy class of* 2*-central involutions, represented by* $z_n = a_nc_n$.

(e) *The centralizer* $H_n = C_{G_n}(z_n)$ *of* z_n *in* G_n *has order* $2^{17}\cdot 3^4\cdot 5$, *and* $H_n = \langle f_n, g_n, i_n, (a_nb_nc_n)^2, (c_nd_ne_nh_n)^3\rangle$.

(f) *The intersection* $D_n = E_n \cap H_n$ *of* E_n *and* H_n *has order* $2^{17}\cdot 3\cdot 5$.

Proof. (a) In [13], the presentation is stated without the subscripts n. In page 110 of [13], it is stated that $G_n \cong \mathrm{Fi}_{22}$ and that G_n has a subgroup $U_n \cong 2.U_6(2)$, with generators as written in the statement. By means of MAGMA's command `CosetAction(G_n,U_n)`, we can check that G_n has a permutation representation PG_n of degree 3510 with stabilizer U_n, with order $|PG_n| = 2^{17}\cdot 3^9\cdot 5^2\cdot 7\cdot 11\cdot 13$. As proved in [9], we have $G_n \cong \mathrm{Fi}_{22}$, and therefore PG_n is a faithful permutation representation of G_n.

(b) In page 110 of [13], it is stated that $G_n \cong \mathrm{Fi}_{22}$ has a subgroup $V_n \cong O_7(3)$, with generators as written in the statement. By means of MAGMA's command `CosetAction(G_n,V_n)`, we can check that G_n has a permutation representation $(PG_n)'$ of degree 14080 with stabilizer V_n.

The statement (c) is as written in page 111 of [13].

(d) The MAGMA command `Classes(E_n)` tells us that E_n has exactly one conjugacy class of 2-central involutions, and it also gives a representative z_n for the class. Now, the command `LookupWord(E_n, z_n : ConjugateCheck:=true)` in MAGMA gives an answer a_nc_n, which is conjugate to z_n. So, we can redefine z_n to be $z_n = a_nc_n$, and use it as the representative for this class of 2-central involutions.

(e) Let $z_n = a_nc_n$ as in (d). Then, the group $H_n = C_{G_n}(z_n)$ is verified by means of MAGMA to have order $2^{17} \cdot 3^4 \cdot 5$. Now, by applying `GetShortGens` the author obtained the five generators $f_n, g_n, i_n, (a_nb_nc_n)^2, (c_nd_ne_nh_n)^3$ for H_n.

(f) The statement is verified by means of MAGMA.

□

In the following theorem the author obtains the presentation for $2\,\mathrm{Fi}_{22}$ as generators and relations, and establishes an isomorphism from $\mathfrak{H}$ to $2\,\mathrm{Fi}_{22}$.

Theorem 4.5. *Keep the notations of Lemma 4.4 and Proposition 4.1 and 4.2. The following statements hold:*

(a) $D/\langle z_1\rangle = D_1 \cong E_n$ *and* $H_2/\langle k_{15}\rangle = H_1 \cong H_n$.

(b) *Let* $\phi_1 : D \to D/\langle z_1\rangle = D_1$ *and* $\phi_2 : H_2 \to H_2/\langle k_{15}\rangle = H_1$ *be the canonical epimorphisms. There exist an element* $r_2 \in G_n$ *and isomorphisms* $\varphi_1 : D_1 \to E_n$ *and* $\varphi_2 : H_1 \to (H_n)^{r_2}$ *such that* $(\varphi_1\circ\phi_1)(p_1) = p_3 = (\varphi_2\circ\phi_2)(p_2)$ *and* $(\varphi_1\circ\phi_1)(q_1) = q_3 = (\varphi_2\circ\phi_2)(q_2)$, *and* $E_n \cap (H_n)^{r_2} = \langle p_3, q_3\rangle$ *has order* $2^{17}\cdot 3\cdot 5$.

(c) *Let* $x_3 = (\varphi_1\circ\phi_1)(x_1)$, $y_3 = (\varphi_1\circ\phi_1)(y_1)$, *and* $h_3 = (\varphi_2\circ\phi_2)(h_2)$. *Then* x_3, y_3 *and* h_3 *can be expressed by words in* $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n$, *as follows:*

$$\begin{aligned} x_3 &= \beta_1(a_n,b_n,c_n,d_n,e_n,f_n,g_n,h_n,i_n) = (r_{n,1}r_{n,2}r_{n,3})^3,\\ y_3 &= \beta_2(a_n,b_n,c_n,d_n,e_n,f_n,g_n,h_n,i_n)\\ &= (s_{n,1}s_{n,2}s_{n,4}s_{n,1}s_{n,4}s_{n,2}s_{n,4})^5(t_{n,1}t_{n,3}^2t_{n,1}t_{n,3}t_{n,1}t_{n,2}t_{n,1}t_{n,3})^{20},\\ h_3 &= \beta_3(a_n,b_n,c_n,d_n,e_n,f_n,g_n,h_n,i_n) = (v_{n,1}v_{n,2})^4, \end{aligned}$$

where

$$\begin{aligned} &r_{n,1} = (w_nu_nt_nu_nx_nt_nx_nt_n)^4, \quad r_{n,2} = (w_nt_nw_nu_nw_nx_nt_nx_n)^4,\\ &r_{n,3} = (x_nw_nt_nu_nx_nw_nt_nx_n)^2, \quad r_{n,4} = (w_nu_nt_nx_nw_nt_nu_nx_nt_n)^4,\\ &s_{n,1} = u_nw_nu_n, \quad s_{n,2} = (u_nw_nx_n)^2, \quad s_{n,3} = (u_nw_nt_nu_n)^2,\\ &s_{n,4} = (u_nt_nw_nx_n)^4, \quad t_{n,1} = s_{n,2}s_{n,4}s_{n,2}^2,\\ &t_{n,2} = (s_{n,4}s_{n,2}s_{n,1}s_{n,3}s_{n,4})^2, \quad t_{n,3} = (s_{n,4}s_{n,2}s_{n,4}s_{n,2}s_{n,1}s_{n,3})^2,\\ &j_n = (c_nd_ne_nh_n)^3, \quad k_n = (c_nd_ne_nf_ng_n)^2, \quad l_n = (a_nb_nc_nd_ne_nh_n)^5,\\ &o_n = (l_nb_nk_nj_nb_ni_n)^6, \quad p_n = (j_nk_nj_nl_nb_ni_nj_ni_n)^5,\\ &v_{n,1} = (p_no_nj_no_nk_n)^4, \quad v_{n,2} = (j_nk_no_nk_n^2p_n)^4, \quad v_{n,3} = k_nj_no_nj_nk_n^2p_n. \end{aligned}$$

(d) $G_n = \langle x_3, y_3, h_3\rangle$.

(e) $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n$ *can be expressed as words in* x_3, y_3, h_3*, denoted by*

$$a_n = \omega_a(x_3, y_3, h_3), \quad b_n = \omega_b(x_3, y_3, h_3), \quad c_n = \omega_c(x_3, y_3, h_3),$$
$$d_n = \omega_d(x_3, y_3, h_3), \quad e_n = \omega_e(x_3, y_3, h_3), \quad f_n = \omega_f(x_3, y_3, h_3),$$
$$g_n = \omega_g(x_3, y_3, h_3), \quad h_n = \omega_h(x_3, y_3, h_3), \quad i_n = \omega_i(x_3, y_3, h_3).$$

The explicit words are

$$\omega_a(x_3, y_3, h_3) = (x_3y_3x_3)^7, \quad \omega_b(x_3, y_3, h_3) = (h_3y_3h_3y_3h_3y_3^3h_3^2y_3^3h_3y_3)^7,$$
$$\omega_c(x_3, y_3, h_3) = (y_3^2x_3y_3x_3y_3^3)^5,$$
$$\omega_d(x_3, y_3, h_3) = (h_3y_3h_3^2y_3h_3y_3h_3y_3h_3y_3^2h_3^2)^{15},$$
$$\omega_e(x_3, y_3, h_3) = (y_3x_3y_3^5x_3)^5,$$
$$\omega_f(x_3, y_3, h_3) = (y_3h_3y_3h_3^2y_3h_3^2y_3^2h_3y_3^4h_3^2)^5,$$
$$\omega_g(x_3, y_3, h_3) = (x_3y_3^2x_3y_3^3x_3)^7, \quad \omega_h(x_3, y_3, h_3) = (y_3^5x_3y_3x_3)^5,$$
$$\omega_i(x_3, y_3, h_3) = (h_3^2y_3^2h_3y_3h_3^2)^7.$$

(f) *The finitely presented group* $G_\ell = \langle a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell, z_\ell\rangle$ *(here* ℓ *stands for "lifted") with set* $\mathcal{R}(G_\ell)$ *of defining relations*

$$a_\ell^2 = b_\ell^2 = c_\ell^2 = d_\ell^2 = e_\ell^2 = f_\ell^2 = g_\ell^2 = h_\ell^2 = i_\ell^2 = 1,$$
$$(a_\ell b_\ell)^3 = 1, (b_\ell c_\ell)^3 = z_\ell, (c_\ell d_\ell)^3 = (d_\ell e_\ell)^3 = 1, (e_\ell f_\ell)^3 = (f_\ell g_\ell)^3 = z_\ell,$$
$$(a_\ell c_\ell)^2 = (a_\ell d_\ell)^2 = (a_\ell e_\ell)^2 = (a_\ell f_\ell)^2 = (a_\ell g_\ell)^2 = (a_\ell h_\ell)^2 = (a_\ell i_\ell)^2 = 1,$$
$$(b_\ell d_\ell)^2 = (b_\ell e_\ell)^2 = (b_\ell f_\ell)^2 = (b_\ell g_\ell)^2 = (b_\ell h_\ell)^2 = (b_\ell i_\ell)^2 = 1,$$
$$(c_\ell e_\ell)^2 = (c_\ell f_\ell)^2 = (c_\ell g_\ell)^2 = (c_\ell h_\ell)^2 = (c_\ell i_\ell)^2 = 1,$$
$$(d_\ell f_\ell)^2 = (d_\ell g_\ell)^2 = (e_\ell g_\ell)^2 = (e_\ell h_\ell)^2 = (e_\ell i_\ell)^2 = 1,$$
$$(d_\ell h_\ell)^3 = (h_\ell i_\ell)^3 = (d_\ell i_\ell)^2 = (f_\ell h_\ell)^2 = (f_\ell i_\ell)^2 = (g_\ell h_\ell)^2 = (g_\ell i_\ell)^2 = 1,$$
$$(d_\ell c_\ell b_\ell d_\ell e_\ell f_\ell d_\ell h_\ell i_\ell)^{10} = (a_\ell b_\ell c_\ell d_\ell e_\ell f_\ell h_\ell)^9 = (b_\ell c_\ell d_\ell e_\ell f_\ell g_\ell h_\ell)^9 = 1,$$
$$z_\ell^2 = (z_\ell, a_\ell) = (z_\ell, b_\ell) = (z_\ell, c_\ell) = (z_\ell, d_\ell) = (z_\ell, e_\ell) = (z_\ell, f_\ell) = 1,$$
$$(z_\ell, g_\ell) = (z_\ell, h_\ell) = (z_\ell, i_\ell) = 1$$

has a faithful permutation representation PG_ℓ *of degree* 28160*, which is isomorphic to* $2\,\mathrm{Fi}_{22}$*, having stabilizer* $\langle b_\ell z_\ell, c_\ell, d_\ell, e_\ell, f_\ell z_\ell, g_\ell, h_\ell, i_\ell\rangle$.

(g) *Let*

$$x_0 = \beta_1(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell),$$
$$y_0 = \beta_2(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell),$$
$$h_0 = \beta_3(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell),$$

where the three words β_1, β_2*, and* β_3 *are the ones obtained in the statement (b). Then* $G_\ell = \langle x_0, y_0, h_0\rangle$.

(h) *There is an isomorphism* $\vartheta : \mathfrak{H} \to G_\ell$ *such that* $\vartheta(\mathfrak{x}_2) = x_0, \vartheta(\mathfrak{y}_2) = y_0$*, and* $\vartheta(\mathfrak{h}_2) = h_0$*, and* $\mathfrak{H}$ *is an irreducible subgroup of* $\mathrm{GL}_{352}(17)$.

Proof. (a) The presentation of $D_1 = D/\langle z_1\rangle$ can be obtained from the presentation of D, as stated in Proposition 4.1(e). Let $\phi_1 : D \to D/\langle z\rangle = D_1$ be the canonical epimorphism. Then, by trying some random short words, we get that a subgroup $\langle \phi_1(y_1)^4, (\phi_1(x_1)\phi_1(y_1)\phi_1(v_1))^2\rangle \cong \mathcal{M}_{22}$ works as a permutation stabilizer for the group D_1, and thus the Magma command

```
CosetAction(D_1,<phi_1(y_1)^4,(phi_1(x_1) phi_1(y_1) phi_1(v_1))^2>
```

gives the faithful permutation representation of D_1 of degree 1024. We can now use this faithful permutation representation for all MAGMA computations in this theorem. From Lemma 3.4 of [9] we already have a presentation and faithful permutation representation of $H_1 = H_2/\langle k_{15}\rangle$ of degree 1024. Then, using the permutation representations, it is verified with MAGMA commands `IsIsomorphic(D_1,E_n)` and `IsIsomorphic(H_1,H_n)` that $D_1 \cong E_n$ and $H_1 \cong H_n$.

(b) Let $\phi_1 : D \to D/\langle z_1\rangle = D_1$ and $\phi_2 : H_2 \to H_2/\langle k_{15}\rangle = H_1$ be the canonical epimorphisms. As proved in (a), there exist isomorphisms $\varphi_1 : D_1 \to E_n$ and $\varphi_0 : H_1 \to H_n$. Next step is to find some inner automorphism $\delta_0 \in \mathrm{Aut}(G_n)$ of G_n such that the new isomorphism $\varphi_2 = \delta_0 \circ \varphi_0 : H_1 \to \delta_0(H_n)$ satisfies $(\varphi_1 \circ \phi_1)(p_1) = (\varphi_2 \circ \phi_2)(p_2)$ and $(\varphi_1 \circ \phi_1)(q_1) = (\varphi_2 \circ \phi_2)(q_2)$. Let φ_1 and φ_0 be fixed. For convenience, let $p_3 = (\varphi_1 \circ \phi_1)(p_1)$, $q_3 = (\varphi_1 \circ \phi_1)(q_1)$, $p'_3 = (\varphi_0 \circ \phi_2)(p_2)$, and $q'_3 = (\varphi_0 \circ \phi_2)(q_2)$.

By means of MAGMA command `IsConjugate(G_n,p'_3,p_3)`, we obtain an element $r_1 \in G_n$ such that $r_1^{-1}p'_3r_1 = p_3$. Fix this r_1. By means of MAGMA, it is checked that $C_{G_n}(p'_3)$ has order 3072, which is relatively small. Then it is verified by the MAGMA command `exists` that there is some element r_3 of $C_{G_n}(p'_3)$ such that $(r_3r_1)^{-1}q'_3(r_3r_1) = q_3$.

Thus we also have $(r_3r_1)^{-1}p'_3(r_3r_1) = r_1^{-1}(r_3^{-1}p'_3r_3)r_1 = r_1^{-1}p'_3r_1 = p_3$. Therefore, letting $r_2 = r_3r_1$ yields $r_2^{-1}p'_3r_2 = p_3$ and $r_2^{-1}q'_3r_2 = q_3$. Now, let $\delta_0 \in \mathrm{Aut}(G_n)$ be the inner automorphism defined as conjugation by r_2. Then δ_0 restricted to H_n is an isomorphism $\delta_{0|H_n} : H_n \to (H_n)^{r_2}$. Thus, $\varphi_2 : H_1 \to (H_n)^{r_2}$ defined by $\varphi_2 = \delta_{0|H_n} \circ \varphi_0$ is an isomorphism, and we can observe that

$$(\varphi_2 \circ \phi_2)(p_2) = (\delta_{0|H_n} \circ \varphi_0 \circ \phi_2)(p_2) = \delta_{0|H_n}(p'_3) = r_2^{-1}p'_3r_2 = p_3, \quad \text{and}$$
$$(\varphi_2 \circ \phi_2)(q_2) = (\delta_{0|H_n} \circ \varphi_0 \circ \phi_2)(q_2) = \delta_{0|H_n}(q'_3) = r_2^{-1}q'_3r_2 = q_3,$$

and therefore $(\varphi_1\circ\phi_1)(p_1) = p_3 = (\varphi_2\circ\phi_2)(p_2)$ and $(\varphi_1\circ\phi_1)(q_1) = q_3 = (\varphi_2\circ\phi_2)(q_2)$, in short. It is checked by means of MAGMA that $E_n \cap (H_n)^{r_2} = \langle p_3, q_3\rangle$, and that $\langle p_3, q_3\rangle$ is of order $2^{17} \cdot 3 \cdot 5$.

(c) Let $\phi_1, \phi_2, \varphi_1, \varphi_2$, and r_2 be as obtained in (b), so that the isomorphisms $\varphi_1 : D_1 \to E_n$ and $\varphi_2 : H_1 \to (H_n)^{r_2}$ satisfy $(\varphi_1 \circ \phi_1)(p_1) = p_3 = (\varphi_2 \circ \phi_2)(p_2)$ and $(\varphi_1 \circ \phi_1)(q_1) = q_3 = (\varphi_2 \circ \phi_2)(q_2)$ and $E_n \cap (H_n)^{r_2} = \langle p_3, q_3\rangle$. Now, let $x_3 = (\varphi_1 \circ \phi_1)(x_1)$, $y_3 = (\varphi_1 \circ \phi_1)(y_1)$, and $h_3 = (\varphi_2 \circ \phi_2)(h_2)$. Then x_3, y_3 and h_3 can be expressed as words in $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n$,and i_n. Thus there are three words by β_1, β_2, and β_3, such that

$$x_3 = \beta_1(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n),$$
$$y_3 = \beta_2(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n),$$
$$h_3 = \beta_3(a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n).$$

The three words β_1, β_2, and β_3 are obtained by Strategy 1.22, as follows.

It is checked by means of MAGMA that $C_{E_n}(x_3)$ of x_3 in E_n has order $2^{13} \cdot 3 = 24576$, which is relatively small. Applying Strategy 1.22 to this group $C_{E_n}(x_3)$, the author found a word for x_3; the results are $C_{E_n}(x_3) = \langle r_{n,1}, r_{n,2}, r_{n,3}, r_{n,4}\rangle$ and $x_3 = (r_{n,1}r_{n,2}r_{n,3})^3$.

Note that y_3 is of order 14 (since y_1 is). By means of MAGMA, it can be checked that the centralizer $C_{E_n}(y_3^7)$ of the involution y_3^7 in E_n has order $2^{16} \cdot 3^2 \cdot 5 \cdot 7$. Applying Strategy 1.22 to this group $C_{E_n}(y_3^7)$, the author found a word for y_3^7; the results are $C_{E_n}(y_3^7) = \langle s_{n,1}, s_{n,2}, s_{n,3}, s_{n,4}\rangle$ and $y_3^7 = (s_{n,1}s_{n,2}s_{n,4}s_{n,1}s_{n,4}s_{n,2}s_{n,4})^5$.

Now, by means of the MAGMA command

```
Subgroups(sub<E_n|s_{n,1},s_{n,2},s_{n,3},s_{n,4}> :Al:=Normal)
```

it can be checked that $C_{E_n}(y_3^7) = \langle E_n \mid s_{n,1}, s_{n,2}, s_{n,3}, s_{n,4}\rangle$ has a unique normal subgroup V_n of order 2^{10}. By means of the MAGMA command

```
HasComplement(C_{E_n}(y_3^7),V_n)
```

we can get a complement C_n of V_n in $C_{E_n}(y_3^7)$. Now, by means of the MAGMA command

```
exists(r_n){r_n:r_n in C_{E_n}(y_3^7)| y_3^2 in (C_n)^{r_2}}
```

we obtain some element $r_n \in C_{E_n}(y_3^7)$ such that $y_3^2 \in (C_n)^{r_2}$. It is also checked be means of MAGMA that $(C_n)^{r_2}$ has order $2^6 \cdot 3^2 \cdot 5 \cdot 7 = 20160$. Now, for such C_n and r_2, the author applied Strategy 1.22 for $(C_n)^{r_2}$ and found a word for y_3^2; the results are $(C_n)^{r_2} = \langle t_{n,1}, t_{n,2}, t_{n,3}\rangle$ and $y_3^2 = (t_{n,1}t_{n,3}^2t_{n,1}t_{n,3}t_{n,1}t_{n,2}t_{n,1}t_{n,3})^5$. Finally, since y_3 is of order 14, we get that

$$
\begin{aligned}
y_3 &= y_3^{15} = (y_3^7)(y_3^2)^4\\
&= (s_{n,1}s_{n,2}s_{n,4}s_{n,1}s_{n,4}s_{n,2}s_{n,4})^5(t_{n,1}t_{n,3}^2t_{n,1}t_{n,3}t_{n,1}t_{n,2}t_{n,1}t_{n,3})^{20},
\end{aligned}
$$

so we found a word β_2.

The application of Strategy 1.22 was not so immediate. The command `GetShortGens` for $(H_n)^{r_2}$ was stopped in the middle by the author, until it gave three elements j_n, k_n, and l_n of $(H_n)^{r_2}$. Now, another application of `GetShortGens` using j_n, k_n, l_n and the original generators for G_n yielded $(H_n)^{r_2} = \langle j_n, k_n, o_n, p_n\rangle$. Note that h_3 is of order 3 (since h_2 is). By means of MAGMA, it can be checked that the normalizer $N_{(H_n)^{r_2}}(\langle h_3\rangle)$ of h_3 in $(H_n)^{r_2}$ has order $2^8 \cdot 3^4 = 20736$, which is relatively small. Using Strategy 1.22 for $N_{(H_n)^{r_2}}(\langle h_3\rangle)$ in $(H_n)^{r_2}$ yielded $N_{(H_n)^{r_2}}(\langle h_3\rangle) = \langle v_{n,1}, v_{n,2}, v_{n,3}\rangle$ and finally $h_3 = (v_{n,1}v_{n,2})^4$.

(d) This can easily be checked by means of MAGMA.

(e) The 9 words can be obtained by the command `LookupWord`.

(f) The objective of this theorem is to verify that the matrix group $\mathfrak{H}$ is isomorphic to $2\,\mathrm{Fi}_{22}$, and also get a nice presentation for it. Thus $\mathfrak{H}$ is expected to have Fi_{22} as a quotient. In (c) and (e) we had $G_n = \langle a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n\rangle = \langle x_3, y_3, h_3\rangle \cong \mathrm{Fi}_{22}$, with words for x_3, y_3, h_3 in terms of $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n$, i_n, and vice versa (words for $a_n, b_n, \ldots, i_n$ in terms of x_3, y_3, h_3). There, the two quotient groups $D_1 = D/\langle z_1\rangle$ and $H_1 = H_2/\langle k_{15}\rangle$ are embedded in $G_n \cong \mathrm{Fi}_{22}$ as $\langle p_3, q_3, x_3, y_3\rangle = \langle x_3, y_3\rangle$ and $\langle p_3, q_3, h_3\rangle$, respectively. Recall that D and H_2 are embedded in the matrix group $\mathfrak{H}$ as $\langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{x}_2, \mathfrak{y}_2\rangle$ and $\langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{h}_2\rangle$, respectively.

Let

$$
\begin{aligned}
\mathfrak{a}_n &= \omega_a(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2), &\quad \mathfrak{b}_n &= \omega_b(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2), &\quad \mathfrak{c}_n &= \omega_c(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2),\\
\mathfrak{d}_n &= \omega_d(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2), &\quad \mathfrak{e}_n &= \omega_e(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2), &\quad \mathfrak{f}_n &= \omega_f(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2),\\
\mathfrak{g}_n &= \omega_g(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2), &\quad \mathfrak{h}_n &= \omega_h(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2), &\quad \mathfrak{i}_n &= \omega_i(\mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2),
\end{aligned}
$$

where the 9 words $\omega_a, \omega_b, \omega_c, \ldots, \omega_i$ are the ones obtained in (e). Recall from Lemma 4.4 that the generators $a_n, b_n, c_n, d_n, e_n, f_n, g_n, h_n$, and i_n of G_n satisfy the set $\mathcal{R}(G_n)$ of defining relations. Now, let's check if these nine matrices $\mathfrak{a}_n$, $\mathfrak{b}_n$, $\mathfrak{c}_n$, $\mathfrak{d}_n$, $\mathfrak{e}_n$, $\mathfrak{f}_n$, $\mathfrak{g}_n$, $\mathfrak{h}_n$, $\mathfrak{i}_n$ also satisfy the set of relations $\mathcal{R}(G_n)$. For example, we had $(b_nc_nd_ne_nf_ng_nh_n)^9 = 1$ as the very last relation in $\mathcal{R}(G_n)$, and thus we check if $(\mathfrak{b}_n\mathfrak{c}_n\mathfrak{d}_n\mathfrak{e}_n\mathfrak{f}_n\mathfrak{g}_n\mathfrak{h}_n)^9$ is the identity matrix in $\mathrm{GL}_{352}(17)$. By means of

MAGMA, it is easy to check if each of these relations is satisfied by the nine matrices $\mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n$.

It turned out that all relations are satisfied, except for the three relations $(b_n c_n)^3$, $(e_n f_n)^3$, and $(f_n g_n)^3$. By means of MAGMA, it is easy to check that $(\mathfrak{b}_n\mathfrak{c}_n)^3 = (\mathfrak{e}_n\mathfrak{f}_n)^3 = (\mathfrak{f}_n\mathfrak{g}_n)^3$. So define a matrix $\mathfrak{z}_n = (\mathfrak{b}_n\mathfrak{c}_n)^3$. Then, we can check that $\mathfrak{z}_n$ has order 2, and commutes with all three matrices $\mathfrak{x}_2$, $\mathfrak{y}_2$, and $\mathfrak{h}_2$, and therefore also with the nine matrices $\mathfrak{a}_n, \mathfrak{b}_n, \dots, \mathfrak{i}_n$; thus, $\mathfrak{z}_n$ is a central involution of $\mathfrak{H}$. Now, match the nine matrices $\mathfrak{a}_n, \mathfrak{b}_n, \dots, \mathfrak{i}_n$ with nine new abstract variables $a_\ell, b_\ell, \dots, i_\ell$, respectively, and $\mathfrak{z}_n$ with another new variable z_ℓ. Then, the ten matrices $\mathfrak{a}_n$, $\mathfrak{b}_n$, ..., $\mathfrak{i}_n$, $\mathfrak{z}_n$ satisfy the set $\mathcal{R}(G_\ell)$ of relations, as written in the statement.

Now, let's verify that the finitely presented group $G_\ell = \langle a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell, z_\ell\rangle$ having $\mathcal{R}(G_\ell)$ as its defining relations is isomorphic to $2\,\mathrm{Fi}_{22}$. We can observe in the defining relations $\mathcal{R}(G_\ell)$ that z_ℓ is in the center of G_ℓ, and that z_ℓ is of order 1 or 2, since $z_\ell^2 = 1$. It is then also easy to observe that $G_\ell/\langle z_\ell\rangle$ is isomorphic to G_n, having exactly same relations. Therefore, since we have $G_n \cong \mathrm{Fi}_{22}$, we now know $G_\ell/\langle z_\ell\rangle \cong \mathrm{Fi}_{22}$. Hence, if $Order(z_\ell) = 2$ implies $G_\ell \cong 2\,\mathrm{Fi}_{22}$, and $Order(z_\ell) = 1$ implies $G_\ell \cong \mathrm{Fi}_{22}$. So, if we prove that G_ℓ has a permutation representation of order $= |2\,\mathrm{Fi}_{22}|$, then we can deduce that G_ℓ is indeed isomorphic to $2\,\mathrm{Fi}_{22}$, and that this permutation representation is faithful.

As in Lemma 4.4(b), G_n has a subgroup $V_n = \langle b_n, c_n, d_n, e_n, f_n, g_n, h_n, i_n\rangle$ which is isomorphic to the simple group $O(7,3)$, and that G_n has a faithful permutation representation $(PG_n)'$ of degree 14080 with permutation stabilizer V_n.

The strategy is to lift V_n to V_ℓ inside G_ℓ by $V_\ell = \langle b_\ell\xi_1,\ c_\ell\xi_2,\ d_\ell\xi_3,\ e_\ell\xi_4,\ f_\ell\xi_5,\ g_\ell\xi_6,\ h_\ell\xi_7,\ i_\ell\xi_8\rangle$ so that $V_\ell \cong O(7,3)$, by picking suitable $\xi_1, \xi_2, \dots, \xi_8$, where each of $\xi_1, \xi_2, \dots, \xi_8$ is either 1 or z_ℓ. Since each of $\xi_1, \xi_2, \dots, \xi_8$ is either 1 or z_ℓ, there are $2^8 = 256$ cases to check. For each choice, the author let $V_\ell = \langle b_\ell\xi_1, c_\ell\xi_2, d_\ell\xi_3, e_\ell\xi_4, f_\ell\xi_5, g_\ell\xi_6, h_\ell\xi_7, i_\ell\xi_8\rangle$, and ran the MAGMA command `CosetAction(G_l, V_l)`. Among the 256 cases, only one case returned a permutation representation of degree 28160, while all others returned that of degree 14080. That single case, namely $\xi_1 = z_\ell, \xi_2 = \xi_3 = \xi_4 = 1, \xi_5 = z_\ell, \xi_6 = \xi_7 = \xi_8 = 1$, is exactly the lifting V_ℓ of V_n which we want.

To summarize, let $V_\ell = \langle b_\ell z_\ell, c_\ell, d_\ell, e_\ell, f_\ell z_\ell, g_\ell, h_\ell, i_\ell\rangle$. Then, the MAGMA command `CosetAction(G_l, V_l)` gives a permutation representation PG_ℓ of degree 28160, which is of order $2^{18}\cdot 3^9\cdot 5^2\cdot 7\cdot 11\cdot 13 = |2\,\mathrm{Fi}_{22}|$. Thus, as asserted, we have $G_\ell \cong 2\,\mathrm{Fi}_{22}$, and this permutation representation PG_ℓ of G_ℓ is faithful.

(g) Let

$$x_0 = \beta_1(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell),$$
$$y_0 = \beta_2(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell),$$
$$h_0 = \beta_3(a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell),$$

where the three words β_1, β_2, and β_3 are the ones we obtained in the statement (b). Then $G_\ell = \langle x_0, y_0, h_0\rangle$ can be checked by means of MAGMA, using the faithful permutation representation PG_ℓ obtained in (f).

(h) Let the three words β_1, β_2, and β_3 be the ones we obtained in the statement (b). Then, it can be checked by means of MAGMA that

$$\mathfrak{x}_2 = \beta_1(\mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n), \quad \mathfrak{y}_2 = \beta_2(\mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n),$$
$$\mathfrak{h}_2 = \beta_3(\mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n),$$

where the nine matrices $\mathfrak{a}_n$, $\mathfrak{b}_n$, $\mathfrak{c}_n$, $\mathfrak{d}_n$, $\mathfrak{e}_n$, $\mathfrak{f}_n$, $\mathfrak{g}_n$, $\mathfrak{h}_n$, and $\mathfrak{i}_n$ are the ones we obtained in the proof of (f) (as words in $\mathfrak{x}_3, \mathfrak{y}_3, \mathfrak{h}_3$). Therefore $\mathfrak{x}_2, \mathfrak{y}_2$, and $\mathfrak{h}_2$ are contained in $\langle \mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n \rangle$. Hence, $\langle \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle = \mathfrak{H}$ is contained in $\langle \mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n \rangle$. It is clear that $\langle \mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n \rangle$ is contained in $\langle \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle = \mathfrak{H}$, since the nine matrices $\mathfrak{a}_n, \mathfrak{b}_n, \ldots, \mathfrak{h}_n$ are words in $\mathfrak{x}_3, \mathfrak{y}_3, \mathfrak{h}_3$. Thus $\mathfrak{H} = \langle \mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n \rangle$. Recall that $\mathfrak{z}_n = (\mathfrak{b}_n \mathfrak{c}_n)^3$. Now we have

$$\mathfrak{H} = \langle \mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n, \mathfrak{z}_n \rangle.$$

Observe that these generating ten matrices $\mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n, \mathfrak{i}_n, \mathfrak{z}_n$ satisfy the relations $R(G_\ell)$. Therefore, the matrix group $\mathfrak{H}$ which is generated by these ten matrices is isomorphic to some quotient group of the finitely presented group G_ℓ which has $R(G_\ell)$ as its defining relations. We proved in (f) that $G_\ell \cong 2\,\mathrm{Fi}_{22}$, and therefore $\mathfrak{H}$ is isomorphic to some quotient group of $2\,\mathrm{Fi}_{22}$. Therefore, $\mathfrak{H}$ is isomorphic to 1, Fi_{22}, or $2\,\mathrm{Fi}_{22}$. Notice that $\mathfrak{z}_n$ is an involution (hence not an identity element) which in the center of this matrix group $\mathfrak{H}$. Among the three choices 1, Fi_{22}, and $2\,\mathrm{Fi}_{22}$, the only group with nontrivial center is $2\,\mathrm{Fi}_{22}$. Therefore $\mathfrak{H} \cong 2\,\mathrm{Fi}_{22}$.

Hence, the natural homomorphism $\vartheta : \mathfrak{H} \to G_\ell$ given by $\vartheta(\mathfrak{a}_n) = a_\ell$, $\vartheta(\mathfrak{b}_n) = b_\ell$, $\vartheta(\mathfrak{c}_n) = c_\ell$, $\vartheta(\mathfrak{d}_n) = d_\ell$,$\vartheta(\mathfrak{e}_n) = e_\ell$, $\vartheta(\mathfrak{f}_n) = f_\ell$, $\vartheta(\mathfrak{g}_n) = g_\ell$,$\vartheta(\mathfrak{h}_n) = h_\ell$, $\vartheta(\mathfrak{i}_n) = i_\ell$, and $\vartheta(\mathfrak{z}_n) = z_\ell$ is an isomorphism. Note that x_0, y_0, and h_0 are expressed by the three words β_1, β_2, and β_3 in terms of $a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell$, and i_ℓ, and also that $\mathfrak{x}_2, \mathfrak{y}_2$, and $\mathfrak{h}_2$ are expressed by the three words β_1, β_2, and β_3 in terms of $\mathfrak{a}_n, \mathfrak{b}_n, \mathfrak{c}_n, \mathfrak{d}_n, \mathfrak{e}_n, \mathfrak{f}_n, \mathfrak{g}_n, \mathfrak{h}_n$, and $\mathfrak{i}_n$. Since ϑ is an isomorphism, it preserves the multiplication of elements, and therefore we have $\vartheta(\mathfrak{x}_2) = x_0, \vartheta(\mathfrak{y}_2) = y_0$, and $\vartheta(\mathfrak{h}_2) = h_0$.

The irreducibility of $\mathfrak{H}$ is checked by means of the Magma command `IsIrreducible`.

□

Corollary 4.6. *Keep the notations in Theorem 4.5. Let* $K = \mathrm{GF}(17)$*. Let* $H := G_\ell = \langle x_0, y_0, h_0 \rangle$*, and* $D_H := \langle x_0, y_0 \rangle$*. Then, H and D_H have orders* $2^{18} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$ *and* $2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$*, respectively. Then the following statements hold:*

(a) *Each Sylow 2-subgroup S of H has a unique maximal elementary abelian normal subgroup A of order 2^{11}. Also, $N_H(A) \cong D = C_E(z)$.*
(b) *There is a Sylow 2-subgroup S such that its maximal elementary abelian normal subgroup A of order 2^{11} satisfies $N_H(A) = \langle x_0, y_0 \rangle = D_H$.*
(c) *There is an isomorphism $\phi_D : D_H \to D$ such that $\phi_D(x_0) = x_1$ and $\phi_D(y_0) = y_1$.*
(d) *A system of representatives w_i of the 114 conjugacy classes of H and the corresponding centralizer orders $|C_H(w_i)|$ are given in Table A.5*
(e) *The character table of H is given in Table B.3.*

Proof. By Theorem 4.5, the finitely presented group $G_\ell = \langle a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell, z_\ell \rangle$ has a faithful permutation representation PG_ℓ of degree 28160, with stabilizer $\langle b_\ell z_\ell, c_\ell, d_\ell, e_\ell, f_\ell z_\ell, g_\ell, h_\ell, i_\ell \rangle$. We also have that x_0, y_0, and h_0 are expressed as words in terms of $a_\ell, b_\ell, c_\ell, d_\ell, e_\ell, f_\ell, g_\ell, h_\ell, i_\ell$, and that $G_\ell = \langle x_0, y_0, h_0 \rangle$. Therefore, it makes sense to say $H := G_\ell = \langle x_0, y_0, h_0 \rangle$, and computations are also possible in G_ℓ, by means of the faithful permutation representation of G_ℓ, and the corresponding permutations for x_0, y_0, and h_0. The orders of H and $D_H := \langle x_0, y_0 \rangle$ are checked with Magma.

(a) By means of the MAGMA commands `S:=SylowSubgroup(H, 2)` and

`Subgroups(S:Al:=''Normal'',IsElementaryAbelian:=true)`,

we obtain a Sylow 2-subgroup S of H, and the unique maximal elementary abelian subgroup A of S of order 2^{11}. By the MAGMA command `IsIsomorphic`, we can also verify that $N_H(A) \cong D$.

(b) Let S be any Sylow 2-subgroup of D_H, and let A be as in (a). Then, it can be checked by the MAGMA that $N_H(A) = D_H$.

(c) Recall that the matrix group $\langle \mathfrak{x}_2, \mathfrak{y}_2 \rangle$ is a faithful representation of $D = \langle x_1, y_1 \rangle$ (by $x_1 \mapsto \mathfrak{x}_2$ and $y_1 \mapsto \mathfrak{y}_2$), since so is $\langle \mathfrak{p}_2, \mathfrak{q}_2, \mathfrak{x}_2, \mathfrak{y}_2 \rangle$. We showed that $\mathfrak{H} = \langle \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle$ is a faithful representation of $H = \langle x_0, y_0, h_0 \rangle$ (by $x_0 \mapsto \mathfrak{x}_2$, $y_0 \mapsto \mathfrak{y}_2$, and $h_0 \mapsto \mathfrak{h}_2$). Therefore, $\langle \mathfrak{x}_2, \mathfrak{y}_2 \rangle$ is a faithful representation of $\langle x_0, y_0 \rangle = D_H$ (by $x_0 \mapsto \mathfrak{x}_2$ and $y_0 \mapsto \mathfrak{y}_2$). Therefore there is an isomorphism ϕ_D from D_H to D such that $\phi_D(x_0) = x_1$ and $\phi_D(y_0) = y_1$.

(d) and (e) Checked by means of MAGMA and Kratzer's Algorithm 5.3.18 of [12].

□

5. Construction of Fischer's simple group Fi_{23}

By Lemma 3.2 and Corollary 4.6 the amalgam $H \leftarrow D \rightarrow E$ constructed in sections 3 and 4 satisfies the main condition of G. Michler's Algorithm 7.4.8 of [12]. Therefore we can apply the Algorithm 7.4.8 of [12] to give here a new existence proof for Fischer's simple group Fi_{23}.

The readers should be aware of the abusive notation used in the following theorem. The symbols $\chi, \tau, \chi_i, \tau_i, \mathfrak{V}, \mathfrak{W}, \mathfrak{V}_i, \mathfrak{W}_i, U_i, S_i, \mathcal{T}$ appearing in the following theorem have nothing to do with those of same notations appearing in Theorem 4.3.

Theorem 5.1. *Keep the notations in Lemma 3.2 and Corollary 4.6. Let* $K = \mathrm{GF}(17)$. *Using the notations of the character tables of B.3, B.2, and B.1 of* H, D, E, *the following statements hold:*

(a) *The smallest degree of a nontrivial pair* $(\chi, \tau) \in mf\,char_{\mathbb{C}}(H) \times mf\,char_{\mathbb{C}}(E)$ *of compatible characters which divides the group order of* Fi_{23} *is* 782.

(b) *There is exactly one compatible pair* $(\chi, \tau) \in mf\,char_{\mathbb{C}}(H) \times mf\,char_{\mathbb{C}}(E)$ *of degree* 782 *of the groups* $H = \langle D_H, h\rangle$ *and* $E = \langle D, e\rangle$:

$$(\chi, \tau) = (\chi_1 + \chi_{\mathbf{3}} + \chi_4, \tau_1 + \tau_2 + \tau_{\mathbf{10}} + \tau_{\mathbf{11}})$$

with common restriction

$$\chi_{|D_H} = \tau_{|D} = \psi_1 + \psi_1 + \psi_2 + \psi_6 + \psi_{\mathbf{9}} + \psi_{\mathbf{10}} + \psi_{15},$$

where irreducible characters with bold face indices denote faithful irreducible characters.

(c) *Let* $\mathfrak{V}$ *and* $\mathfrak{W}$ *be the up-to-isomorphism uniquely determined faithful semi-simple multiplicity-free* 782*-dimensional modules of* H *and* E *over* $F = \mathrm{GF}(17)$ *corresponding to the compatible pair* χ, τ, *respectively.*

Let $\kappa_{\mathfrak{V}} : H \rightarrow \mathrm{GL}_{782}(17)$ *and* $\kappa_{\mathfrak{W}} : E \rightarrow \mathrm{GL}_{782}(17)$ *be the representations of* H *and* E *afforded by the modules* $\mathfrak{V}$ *and* $\mathfrak{W}$, *respectively.*

Let $\mathfrak{x} = \kappa_{\mathfrak{V}}(x_0)$, $\mathfrak{y} = \kappa_{\mathfrak{V}}(y_0)$, $\mathfrak{h} = \kappa_{\mathfrak{V}}(h_0)$ *in* $\kappa_{\mathfrak{V}}(H) \leq \mathrm{GL}_{782}(17)$. *Then the following assertions hold:*

$\mathfrak{V}_{|D_H} \cong \mathfrak{W}_{|D}$, *and there is a transformation matrix* $\mathcal{T} \in \mathrm{GL}_{782}(17)$ *such that*

$$\mathfrak{x} = \mathcal{T}^{-1}\kappa_{\mathfrak{W}}(x)\mathcal{T}, \quad \mathfrak{y} = \mathcal{T}^{-1}\kappa_{\mathfrak{W}}(y)\mathcal{T}.$$

(d) *Let* $Y = \mathrm{GL}_{782}(17)$, $\mathfrak{D} = \langle \mathfrak{x}, \mathfrak{y}\rangle$, $\mathfrak{H} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h}\rangle$. *Let* $\mathcal{D} = C_Y(\mathfrak{D})$ *and* $\mathcal{H} = C_Y(\mathfrak{H})$. *Let* $\mathfrak{e}_1 = \mathcal{T}^{-1}\kappa_{\mathfrak{W}}(e)\mathcal{T}$. *Let* $\mathfrak{E} = \langle \mathfrak{D}, \mathfrak{e}_1\rangle$ *and* $\mathcal{E} = C_Y(\mathfrak{E})$. *Then the following statements hold:*

(1) *There is an isomorphism*

$$\gamma : \mathcal{D} \rightarrow \mathcal{D}_1 = \mathrm{GL}_2(17) \times K^{*5} \leq \mathrm{GL}_7(17).$$

(2) $\mathcal{H}_1 = \gamma(\mathcal{H})$ *is generated by the three diagonal matrices* $a_1 = diag(3, 1, 1, 1, 1, 1, 1)$, $a_2 = diag(1, 1, 1, 1, 3, 3, 1)$ *and* $a_3 = diag(1, 3, 3, 3, 1, 1, 3)$.

(3) $\mathcal{E}_1 = \gamma(\mathcal{E})$ *is generated by the four diagonal matrices* $b_1 = a_1$, $b_2 = diag(1, 3, 3, 1, 1, 1, 1)$, $b_3 = diag(1, 1, 1, 3, 3, 1, 1)$ *and* $b_4 = diag(1, 1, 1, 1, 1, 3, 3)$.

(4) $\mathcal{D}$ *has* 321×16 $\mathcal{H}$-$\mathcal{E}$ *double cosets.*

(5) *The free product* $H *_D E$ *of* H *and* E *with amalgamated subgroup* D *has exactly one irreducible* 782*-dimensional representation over* K

whose Sylow 2-subgroups have the same exponent as the ones of H. It corresponds to the $\mathcal{H}$-$\mathcal{E}$ double coset representative

$$\mathcal{F} = diag(u, 1^{21}, 1^{77}, 1^{176}, 1^{176}, 16^{330}) \in \mathrm{GL}_{782}(17) \quad where$$

$$u = \begin{pmatrix} 1 & 1 \\ 9 & 14 \end{pmatrix}.$$

Let $\mathfrak{e} = \mathcal{F}^{-1}\mathfrak{e}_1\mathcal{F}$ and $\mathfrak{G} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h}, \mathfrak{e} \rangle$. The proof of this Sylow 2-subgroup test for $\mathfrak{G}$ is split into two parts. The first half is the order test for the elements $\mathfrak{he}$ and $\mathfrak{yeh}$; these two matrices have to have orders of elements in Fi_{23} (it turned out that $\mathfrak{he}$ has order 12, and $\mathfrak{yeh}$ has order 13). The second half of the proof is done in (e) of this theorem.

(6) *The four generating matrices of $\mathfrak{G}$ are documented in* [10].

(e) *$\mathfrak{G}$ has a faithful permutation representation $P\mathfrak{G}$ of degree 31671 with stabilizer $\mathfrak{H} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h} \rangle$.*

(f) *$\mathfrak{G}$ is a finite simple group of order $2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$ with centralizer $C_{\mathfrak{G}}(\mathfrak{z}) = \mathfrak{H} \cong H$ of the involution $\mathfrak{z} = (\mathfrak{x}\mathfrak{y}^2)^7$.*

(g) *$\mathfrak{G}$ has 98 conjugacy classes ${\mathfrak{g}_i}^{\mathfrak{G}}$ with representatives $\mathfrak{g}_i$ and centralizer orders $|C_{\mathfrak{G}}(\mathfrak{g}_i)|$ as given in Table A.6.*

(h) *The character table of $\mathfrak{G}$ coincides with that of Fi_{23} in the Atlas [4], its p. 178-179.*

Proof. (a) The character tables of the groups H, D, and E are stated in the Appendix B.3, B.2, and B.1. In the following we use their notations. Using Magma and the character tables of H, D, and E and the fusion of the classes of $D_H \cong D$ in H and D in E, an application of Kratzer's Algorithm 7.3.10 of [12] yields the compatible pair stated in assertion (a), dividing the group order of Fi_{23}; the group order of Fi_{23} is taken from Atlas [4].

(b) The application of Kratzer's Algorithm 7.3.10 of [12] also shows that the pairs (χ, τ) of (b) is the only compatible pair of degree 782 with respect to the fusion of the D-classes into the H- and into the E-classes.

(c) In order to construct the faithful irreducible representation $\mathfrak{V}$ corresponding to the character $\chi = \chi_1 + \chi_3 + \chi_4$ of degree 782, the author employed the Magma command `LowIndexSubgroups(PH, 4000)` using the faithful permutation representation PH of H of degree 28160 (see statements (f) and (g) of Theorem 4.5 for construction of PH). Magma found a subgroup U_1 of H such that the followings hold.

U_1 is of index 3510 in H, and χ_4 (dimension 429) is a constituent of the permutation character $(1_{U_1})^H$. By `GetShortGens` the author obtained $U_1 = \langle (y_0)^7$, $(y_0h_0y_0^2h_0)^7$, $(y_0h_0y_0h_0y_0)^7$, $(h_0y_0^2h_0y_0)^7$, $(h_0y_0h_0y_0^2)^7$, $(y_0^3h_0^2y_0^2)^{11} \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_U)^H$ the author obtained the irreducible KH-module $\mathfrak{V}_4$ corresponding to χ_4.

Note that χ_1 is the trivial character. Notice also that χ_3 is the unique irreducible character of H of degree 352, and that we already have an irreducible representation of H of degree 352, namely $\mathfrak{H} = \langle \mathfrak{x}_2, \mathfrak{y}_2, \mathfrak{h}_2 \rangle$. Thus, we can use $\mathfrak{H}$ as a representation of H corresponding to χ_3.

In order to construct the faithful irreducible representation $\mathfrak{W}$ corresponding to the character $\tau = \tau_1 + \tau_2 + \tau_{10} + \tau_{11}$ of degree 782, the author employed the Magma command `LowIndexSubgroups(PE, 1500)` using the faithful permutation presentation PE of E of degree 1012 (see Lemma 3.2 for construction of PE). MAGMA found subgroups S_1, S_2, and S_3 such that the followings hold.

S_1 is of index 506 in E, and τ_2 (dimension 22) and τ_{10} (dimension 253) are constituents of the permutation character $(1_{S_1})^E$. By `GetShortGens` we get $S_1 = \langle (y_1^3 e_1 y_1)^3, (y_1^2 x_1 y_1 x_1 y_1)^3, (y_1^4 e_1 x_1)^4, (y_1 e_1 x_1 y_1^3)^4 \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{S_1})^e$ the author obtained the irreducible KE-modules $\mathfrak{W}_2$ and $\mathfrak{W}_{10}$ corresponding to τ_2 and τ_{10}, respectively.

S_2 is of index 1012 in E, and τ_{11} (dimension 506) is a constituent of the permutation character $(1_{S_2})^E$. By `GetShortGens` we get $S_2 = \langle (x_1 e_1 y_1)^4, (x_1 y_1^3 x_1 e_1)^7, (y_1^2 x_1 y_1 x_1 y_1)^2 \rangle$. By applying the Meat-axe Algorithm to the permutation module $(1_{S_2})^E$ the author obtained the irreducible KE-module $\mathfrak{W}_{11}$ corresponding to τ_{11}.

Therefore we can obtain the representations $\kappa_{\mathfrak{V}} : H \to \mathrm{GL}_{782}(17)$ and $\kappa_{\mathfrak{W}} : E \to \mathrm{GL}_{782}(17)$ of H and E afforded by the modules $\mathfrak{V}$ and $\mathfrak{W}$, respectively, as follows. As before, $\mathfrak{V} = \mathfrak{V}_1 \oplus \mathfrak{V}_3 \oplus \mathfrak{V}_4$, where $\mathfrak{V}_1$ is the trivial KH-module, and $\mathfrak{V}_3$ corresponds to the representation $\kappa_{\mathfrak{V}_3} : H \to \mathrm{GL}_{352}(17)$ given by $\kappa_{\mathfrak{V}_3}(x_0) = \mathfrak{x}_2$, $\kappa_{\mathfrak{V}_3}(y_0) = \mathfrak{y}_2$, and $\kappa_{\mathfrak{V}_3}(h_0) = \mathfrak{h}_2$. Then, the matrices for $\kappa_{\mathfrak{V}}$ can be obtained by diagonal joining, for example,

$$\kappa_{\mathfrak{V}}(x_0) = \mathrm{diag}(\kappa_{\mathfrak{V}_1}(x_0), \kappa_{\mathfrak{V}_3}(x_0), \kappa_{\mathfrak{V}_4}(x_0)),$$

where $\kappa_{\mathfrak{V}_1} : H \to \mathrm{GL}_1(17)$ and $\kappa_{\mathfrak{V}_4} : H \to \mathrm{GL}_{429}(17)$ are the representations of H afforded by the KH-modules $\mathfrak{V}_1$ and $\mathfrak{V}_4$, respectively. And similarly for y_0 and h_0, too. For E side, let $\mathfrak{W} = \mathfrak{W}_1 \oplus \mathfrak{W}_2 \oplus \mathfrak{W}_{10} \oplus \mathfrak{W}_{11}$, where $\mathfrak{W}_1$ is the trivial KE-module. Then, the matrices for $\kappa_{\mathfrak{W}}$ can be obtained by diagonal joining, for example,

$$\kappa_{\mathfrak{W}}(x_1) = \mathrm{diag}(\kappa_{\mathfrak{W}_1}(x_1), \kappa_{\mathfrak{W}_2}(x_1), \kappa_{\mathfrak{W}_{10}}(x_1), \kappa_{\mathfrak{W}_{11}}(x_1)),$$

where $\kappa_{\mathfrak{W}_1} : E \to \mathrm{GL}_1(17)$, $\kappa_{\mathfrak{W}_2} : E \to \mathrm{GL}_{22}(17)$, $\kappa_{\mathfrak{W}_{10}} : E \to \mathrm{GL}_{253}(17)$, and $\kappa_{\mathfrak{W}_{11}} : E \to \mathrm{GL}_{506}(17)$ are the representations of E afforded by the KE-modules $\mathfrak{W}_1$, $\mathfrak{W}_2$, $\mathfrak{W}_{10}$, and $\mathfrak{W}_{11}$, respectively. And similarly for y_1 and e_1, too.

$\chi_{|D_H} = \tau_{|D}$ means $\mathfrak{V}_{|D_H} \cong \mathfrak{W}_{|D}$. Recall that two representations of a group being isomorphic means that there is some matrix $\mathcal{T}$ such that for every element of the group, the matrix for the element corresponding to one of the two representations is conjugate to the matrix for the same element corresponding to the other representation by $\mathcal{T}$. However, here we have two isomorphic groups D_H and D instead of identical groups. So, employing an isomorphism $\phi_D : D_H \to D$ as obtained in Corollary 4.6(c), now we can see that $\mathfrak{V}_{|D_H} \cong \mathfrak{W}_{|D}$ means that there is some $\mathcal{T} \in \mathrm{GL}_{782}(17)$ such that $\kappa_{\mathfrak{V}|D_H}(\phi_D(g)) = \mathcal{T}_1^{-1}\kappa_{\mathfrak{W}|T}(g)\mathcal{T}_1$ for all $g \in D_H$.

Assuming we have such $\mathcal{T}$, we get

$$\mathfrak{x} = \kappa_{\mathfrak{V}|D_H}(x_0) = \kappa_{\mathfrak{V}|D_H}(\phi_D(x_1)) = \mathcal{T}^{-1}\kappa_{\mathfrak{W}|D}(x_1)\mathcal{T}, \quad \text{and}$$
$$\mathfrak{y} = \kappa_{\mathfrak{V}|D_H}(y_0) = \kappa_{\mathfrak{V}|D_H}(\phi_D(y_1)) = \mathcal{T}^{-1}\kappa_{\mathfrak{W}|D}(y_1)\mathcal{T},$$

thus $\mathfrak{x} = \mathcal{T}^{-1}\kappa_{\mathfrak{W}}(x_1)\mathcal{T}$ and $\mathfrak{y} = \mathcal{T}^{-1}\kappa_{\mathfrak{W}}(y_1)\mathcal{T}$, as desired in the statement (1). Knowing that such $\mathcal{T}$ exists, we can apply the Parker's isomorphism test of Proposition 6.1.6 of [12] by means of the MAGMA command

```
IsIsomorphic(GModule(sub<Y|W(x1),W(y1)>),GModule(sub<Y|V(x0),V(y0)>)),
```

which gives the boolean value, which is `true` in this case, and the desired transformation matrix $\mathcal{T}$.

By assertion (b) and Corollary 7.2.4 of [12] this transformation matrix $\mathcal{T}$ has to be multiplied by a block diagonal matrix $\mathcal{S}_0$ of $\mathrm{GL}_{782}(17)$. In order to calculate its entries one has to get the composition factors of the restrictions $\chi_{i|D_H}$ and $\tau_{j|D}$ to D_H and D, respectively. From the fusion and the 3 character tables B.3, B.2 and B.1 follows that:

$$\chi_{1|D_H} = \psi_1, \quad \chi_{3|D_H} = \psi_9 + \psi_{10}, \quad \chi_{4|D_H} = \psi_1 + \psi_2 + \psi_6 + \psi_{15}, \quad \text{and}$$
$$\tau_{1|D} = \psi_1, \quad \tau_{2|D} = \psi_1 + \psi_2, \quad \tau_{10|D} = \psi_6 + \psi_9, \quad \tau_{11|D} = \psi_{10} + \psi_{15}.$$

Thus, by applying the Meat-axe Algorithm to $\chi_4|_{D_H}$, we get the KD_H-modules $\mathfrak{U}_2, \mathfrak{U}_6$, and $\mathfrak{U}_{15}$ of $D_H \cong D$, such that $\mathfrak{V}_{4|D_H} \cong \mathfrak{U}_1 \oplus \mathfrak{U}_2 \oplus \mathfrak{U}_6 \oplus \mathfrak{U}_{15}$, where $\mathfrak{U}_1$ is the trivial KD_H-module. We already have the constituents KD_H-modules $\mathfrak{U}_9$ and $\mathfrak{U}_{10}$ of the restriction of $\mathfrak{V}$ to D_H, so that $\mathfrak{V}_{3|D_H} \cong \mathfrak{U}_9 \oplus \mathfrak{U}_{10}$. Thus we also have $\mathfrak{W}_{2|D} \cong \mathfrak{U}_1 \oplus \mathfrak{U}_2$, $\mathfrak{W}_{10|D} \cong \mathfrak{U}_6 \oplus \mathfrak{U}_9$, and $\mathfrak{W}_{11|D} \cong \mathfrak{U}_{10} \oplus \mathfrak{U}_{15}$. It is clear that $\mathfrak{V}_{1|D_H} = \mathfrak{U}_1 = \mathfrak{W}_{1|D}$. Notice that the KD_H-modules can be thought of as KD-modules and vice versa, via the isomorphism $\phi_D : D_H \to D$. Now we have

$$\mathfrak{V}_{|D_H} \cong \mathfrak{U}_1 \oplus \mathfrak{U}_1 \oplus \mathfrak{U}_2 \oplus \mathfrak{U}_6 \oplus \mathfrak{U}_9 \oplus \mathfrak{U}_{10} \oplus \mathfrak{U}_{15} \cong \mathfrak{W}_{|D}.$$

Therefore, there is a transformation matrix $\mathcal{S}_0 \in \mathrm{GL}_{782}(17)$ such that

$$\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(x_0)\mathcal{S}_0 = \mathrm{diag}(\kappa_{\mathfrak{U}_1}(x_0), \kappa_{\mathfrak{U}_1}(x_0), \kappa_{\mathfrak{U}_2}(x_0), \kappa_{\mathfrak{U}_6}(x_0), \kappa_{\mathfrak{U}_9}(x_0), \kappa_{\mathfrak{U}_{10}}(x_0), \kappa_{\mathfrak{U}_{15}}(x_0)),$$
and
$$\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(y_0)\mathcal{S}_0 = \mathrm{diag}(\kappa_{\mathfrak{U}_1}(y_0), \kappa_{\mathfrak{U}_1}(y_0), \kappa_{\mathfrak{U}_2}(y_0), \kappa_{\mathfrak{U}_6}(y_0), \kappa_{\mathfrak{U}_9}(y_0), \kappa_{\mathfrak{U}_{10}}(y_0), \kappa_{\mathfrak{U}_{15}}(y_0)),$$

where $\kappa_{\mathfrak{U}_1} : D \to \mathrm{GL}_1(17)$, $\kappa_{\mathfrak{U}_2} : D \to \mathrm{GL}_{21}(17)$, $\kappa_{\mathfrak{U}_6} : D \to \mathrm{GL}_{77}(17)$, $\kappa_{\mathfrak{U}_9} : D \to \mathrm{GL}_{176}(17)$, $\kappa_{\mathfrak{U}_{10}} : D \to \mathrm{GL}_{176}(17)$, and $\kappa_{\mathfrak{U}_{15}} : D \to \mathrm{GL}_{330}(17)$ are the representations of D afforded by the modules $\mathfrak{U}_1$, $\mathfrak{U}_2$, $\mathfrak{U}_6$, $\mathfrak{U}_9$, $\mathfrak{U}_{10}$, and $\mathfrak{U}_{15}$, respectively. This matrix $\mathcal{S}_0$ can be obtained by applying Parker's isomorphism test to the two modules $\mathfrak{V}$ and $\mathfrak{U}_1 \oplus \mathfrak{U}_1 \oplus \mathfrak{U}_2 \oplus \mathfrak{U}_6 \oplus \mathfrak{U}_9 \oplus \mathfrak{U}_{10} \oplus \mathfrak{U}_{15}$. To be precise, the author did this Parker's isomorphism test first for lower right 781 by 781 submatrices of the relevant matrices, and then enlarged the transformation matrix by diagonally joining 1 in the upper left corner. By this way we can be guaranteed to have right restrictions as we expect. We can assume that we started with $\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(x_0)\mathcal{S}_0$, $\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(y_0)\mathcal{S}_0$, and $\mathcal{S}_0^{-1}\kappa_{\mathfrak{V}}(h_0)\mathcal{S}_0$ as $\mathfrak{x}$, $\mathfrak{y}$, and $\mathfrak{h}$, at the beginning of the statement (c). Hence, $\mathfrak{x}$ and $\mathfrak{y}$ are now assumed to be in the block diagonal form as follows, from the beginning (then $\mathfrak{h}$ would be in a certain block form; although not block diagonal, it still carries the structure of $\mathfrak{V}_1 \oplus \mathfrak{V}_3 \oplus \mathfrak{V}_4$):

$$\mathfrak{x} = \mathrm{diag}(\kappa_{\mathfrak{U}_1}(x_0), \kappa_{\mathfrak{U}_1}(x_0), \kappa_{\mathfrak{U}_2}(x_0), \kappa_{\mathfrak{U}_6}(x_0), \kappa_{\mathfrak{U}_9}(x_0), \kappa_{\mathfrak{U}_{10}}(x_0), \kappa_{\mathfrak{U}_{15}}(x_0)), \text{ and}$$
$$\mathfrak{y} = \mathrm{diag}(\kappa_{\mathfrak{U}_1}(y_0), \kappa_{\mathfrak{U}_1}(y_0), \kappa_{\mathfrak{U}_2}(y_0), \kappa_{\mathfrak{U}_6}(y_0), \kappa_{\mathfrak{U}_9}(y_0), \kappa_{\mathfrak{U}_{10}}(y_0), \kappa_{\mathfrak{U}_{15}}(y_0)).$$

(d) Let $Y = \mathrm{GL}_{782}(17)$, $\mathcal{D} = C_Y(\mathfrak{D})$, $\mathcal{H} = C_Y(\mathfrak{H})$ and $\mathcal{E} = C_Y(\mathfrak{E})$. By (b) the restrictions of the compatible characters (χ, τ) are not multiplicity free. Therefore Theorem 7.2.2 of [12] asserts that one has to determine the $\mathcal{H}$-$\mathcal{E}$ double cosets of $\mathcal{D}$ in order to find all the suitable representations of degree 782 of the free product $H *_D E$ with amalgamated subgroup D. How a double coset representative gives rise to a representation will be explained later in this proof.

For each integer k let v^k denote the diagonal matrix of $\mathrm{GL}_k(17)$ having all diagonal entries equal to $v \in K^*$.

From (c) we know that each element of $\mathfrak{D} = \langle \mathfrak{x}_0, \mathfrak{y}_0 \rangle$ is of the block diagonal form as follows:

$$\mathfrak{g} = \mathrm{diag}(\kappa_{\mathfrak{U}_1}(g), \kappa_{\mathfrak{U}_1}(g), \kappa_{\mathfrak{U}_2}(g), \kappa_{\mathfrak{U}_6}(g), \kappa_{\mathfrak{U}_9}(g), \kappa_{\mathfrak{U}_{10}}(g), \kappa_{\mathfrak{U}_{15}}(g)),$$

for any element $g \in D_H$. Note that all of $\kappa_{\mathfrak{U}_1}, \kappa_{\mathfrak{U}_2}, \kappa_{\mathfrak{U}_6}, \kappa_{\mathfrak{U}_9}, \kappa_{\mathfrak{U}_{10}}$, and $\kappa_{\mathfrak{U}_{15}}$ are irreducible representations of D_H. Therefore, by the Theorem of Artin-Wedderburn (Theorem 2.1.27 of [12]) we know that each element $\mathcal{V}$ of $\mathcal{D} = C_Y(\mathfrak{D})$ can be represented as a blocked diagonal matrix

$$\mathcal{V} = diag(a, b^{21}, c^{77}, d^{176}, e^{176}, f^{330}) \in \mathrm{GL}_{782}(17)$$

where $a \in \mathrm{GL}_2(17)$ and all $b, c, d, e, f \in K^*$ are uniquely determined by $\mathcal{V}$.

The map $\gamma : \mathcal{D} \to \mathcal{D}_1 := \mathrm{GL}_2(17) \times K^{*5}$ defined by

$$\gamma(\mathcal{V}) = diag(a, b, c, d, e, f) \in \mathrm{GL}_7(17)$$

is an isomorphism. Thus assertion (1) of (d) holds.

Let $\mathcal{H}_1 = \gamma(\mathcal{H})$, $\mathcal{E}_1 = \gamma(\mathcal{E})$. Recall from the proof of (c) that

$$\chi_{1|D_H} = \psi_1, \quad \chi_{3|D_H} = \psi_9 + \psi_{10}, \quad \chi_{4|D_H} = \psi_1 + \psi_2 + \psi_6 + \psi_{15}, \quad \text{and}$$
$$\tau_{1|D} = \psi_1, \quad \tau_{2|D} = \psi_1 + \psi_2, \quad \tau_{10|D} = \psi_6 + \psi_9, \quad \tau_{11|D} = \psi_{10} + \psi_{15}.$$

Since $3 \in K$ generates the multiplicative group K^* of K it follows from these restrictions that $\mathcal{H}_1 = \langle a_1, a_2, a_3 \rangle$ and $\mathcal{E}_1 = \langle b_1, b_2, b_3, b_4 \rangle$ where the generators a_i and b_j denote the diagonal matrices of $\mathrm{GL}_7(17)$ given in assertions (2) and (3) of statement (d), respectively. In particular, $\mathcal{H}_1$ and $\mathcal{E}_1$ are abelian.

Recall that we have to determine the $\mathcal{H}$-$\mathcal{E}$ double cosets of $\mathcal{D}$. Since γ is an isomorphism it suffices to determine the $\mathcal{H}_1$-$\mathcal{E}_1$ double cosets of $\mathcal{D}_1$. Let $\mathcal{A}$ and $\mathcal{B}$ be the direct factors $\mathrm{GL}_2(17)$ and K^{*5} of $\mathcal{D}_1$, respectively, so $\mathcal{D}_1 = \mathcal{A} \times \mathcal{B}$. Let $\Phi_{\mathcal{A}} : \mathcal{D}_1 \to \mathcal{A}$ and $\Phi_{\mathcal{B}} : \mathcal{D}_1 \to \mathcal{B}$ be the projection mappings, i.e. $\Phi_{\mathcal{A}}(ab) = a$ and $\Phi_{\mathcal{B}}(ab) = b$ for all $a \in \mathcal{A}$ and $b \in \mathcal{B}$. Thus, for any element g of $\mathcal{D}_1$, the projection $\Phi_{\mathcal{A}}(g)$ is just the upper left 2 by 2 submatrix of g, and $\Phi_{\mathcal{B}}(g)$ the lower right 5 by 5 submatrix of g.

Let λ_1, λ_2 be two representatives for a single $\mathcal{H}_1$-$\mathcal{E}_1$ double coset, i.e. $\mathcal{H}_1\lambda_1\mathcal{E}_1 = \mathcal{H}_1\lambda_2\mathcal{E}_1$. Then $\lambda_2 \in \mathcal{H}_1\lambda_1\mathcal{E}_1$, and therefore $\lambda_2 = a\lambda_1 b$ for some $a \in \mathcal{A}$ and $b \in \mathcal{B}$. Note that any element of $\mathcal{D}_1$ can uniquely be written as multiplication of an element in $\mathcal{A}$ and an element in $\mathcal{B}$. Therefore, there exist $\xi_1, \delta_1, \theta_1$ in $\mathcal{A}$ and $\xi_2, \delta_2, \theta_2$ in $\mathcal{B}$ such that $\lambda = \theta_1\theta_2$, $a = \xi_1\xi_2$ and $b = \psi_1\delta_2$. Then, $\lambda_2 = a\lambda_1 b$ implies $\lambda_2 = (\xi_1\xi_2)(\theta_1\theta_2)(\delta_1\delta_2)$.

Observe that elements of $\mathcal{B}$ commutes both with elements of $\mathcal{A}$ and those of $\mathcal{B}$. Therefore, $\lambda_2 = (\xi_1\xi_2)(\theta_1\theta_2)(\psi_1\psi_2)$ implies $\lambda_2 = (\xi_1\theta_1\psi_1)(\xi_2\theta_2\psi_2)$. Since $\xi_1\theta_1\psi_1 \in \mathcal{A}$, $\xi_2\theta_2\psi_2 \in \mathcal{B}$, and $\Phi_{\mathcal{B}}(\lambda_1) = \theta_2$, we have $\Phi_{\mathcal{B}}(\lambda_2) = \xi_2\theta_2\psi_2 = \xi_2\Phi_{\mathcal{B}}(\lambda_1)\psi_2$. Let $\mathcal{H}_2 = \Phi_{\mathcal{B}}(\mathcal{H}_1)$ and $\mathcal{E}_2 = \Phi_{\mathcal{B}}(\mathcal{E}_1)$. Then, $\Phi_{\mathcal{B}}(\lambda_2) = \xi_2\Phi_{\mathcal{B}}(\lambda_1)\psi_2$ means that $\Phi_{\mathcal{B}}(\lambda_1)$ and $\Phi_{\mathcal{B}}(\lambda_2)$ represent a single $\mathcal{H}_2$-$\mathcal{E}_2$ double coset in $\mathcal{B} = \Phi_{\mathcal{B}}(\mathcal{D}_1)$.

Since $\mathcal{B}$ is an abelian group, we get that $\mathcal{H}_2\mathcal{E}_2$ is a group, and that $\mathcal{H}_2$-$\mathcal{E}_2$ double cosets are just $\mathcal{H}_2\mathcal{E}_2$-cosets. Observe that the group $\mathcal{H}_2\mathcal{E}_2$ are generated by $\Phi_{\mathcal{B}}(a_i)$ and $\Phi_{\mathcal{B}}(b_j)$, where

$$\Phi_{\mathcal{B}}(a_1) = \mathrm{Id}(\mathrm{GL}_5(17)), \Phi_{\mathcal{B}}(a_2) = \mathrm{diag}(1,1,3,3,1), \Phi_{\mathcal{B}}(a_3) = \mathrm{diag}(3,3,1,1,3),$$
$$\Phi_{\mathcal{B}}(b_1) = \mathrm{Id}(\mathrm{GL}_5(17)), \Phi_{\mathcal{B}}(b_2) = \mathrm{diag}(3,1,1,1,1),$$
$$\Phi_{\mathcal{B}}(b_3) = \mathrm{diag}(1,3,3,1,1), \Phi_{\mathcal{B}}(b_4) = \mathrm{diag}(1,1,1,3,3).$$

It is checked in MAGMA that the group $\mathcal{H}_2\mathcal{E}_2$ has order 16^4, and that the set of sixteen matrices $\zeta_j = \mathrm{diag}(1,1,1,1,j)$, where j runs through $1, 2, 3, \ldots, 16$, serves as a complete set of representatives of distinct $\mathcal{H}_2\mathcal{E}_2$-cosets in $\mathcal{B}$.

What is just proved is that for any $\mathcal{H}_1$-$\mathcal{E}_1$ double coset representative λ, we can find some $a \in \mathcal{H}_1$ and $b \in \mathcal{E}_1$ such that $\Phi_{\mathcal{B}}(a)\Phi_{\mathcal{B}}(\lambda)\Phi_{\mathcal{B}}(b)$ is of the form ζ_j, for some j (in the above argument, we had $\xi_2 = \Phi_{\mathcal{B}}(a)$ and $\psi_2 = \Phi_{\mathcal{B}}(b)$). Now, $a\lambda b$ represents the same $\mathcal{H}_1$-$\mathcal{E}_1$ double coset as λ, and we now have that $\Phi_{\mathcal{B}}(a\lambda b) = \Phi_{\mathcal{B}}(a)\Phi_{\mathcal{B}}(\lambda)\Phi_{\mathcal{B}}(b)$ is of the form ζ_j. Hence, we can now assume that any double coset representative λ satisfies $\Phi_{\mathcal{B}}(\lambda) = \zeta_j$ for some j. Now, suppose that λ_1 and λ_2 represent a single $\mathcal{H}_1$-$\mathcal{E}_1$ double coset. We proved above that $\Phi_{\mathcal{B}}(\lambda_1) = \zeta_j = \Phi_{\mathcal{B}}(\lambda_2)$ for some j.

Fix any $j \in \{1, 2, 3, \ldots, 16\}$. Let's find out how many distinct $\mathcal{H}_1$-$\mathcal{E}_1$ double coset representative λ there are such that $\Phi_{\mathcal{B}}(\lambda) = \zeta_j$. Let λ_1 and λ_2 be $\mathcal{H}_1$-$\mathcal{E}_1$ double coset representatives such that $\Phi_{\mathcal{B}}(\lambda_1) = \zeta_j = \Phi_{\mathcal{B}}(\lambda_2)$ for this fixed j. Then $\lambda_2 \in \mathcal{H}_1\lambda_1\mathcal{E}_1$, and therefore $\lambda_2 = a\lambda_1 b$ for some $a \in \mathcal{H}_1$ and $b \in \mathcal{E}_1$. Observe that $\mathcal{H}_1$ is abelian and is generated by a_1, a_2, a_3. Therefore $a = a_1^{\pi_1}a_2^{\pi_2}a_3^{\pi_3}$ for some π_1, π_2, π_3 in $\{1, 2, 3, \ldots, 16\}$. Similarly, $\mathcal{E}_1$ is abelian and is generated by b_1, b_2, b_3, b_4. Therefore $b = b_1^{\sigma_1}b_2^{\sigma_2}b_3^{\sigma_3}b_4^{\sigma_4}$ for some $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ in $\{1, 2, 3, \ldots, 16\}$. We can observe by computation that

$$\begin{aligned} a &= a_1^{\pi_1}a_2^{\pi_2}a_3^{\pi_3} = \operatorname{diag}(3^{\pi_1}, 3^{\pi_3}, 3^{\pi_3}, 3^{\pi_3}, 3^{\pi_2}, 3^{\pi_2}, 3^{\pi_3}), \quad \text{and} \\ b &= b_1^{\sigma_1}b_2^{\sigma_2}b_3^{\sigma_3}b_4^{\sigma_4} = \operatorname{diag}(3^{\sigma_1}, 3^{\sigma_2}, 3^{\sigma_2}, 3^{\sigma_3}, 3^{\sigma_3}, 3^{\sigma_4}, 3^{\sigma_4}). \end{aligned}$$

For $i = 1, 2$, we know that $\lambda_i = \operatorname{diag}(\Phi_{\mathcal{A}}(\lambda_i), \Phi_{\mathcal{B}}(\lambda_i))$ and that $\Phi_{\mathcal{B}}(\lambda_i) = \zeta_j = \operatorname{diag}(1, 1, 1, 1, j)$. Let $\Phi_{\mathcal{A}}(\lambda_1) = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)$ and $\Phi_{\mathcal{A}}(\lambda_2) = \left(\begin{smallmatrix} A' & B' \\ C' & D' \end{smallmatrix}\right)$. Thus, we can observe that

$$\begin{aligned} a\lambda_1 b &= a[\operatorname{diag}(\Phi_{\mathcal{A}}(\lambda_1), \Phi_{\mathcal{B}}(\lambda_1))]b = a[\operatorname{diag}(\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right), 1, 1, 1, 1, j)]b \\ &= \operatorname{diag}(\left(\begin{smallmatrix} 3^{\pi_1} & 0 \\ 0 & 3^{\pi_3} \end{smallmatrix}\right)\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)\left(\begin{smallmatrix} 3^{\sigma_1} & 0 \\ 0 & 3^{\sigma_2} \end{smallmatrix}\right), 3^{\pi_3+\sigma_2}, 3^{\pi_3+\sigma_3}, 3^{\pi_2+\sigma_3}, 3^{\pi_2+\sigma_4}, j3^{\pi_3+\sigma_4}) \end{aligned}$$

We also have

$$\lambda_2 = \operatorname{diag}(\Phi_{\mathcal{A}}(\lambda_2), \Phi_{\mathcal{B}}(\lambda_2)) = \operatorname{diag}(\left(\begin{smallmatrix} A' & B' \\ C' & D' \end{smallmatrix}\right), 1, 1, 1, 1, j),$$

and therefore from $\lambda_2 = a\lambda_1 b$ we get $\left(\begin{smallmatrix} 3^{\pi_1} & 0 \\ 0 & 3^{\pi_3} \end{smallmatrix}\right)\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)\left(\begin{smallmatrix} 3^{\sigma_1} & 0 \\ 0 & 3^{\sigma_2} \end{smallmatrix}\right) = \left(\begin{smallmatrix} A' & B' \\ C' & D' \end{smallmatrix}\right)$ and $3^{\pi_3+\sigma_2} = 3^{\pi_3+\sigma_3} = 3^{\pi_2+\sigma_3} = 3^{\pi_2+\sigma_4} = 3^{\pi_3+\sigma_4} = 1$. It is easy to see that $3^{\pi_3+\sigma_2} = 3^{\pi_3+\sigma_3} = 3^{\pi_2+\sigma_3} = 3^{\pi_2+\sigma_4} = 3^{\pi_3+\sigma_4} = 1$ implies

$$\pi_3 = -\sigma_2, \quad \pi_3 = -\sigma_3, \quad \pi_2 = -\sigma_3, \quad \pi_2 = -\sigma_4, \quad \pi_3 = -\sigma_4,$$

so that $\pi_2 = \pi_3 = -\sigma_2 = -\sigma_3 = -\sigma_4$.

Now let's look at

$$\begin{pmatrix} 3^{\pi_1} & 0 \\ 0 & 3^{\pi_3} \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} 3^{\sigma_1} & 0 \\ 0 & 3^{\sigma_2} \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}.$$

Since π_1, π_3, σ_1 are arbitrary and $\sigma_2 = -\pi_3$, the above equation can be rewritten as

$$\begin{pmatrix} s & 0 \\ 0 & u \end{pmatrix}\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} t & 0 \\ 0 & u^{-1} \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix},$$

where s, t, u are any elements in K^*. By computation we get $\left(\begin{smallmatrix} s & 0 \\ 0 & u \end{smallmatrix}\right)\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)\left(\begin{smallmatrix} t & 0 \\ 0 & u^{-1} \end{smallmatrix}\right) = \left(\begin{smallmatrix} (stA) & (su^{-1}B) \\ (tuC) & D \end{smallmatrix}\right)$. Hence, the above equation becomes

$$(*): \quad \begin{pmatrix} (stA) & (su^{-1}B) \\ (tuC) & D \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}.$$

Define a relation $\sim$ by : $\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \sim \left(\begin{smallmatrix} A' & B' \\ C' & D' \end{smallmatrix}\right)$ if $(*)$ holds. Then, it is easy to prove that $\sim$ is an equivalence relation. It is easy to observe that we have $\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \sim \left(\begin{smallmatrix} A' & B' \\ C' & D' \end{smallmatrix}\right)$ if and only if $D = D'$ and $\left(\begin{smallmatrix} A' & B' \\ C' & D \end{smallmatrix}\right)$ can be obtained by multiplying some numbers$\in K^*$ to the first column and the first row of $\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)$. Therefore, it is just an easy computation by hand to get the complete list of representatives for distinct equivalence classes of $\sim$. The list of representatives is presented as follows.

For cases $D = 0$,

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 1 & 1 \\ c & 0 \end{pmatrix}$$

are the representatives for all possible distinct classes, where c runs through K^*.

This gives $1 + 16 = 17$ cases. For cases $D \neq 0$ and at least one of A, B, C is 0,

$$\begin{pmatrix} 1 & 0 \\ 1 & c \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 0 & c \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & c \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 1 & 0 \\ 0 & c \end{pmatrix}$$

are the representatives for all possible distinct classes, where c runs through K^*. This gives $4 \times 16 = 64$ cases.

For cases when A, B, C, D are all nonzero,

$$\begin{pmatrix} 1 & 1 \\ c & d \end{pmatrix}$$

are the representatives for all possible distinct classes, where c and d runs through K^* and $c \neq d$. Here we have $16^2 - 16 = 240$ cases. Now we have the complete list of equivalence classes of $\sim$, and there are $17 + 64 + 240 = 321$ classes.

What is just proved is that, for any fixed $j \in \{1, 2, 3, \ldots, 16\}$, there are 321 distinct $\mathcal{H}_1$-$\mathcal{E}_1$ double coset representatives λ such that $\Phi_{\mathcal{B}}(\lambda) = \zeta_j$. Therefore, the number of $\mathcal{H}_1$-$\mathcal{E}_1$ double cosets in $\mathcal{D}_1$ is exactly $16 \times 321 = 5136$ as desired in (4). We also have a method to enumerate all the double coset representatives; namely, each double coset representative is of the form

$$M_{k,j} = \mathrm{diag}(m_k, 1, 1, 1, 1, j),$$

where j ranges in K^* and $m_k \in \mathrm{GL}_2(17)$ runs through the 321 choices for $\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)$ described above. Since there are 321 choices for m_k, the subscript k can be considered to range in $\{1, 2, \ldots, 321\}$. For each k, let $\mathfrak{m}_{k,j} = \gamma^{-1}(M_{k,j})$, $\mathfrak{e}_{k,j} = \mathfrak{m}_{k,j}^{-1}\mathfrak{e}_1\mathfrak{m}_{k,j}$, and

$$\mathfrak{G}_{k,j} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h}, \mathfrak{e}_{k,j} \rangle.$$

As stated in Theorem 7.2.2 of [12], this matrix group $\mathfrak{G}_{k,j}$ is the representation which corresponds to the double coset representative $\mathfrak{m}_{k,j}$.

As mentioned in the statement (5), only first half of the Sylow 2-subgroup test of Step 5(c) of Algorithm 7.4.8 of [12] for each $\mathfrak{G}_{k,j}$ is done here. For each k, j, the author checked the orders of the matrices $\mathfrak{h}\mathfrak{e}_{k,j}$ and $\mathfrak{y}\mathfrak{e}_{k,j}\mathfrak{h}$ with Magma. If either of the two matrices has an order which can't be the order of an element of Fi_{23} (information on largest order of elements in Fi_{23} is obtained in Atlas [4]), then the particular pair (k, j) is discarded. Through this test, only one case survived, namely the case is with $j = 16$ and $k = k_0$ such that

$$m_{k_0} = \begin{pmatrix} 1 & 1 \\ 9 & 14 \end{pmatrix}.$$

For this k_0, let $\mathfrak{e} = \mathfrak{e}_{k,j}$ and $\mathfrak{G} = \mathfrak{G}_{k_0,16} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h}, \mathfrak{e} \rangle$. It is checked with MAGMA that the two matrices $\mathfrak{h}\mathfrak{e}$ and $\mathfrak{y}\mathfrak{e}\mathfrak{h}$ have orders 12 and 13, respectively. So (5) is done. The four generating matrices $\mathfrak{x}, \mathfrak{y}, \mathfrak{h}, \mathfrak{e}$ are documented in the author's website [10] as mentioned in (6).

(e) Using the algorithm described in the proof of Theorem 6.2.1 of [12] implemented in Magma, a faithful permutation representation $P\mathfrak{G}$ of $\mathfrak{G}$ of degree 31671 with stabilizer $\mathfrak{H}$ is obtained. In order to use this algorithm for getting permutation representation, the author had to look for a short-word matrix in terms of $\mathfrak{x}, \mathfrak{y}, \mathfrak{h}, \mathfrak{e}$ which has order 23 (since 23 divides the order of $\mathfrak{G}$, but not the order of the permutation stabilizer $\mathfrak{H}$); the matrix $\mathfrak{x}\mathfrak{y}\mathfrak{e}\mathfrak{h}^2$ of order 23 is used.

In particular, it is shown by means of MAGMA that $|\mathfrak{G}| = 2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$, using the faithful permutation representation $P\mathfrak{G}$.

(f) Let $\mathfrak{z} = (\mathfrak{x}\mathfrak{y}^2)^7$. Using the faithful permutation representation $P\mathfrak{G}$ of degree 31671, it is verified that $C_{\mathfrak{G}}(\mathfrak{z}) = \mathfrak{H} \cong H$.

(g) Using the faithful permutation representation $P\mathfrak{G}$ of degree 31671 and Kratzer's Algorithm 5.3.18 of [12], the representatives of all the conjugacy classes of $\mathfrak{G}$ are obtained using MAGMA, see Table A.6.

(h) Furthermore, character table of $\mathfrak{G}$ is computed by means of the above permutation representation $P\mathfrak{G}$ and MAGMA. It coincides with the one of Fi_{23} in [4], p. 178 -179. The character table of $\mathfrak{G}$ implies that $\mathfrak{G}$ is a simple group. This completes the proof.

□

Remark 5.2. *Let $E_1 = V_1 \rtimes \mathcal{M}_{23}$ be the split extension of $\mathcal{M}_{23}$ by its simple module V_1 of dimension 11 over $F = \mathrm{GF}(2)$. Then E_1 has a unique class of 2-central involutions represented by some element z'_1. However, when applying the Algorithm 2.1 to E_1, an overgroup of $C_{E_1}(z'_1)$ of odd index satisfying all conditions of Algorithm 2.1 was not found. Hence Algorithm 2.1 can't be applied for further steps.*

Remark 5.3. *Let $E_2 = V_2 \rtimes \mathcal{M}_{23}$ be the split extension of $\mathcal{M}_{23}$ by its simple module V_2 of dimension 11 over $F = \mathrm{GF}(2)$. Then E_2 has three classes of 2-central involutions represented by some elements z'_2, z'_3, and z'_4. However, when applying the Algorithm 2.1 to E_2, an overgroup of $C_{E_i}(z'_i)$ of odd index satisfying all conditions of Algorithm 2.1 was not found, for any $i = 2, 3, 4$. Hence Algorithm 2.1 can't be applied for further steps.*

Appendix A. Representatives of conjugacy classes

A.1. *Conjugacy classes of* $E(\mathrm{Fi}_{23}) = E = \langle x_1, y_1, e_1 \rangle$, *with subscripts dropped*

Class	*Representative*	$\lvert Centralizer \rvert$	2P	3P	5P	7P	11P	23P
1	1	$2^{18}\cdot 3^2\cdot 5\cdot 7\cdot 11\cdot 23$	1	1	1	1	1	1
2_1	$(y)^7$	$2^{18}\cdot 3^2\cdot 5\cdot 7\cdot 11$	1	2_1	2_1	2_1	2_1	2_1
2_2	$(y^3e)^6$	$2^{18}\cdot 3^2\cdot 5\cdot 7$	1	2_2	2_2	2_2	2_2	2_2
2_3	$(xye)^8$	$2^{18}\cdot 3^2\cdot 5$	1	2_3	2_3	2_3	2_3	2_3
2_4	e	$2^{14}\cdot 3\cdot 7$	1	2_4	2_4	2_4	2_4	2_4
2_5	x	$2^{14}\cdot 3$	1	2_5	2_5	2_5	2_5	2_5
3	$(y^3e)^4$	$2^7\cdot 3^2\cdot 5$	3	1	3	3	3	3
4_1	$(xy^2xey^2)^7$	$2^{11}\cdot 3\cdot 7$	2_2	4_1	4_1	4_1	4_1	4_1
4_2	$(xye)^4$	$2^{12}\cdot 3$	2_3	4_2	4_2	4_2	4_2	4_2
4_3	$(y^2e)^2$	$2^{12}\cdot 3$	2_3	4_3	4_3	4_3	4_3	4_3
4_4	$(y^3e)^3$	$2^{11}\cdot 3$	2_2	4_4	4_4	4_4	4_4	4_4
4_5	$(xey)^2$	2^9	2_5	4_5	4_5	4_5	4_5	4_5
4_6	$(xyey^3)^2$	2^9	2_5	4_6	4_6	4_6	4_6	4_6
4_7	xe	2^8	2_4	4_7	4_7	4_7	4_7	4_7
5	$(xyxye)^2$	$2^3\cdot 3\cdot 5$	5	5	1	5	5	5
6_1	$(xyxey^2)^5$	$2^7\cdot 3^2\cdot 5$	3	2_3	6_1	6_1	6_1	6_1
6_2	$(xyexe)^2$	$2^7\cdot 3^2$	3	2_3	6_2	6_2	6_2	6_2
6_3	$xyey^3e$	$2^7\cdot 3^2$	3	2_1	6_3	6_3	6_3	6_3
6_4	$(y^3e)^2$	$2^6\cdot 3^2$	3	2_2	6_4	6_4	6_4	6_4
6_5	$xyxexey^2$	$2^6\cdot 3^2$	3	2_3	6_5	6_5	6_5	6_5
6_6	y^4e	$2^5\cdot 3$	3	2_4	6_6	6_6	6_6	6_6
6_7	$xyxeyxe$	$2^5\cdot 3$	3	2_5	6_7	6_7	6_7	6_7
7_1	$(y)^2$	$2^3\cdot 7$	7_1	7_2	7_2	1	7_1	7_1
7_2	$(ye)^2$	$2^3\cdot 7$	7_2	7_1	7_1	1	7_2	7_2
8_1	$(xye)^2$	2^7	4_2	8_1	8_1	8_1	8_1	8_1
8_2	y^2e	2^7	4_3	8_2	8_2	8_2	8_2	8_2
8_3	xy^2ey^2e	2^7	4_3	8_3	8_3	8_3	8_3	8_3
8_4	xey	2^5	4_5	8_4	8_4	8_4	8_4	8_4
8_5	$xyey^3$	2^5	4_6	8_5	8_5	8_5	8_5	8_5
10_1	$(xyxey^2)^3$	$2^3\cdot 3\cdot 5$	5	10_1	2_3	10_1	10_1	10_1
10_2	$xyxye$	$2^3\cdot 5$	5	10_2	2_2	10_2	10_2	10_2
10_3	$xyxyey$	$2^3\cdot 5$	5	10_3	2_1	10_3	10_3	10_3
11_1	$(xy^3)^2$	$2\cdot 11$	11_2	11_1	11_1	11_2	1	11_1
11_2	$(xy^3)^4$	$2\cdot 11$	11_1	11_2	11_2	11_1	1	11_2
12_1	$xyexe$	$2^5\cdot 3$	6_2	4_2	12_1	12_1	12_1	12_1
12_2	$xyxyxey^2$	$2^5\cdot 3$	6_2	4_3	12_2	12_2	12_2	12_2
12_3	y^3e	$2^4\cdot 3$	6_4	4_4	12_3	12_3	12_3	12_3
12_4	y^3ey^2e	$2^4\cdot 3$	6_4	4_1	12_4	12_4	12_4	12_4
14_1	$xyey^2$	$2^3\cdot 7$	7_2	14_2	14_2	2_2	14_1	14_1
14_2	xy^5	$2^3\cdot 7$	7_1	14_1	14_1	2_2	14_2	14_2
14_3	y	$2^2\cdot 7$	7_1	14_4	14_4	2_1	14_3	14_3
14_4	ye	$2^2\cdot 7$	7_2	14_3	14_3	2_1	14_4	14_4
14_5	$xyxe$	$2^2\cdot 7$	7_2	14_6	14_6	2_4	14_5	14_5
14_6	xy^3xe	$2^2\cdot 7$	7_1	14_5	14_5	2_4	14_6	14_6
15_1	xy^3e	$2\cdot 3\cdot 5$	15_1	5	3	15_2	15_2	15_1
15_2	$(xy^3ey)^2$	$2\cdot 3\cdot 5$	15_2	5	3	15_1	15_1	15_2
16_1	xye	2^5	8_1	16_1	16_2	16_2	16_1	16_2
16_2	xey^3	2^5	8_1	16_2	16_1	16_1	16_2	16_1
22_1	xy^3	$2\cdot 11$	11_1	22_1	22_1	22_2	2_1	22_1
22_2	xey^2	$2\cdot 11$	11_2	22_2	22_2	22_1	2_1	22_2
23_1	$xeyey$	23	23_1	23_1	23_2	23_2	23_2	1
23_2	xy^2eyxe	23	23_2	23_2	23_1	23_1	23_1	1
28_1	xy^2xey^2	$2^2\cdot 7$	14_1	28_2	28_2	4_1	28_1	28_1
28_2	xey^3ey	$2^2\cdot 7$	14_2	28_1	28_1	4_1	28_2	28_2
30_1	$xyxey^2$	$2\cdot 3\cdot 5$	15_1	10_1	6_1	30_2	30_2	30_1
30_2	xy^3ey	$2\cdot 3\cdot 5$	15_2	10_1	6_1	30_1	30_1	30_2

A.2. *Conjugacy classes of* $D(\mathrm{Fi}_{23}) = D = \langle x_1, y_1 \rangle \cong \langle x_0, y_0 \rangle$*, with subscripts dropped*

Class	*Representative*	$\lvert Centralizer \rvert$	2P	3P	5P	7P	11P
1	1	$2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	1	1	1	1	1
2_1	$(xy^2)^7$	$2^{18} \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	1	2_1	2_1	2_1	2_1
2_2	$(y)^7$	$2^{17} \cdot 3^2 \cdot 5 \cdot 7$	1	2_2	2_2	2_2	2_2
2_3	$(xy^5)^7$	$2^{17} \cdot 3^2 \cdot 5 \cdot 7$	1	2_3	2_3	2_3	2_3
2_4	$(xyxy^2)^8$	$2^{18} \cdot 3 \cdot 5$	1	2_4	2_4	2_4	2_4
2_5	$(xyxy^3)^6$	$2^{18} \cdot 3 \cdot 5$	1	2_5	2_5	2_5	2_5
2_6	$(xyxy^2xy^2)^4$	$2^{16} \cdot 3^2$	1	2_6	2_6	2_6	2_6
2_7	x	$2^{14} \cdot 3$	1	2_7	2_7	2_7	2_7
2_8	$(xy^3xy^4)^3$	$2^{14} \cdot 3$	1	2_8	2_8	2_8	2_8
2_9	$(xy^2xy^3xy^2xy^5)^2$	2^{13}	1	2_9	2_9	2_9	2_9
3	$(xyxy^3)^4$	$2^7 \cdot 3^2$	3	1	3	3	3
4_1	$(xyxy^3)^3$	$2^{11} \cdot 3$	2_5	4_1	4_1	4_1	4_1
4_2	$(xyxy^3xyxy^4)^3$	$2^{11} \cdot 3$	2_5	4_2	4_2	4_2	4_2
4_3	$(xyxy^2)^4$	2^{12}	2_4	4_3	4_3	4_3	4_3
4_4	$(xyxyxy^2xy^6)^2$	2^{12}	2_4	4_4	4_4	4_4	4_4
4_5	$(xyxy^2xy^2)^2$	$2^{10} \cdot 3$	2_6	4_5	4_5	4_5	4_5
4_6	$(xyxy^2xyxy^6)^3$	$2^{10} \cdot 3$	2_6	4_6	4_6	4_6	4_6
4_7	$xy^2xy^7xy^5$	2^{10}	2_5	4_7	4_7	4_7	4_7
4_8	$(xyxy^3xy^2)^2$	2^9	2_7	4_8	4_8	4_8	4_8
4_9	$(xyxy^2xy^4)^2$	2^9	2_7	4_9	4_9	4_9	4_9
4_{10}	$xy^2xy^3xy^5$	2^8	2_8	4_{10}	4_{10}	4_{10}	4_{10}
4_{11}	$xy^2xy^3xy^2xy^5$	2^8	2_9	4_{11}	4_{11}	4_{11}	4_{11}
4_{12}	$xyxyxy^4xy^4xy^2$	2^8	2_9	4_{12}	4_{12}	4_{12}	4_{12}
4_{13}	$xyxyxyxy^2xy^2$	2^7	2_8	4_{13}	4_{13}	4_{13}	4_{13}
5	$(xyxy^5)^2$	$2^3 \cdot 5$	5	5	1	5	5
6_1	$(xyxy^2xyxy^6)^2$	$2^7 \cdot 3^2$	3	2_6	6_1	6_1	6_1
6_2	$xy^2xy^2xy^6xy^3$	$2^7 \cdot 3^2$	3	2_6	6_2	6_2	6_2
6_3	$xyxy^2xy^3xy^2xy^5$	$2^7 \cdot 3^2$	3	2_1	6_3	6_3	6_3
6_4	$xyxyxy^2xyxyxy^3$	$2^5 \cdot 3^2$	3	2_6	6_4	6_4	6_4
6_5	$xyxy^2xy^6xy^4$	$2^5 \cdot 3^2$	3	2_6	6_5	6_5	6_5
6_6	$xyxy^3xyxy^4xy^3$	$2^5 \cdot 3^2$	3	2_3	6_6	6_6	6_6
6_7	$xyxyxy^4xy^3xyxy^2$	$2^5 \cdot 3^2$	3	2_2	6_7	6_7	6_7
6_8	$(xyxy^3)^2$	$2^6 \cdot 3$	3	2_5	6_8	6_8	6_8
6_9	$xyxy^5xy^2$	$2^6 \cdot 3$	3	2_4	6_9	6_9	6_9
6_{10}	xy^3xy^4	$2^5 \cdot 3$	3	2_8	6_{10}	6_{10}	6_{10}
6_{11}	xy^4xy^6	$2^5 \cdot 3$	3	2_7	6_{11}	6_{11}	6_{11}
7_1	$(y)^2$	$2^2 \cdot 7$	7_1	7_2	7_2	1	7_1
7_2	$(xy^2)^2$	$2^2 \cdot 7$	7_2	7_1	7_1	1	7_2
8_1	$(xyxy^2)^2$	2^7	4_3	8_1	8_1	8_1	8_1
8_2	$xyxyxy^2xy^6$	2^7	4_4	8_2	8_2	8_2	8_2
8_3	$xyxy^6xy^3xy^4$	2^7	4_4	8_3	8_3	8_3	8_3
8_4	$xyxy^2xy^2$	2^6	4_5	8_4	8_4	8_4	8_4
8_5	$xy^2xy^3xy^3$	2^6	4_3	8_5	8_5	8_5	8_5
8_6	$xy^2xy^2xy^2xy^3$	2^6	4_5	8_6	8_6	8_6	8_6
8_7	$xyxy^3xy^2$	2^5	4_8	8_7	8_7	8_7	8_7
8_8	$xyxy^2xy^4$	2^5	4_9	8_8	8_8	8_8	8_8
10_1	$xyxy^5$	$2^3 \cdot 5$	5	10_2	2_3	10_2	10_1
10_2	$xyxy^6$	$2^3 \cdot 5$	5	10_1	2_3	10_1	10_2
10_3	$xyxy^3xy^3$	$2^3 \cdot 5$	5	10_3	2_5	10_3	10_3
10_4	$xyxy^2xyxy^3$	$2^3 \cdot 5$	5	10_6	2_2	10_6	10_4
10_5	$xy^2xy^3xy^4$	$2^3 \cdot 5$	5	10_5	2_4	10_5	10_5
10_6	$xy^2xy^4xy^3$	$2^3 \cdot 5$	5	10_4	2_2	10_4	10_6
10_7	$xyxyxy^4xy^3xy^2$	$2^3 \cdot 5$	5	10_7	2_1	10_7	10_7
11_1	$(xy^3)^2$	$2 \cdot 11$	11_2	11_1	11_1	11_2	1
11_2	$(xy^3)^4$	$2 \cdot 11$	11_1	11_2	11_2	11_1	1
12_1	$xyxy^2xyxy^6$	$2^5 \cdot 3$	6_1	4_6	12_1	12_1	12_1
12_2	$xyxyxy^2xy^7$	$2^5 \cdot 3$	6_1	4_5	12_2	12_2	12_2
12_3	$xyxy^3$	$2^4 \cdot 3$	6_8	4_1	12_3	12_3	12_3
12_4	$xyxy^3xyxy^4$	$2^4 \cdot 3$	6_8	4_2	12_4	12_4	12_4
14_1	y	$2^2 \cdot 7$	7_1	14_6	14_6	2_2	14_1
14_2	xy^2	$2^2 \cdot 7$	7_2	14_5	14_5	2_1	14_2
14_3	xy^5	$2^2 \cdot 7$	7_1	14_4	14_4	2_3	14_3
14_4	xy^2xy^3	$2^2 \cdot 7$	7_2	14_3	14_3	2_3	14_4
14_5	xy^2xy^4	$2^2 \cdot 7$	7_1	14_2	14_2	2_1	14_5
14_6	xy^3xy^5	$2^2 \cdot 7$	7_2	14_1	14_1	2_2	14_6
16_1	$xyxy^2$	2^5	8_1	16_1	16_2	16_2	16_1
16_2	$xyxyxy^2$	2^5	8_1	16_2	16_1	16_1	16_2
22_1	xy^3	$2 \cdot 11$	11_1	22_1	22_1	22_2	2_1
22_2	$xyxyxy^2xy^2$	$2 \cdot 11$	11_2	22_2	22_2	22_1	2_1

A.3. *Conjugacy classes of* $H_2 = \langle p_2, q_2, h_2 \rangle$, *with subscripts dropped*

Class	*Representative*	$\lvert Centralizer \rvert$	2P	3P	5P
1	1	$2^{18} \cdot 3^4 \cdot 5$	1	1	1
2_1	$(pq)^8$	$2^{18} \cdot 3^4 \cdot 5$	1	2_1	2_1
2_2	$(p^2q)^5$	$2^{18} \cdot 3^4 \cdot 5$	1	2_2	2_2
2_3	$(p^2h^2q)^5$	$2^{18} \cdot 3^4 \cdot 5$	1	2_3	2_3
2_4	$(q)^3$	$2^{17} \cdot 3^4 \cdot 5$	1	2_4	2_4
2_5	$(p^2q^2h)^5$	$2^{17} \cdot 3^4 \cdot 5$	1	2_5	2_5
2_6	$(qh)^6$	$2^{16} \cdot 3^2$	1	2_6	2_6
2_7	$(p^2hqh)^6$	$2^{16} \cdot 3^2$	1	2_7	2_7
2_8	$(p^2qp^2h)^3$	$2^{16} \cdot 3^2$	1	2_8	2_8
2_9	$(phphq^2h)^3$	$2^{16} \cdot 3^2$	1	2_9	2_9
2_{10}	$(pqh)^6$	$2^{17} \cdot 3$	1	2_{10}	2_{10}
2_{11}	$(pqhq)^6$	$2^{17} \cdot 3$	1	2_{11}	2_{11}
2_{12}	$(pqph)^6$	$2^{15} \cdot 3^2$	1	2_{12}	2_{12}
2_{13}	$(p^2hq^2)^3$	$2^{15} \cdot 3^2$	1	2_{13}	2_{13}
2_{14}	$(p^2h^2qh)^4$	$2^{16} \cdot 3$	1	2_{14}	2_{14}
2_{15}	$(ph)^5$	$2^{11} \cdot 3^2 \cdot 5$	1	2_{15}	2_{15}
2_{16}	$(phqh)^5$	$2^{11} \cdot 3^2 \cdot 5$	1	2_{16}	2_{16}
2_{17}	$(p)^2$	$2^{13} \cdot 3$	1	2_{17}	2_{17}
2_{18}	$(pq^2)^6$	$2^{13} \cdot 3$	1	2_{18}	2_{18}
2_{19}	$(p^2qp^2qh)^3$	$2^{13} \cdot 3$	1	2_{19}	2_{19}
2_{20}	$(p^2qpqph)^3$	$2^{13} \cdot 3$	1	2_{20}	2_{20}
2_{21}	p^2q^3	$2^{12} \cdot 3$	1	2_{21}	2_{21}
2_{22}	$(pqpq^2h)^3$	$2^{12} \cdot 3$	1	2_{22}	2_{22}
2_{23}	p^3h^2qhqpq	2^{13}	1	2_{23}	2_{23}
2_{24}	$(p^2hph^2)^3$	$2^{10} \cdot 3$	1	2_{24}	2_{24}
3_1	h	$2^8 \cdot 3^4$	3_1	1	3_1
3_2	$(pqh)^4$	$2^9 \cdot 3^3$	3_2	1	3_2
3_3	$(q)^2$	$2^6 \cdot 3^3$	3_3	1	3_3
4_1	$(p^2qph)^6$	$2^{12} \cdot 3^3$	2_1	4_1	4_1
4_2	$(phq)^5$	$2^{11} \cdot 3^2 \cdot 5$	2_2	4_2	4_2
4_3	$(p^3hqh)^3$	$2^{11} \cdot 3^2 \cdot 5$	2_2	4_3	4_3
4_4	$(pq)^4$	$2^{12} \cdot 3$	2_1	4_4	4_4
4_5	$(pqh)^3$	$2^{11} \cdot 3$	2_{10}	4_5	4_5
4_6	$(q^2h)^3$	$2^{11} \cdot 3$	2_{10}	4_6	4_6
4_7	$(pqphq)^3$	$2^{11} \cdot 3$	2_{10}	4_7	4_7
4_8	$(pq^4h)^3$	$2^{11} \cdot 3$	2_{11}	4_8	4_8
4_9	$(pqphpqh)^3$	$2^{11} \cdot 3$	2_{11}	4_9	4_9
4_{10}	$(p^2qpq^2h^2)^3$	$2^{11} \cdot 3$	2_{10}	4_{10}	4_{10}
4_{11}	$(qh)^3$	$2^{10} \cdot 3$	2_6	4_{11}	4_{11}
4_{12}	$(qh^2)^3$	$2^{10} \cdot 3$	2_{10}	4_{12}	4_{12}
4_{13}	$(pqhq)^3$	$2^{10} \cdot 3$	2_{11}	4_{13}	4_{13}
4_{14}	$(p^2hqh)^3$	$2^{10} \cdot 3$	2_7	4_{14}	4_{14}
4_{15}	$(pq^2h^2)^3$	$2^{10} \cdot 3$	2_{10}	4_{15}	4_{15}
4_{16}	p^3hph	$2^{10} \cdot 3$	2_7	4_{16}	4_{16}
4_{17}	$(p^2hph^2q)^3$	$2^{10} \cdot 3$	2_2	4_{17}	4_{17}
4_{18}	$(p^2hphphq)^3$	$2^{10} \cdot 3$	2_6	4_{18}	4_{18}
4_{19}	$(p^2hphq)^2$	2^{11}	2_{11}	4_{19}	4_{19}
4_{20}	$(pqpqhq)^2$	2^{11}	2_{11}	4_{20}	4_{20}
4_{21}	$(pq^2)^3$	$2^9 \cdot 3$	2_{18}	4_{21}	4_{21}
4_{22}	$(p^3q)^3$	$2^9 \cdot 3$	2_{17}	4_{22}	4_{22}
4_{23}	$(pqph)^3$	$2^9 \cdot 3$	2_{12}	4_{23}	4_{23}
4_{24}	$(p^3qh)^3$	$2^9 \cdot 3$	2_{18}	4_{24}	4_{24}
4_{25}	$(p^2qhq)^3$	$2^9 \cdot 3$	2_{12}	4_{25}	4_{25}
4_{26}	$(p^2hph)^3$	$2^9 \cdot 3$	2_{17}	4_{26}	4_{26}
4_{27}	$(p^2h^2qh)^2$	2^{10}	2_{14}	4_{27}	4_{27}
4_{28}	p^3qph^2qh	2^{10}	2_{14}	4_{28}	4_{28}
4_{29}	p	2^9	2_{17}	4_{29}	4_{29}
4_{30}	pq^3	2^9	2_{18}	4_{30}	4_{30}
4_{31}	$phqh^2$	2^9	2_{17}	4_{31}	4_{31}
4_{32}	p^3qp^2h	2^9	2_{11}	4_{32}	4_{32}
4_{33}	$p^2q^2ph^2$	2^9	2_{10}	4_{33}	4_{33}
4_{34}	p^2qhqhq	2^9	2_{17}	4_{34}	4_{34}
4_{35}	p^2hqhq^2	2^9	2_{17}	4_{35}	4_{35}
4_{36}	$pqphqph$	2^9	2_{18}	4_{36}	4_{36}
4_{37}	pq^2hphqh	2^9	2_{17}	4_{37}	4_{37}
4_{38}	p^2hphpq^2h	2^9	2_{17}	4_{38}	4_{38}
4_{39}	$pq^2h^2q^2ph$	2^9	2_{10}	4_{39}	4_{39}
4_{40}	pq^2phq	2^8	2_{17}	4_{40}	4_{40}
4_{41}	$pqpqpqph$	2^8	2_{17}	4_{41}	4_{41}
5	$(ph)^2$	$2^4 \cdot 5$	5	5	1
6_1	q^3h	$2^8 \cdot 3^4$	3_1	2_4	6_5

Conjugacy classes of $H_2 = \langle p_2, q_2, h_2 \rangle$, *with subscripts dropped (continued)*

Class	*Representative*	*Centralizer*	2P	3P	5P
6_2	$(pqpqh)^3$	$2^8 \cdot 3^4$	3_1	2_5	6_3
6_3	$(pq^2ph)^3$	$2^8 \cdot 3^4$	3_1	2_5	6_2
6_4	$(phphq)^2$	$2^8 \cdot 3^4$	3_1	2_1	6_4
6_5	q^3h^2	$2^8 \cdot 3^4$	3_1	2_4	6_1
6_6	$(p^2qhqh)^3$	$2^8 \cdot 3^4$	3_1	2_3	6_6
6_7	$(pqhqph^2)^3$	$2^8 \cdot 3^4$	3_1	2_2	6_7
6_8	$(p^2qph)^4$	$2^9 \cdot 3^3$	3_2	2_1	6_8
6_9	$(p^3hqh)^2$	$2^9 \cdot 3^3$	3_2	2_2	6_9
6_{10}	$p^2hph^2pq^2$	$2^9 \cdot 3^3$	3_2	2_3	6_{10}
6_{11}	p^3qhpq	$2^8 \cdot 3^3$	3_2	2_5	6_{11}
6_{12}	$p^2hpqhpq$	$2^8 \cdot 3^3$	3_2	2_4	6_{12}
6_{13}	$(qh)^2$	$2^8 \cdot 3^2$	3_1	2_6	6_{13}
6_{14}	$(pqph)^2$	$2^8 \cdot 3^2$	3_1	2_{12}	6_{16}
6_{15}	$(p^2hqh)^2$	$2^8 \cdot 3^2$	3_1	2_7	6_{15}
6_{16}	$(pq^2ph^2)^2$	$2^8 \cdot 3^2$	3_1	2_{12}	6_{14}
6_{17}	$phqph^2$	$2^8 \cdot 3^2$	3_1	2_{13}	6_{18}
6_{18}	$phqhph^2$	$2^8 \cdot 3^2$	3_1	2_{13}	6_{17}
6_{19}	pq^3h^2phq	$2^8 \cdot 3^2$	3_1	2_8	6_{19}
6_{20}	$p^3hpqh^2q^2$	$2^8 \cdot 3^2$	3_1	2_9	6_{20}
6_{21}	$(pq^2hq^2)^2$	$2^6 \cdot 3^3$	3_3	2_2	6_{21}
6_{22}	p^2hphpq	$2^6 \cdot 3^3$	3_3	2_3	6_{22}
6_{23}	q^2hqhqh	$2^6 \cdot 3^3$	3_3	2_1	6_{23}
6_{24}	p^2qh^2	$2^7 \cdot 3^2$	3_2	2_6	6_{24}
6_{25}	p^2qp^2h	$2^7 \cdot 3^2$	3_2	2_8	6_{25}
6_{26}	p^2hphph	$2^7 \cdot 3^2$	3_2	2_7	6_{26}
6_{27}	$q^2h^2qhqh^2$	$2^7 \cdot 3^2$	3_2	2_9	6_{27}
6_{28}	q	$2^5 \cdot 3^3$	3_3	2_4	6_{28}
6_{29}	p^3qp^2hph	$2^5 \cdot 3^3$	3_3	2_5	6_{29}
6_{30}	$(pqh)^2$	$2^8 \cdot 3$	3_2	2_{10}	6_{30}
6_{31}	$(pqhq)^2$	$2^8 \cdot 3$	3_2	2_{11}	6_{31}
6_{32}	p^2hq^2	$2^6 \cdot 3^2$	3_2	2_{13}	6_{32}
6_{33}	$phqhqh^2$	$2^6 \cdot 3^2$	3_2	2_{15}	6_{33}
6_{34}	$ph^2q^2h^2$	$2^6 \cdot 3^2$	3_2	2_{16}	6_{34}
6_{35}	p^2qpq^2ph	$2^6 \cdot 3^2$	3_2	2_{12}	6_{35}
6_{36}	p^3h^2pq	$2^7 \cdot 3$	3_2	2_{14}	6_{36}
6_{37}	$qhqh^2$	$2^5 \cdot 3^2$	3_3	2_6	6_{37}
6_{38}	p^3qpq	$2^5 \cdot 3^2$	3_3	2_{13}	6_{41}
6_{39}	$pqhq^2ph$	$2^5 \cdot 3^2$	3_3	2_{12}	6_{43}
6_{40}	$phphq^2h$	$2^5 \cdot 3^2$	3_3	2_9	6_{40}
6_{41}	$p^2q^2h^2qh$	$2^5 \cdot 3^2$	3_3	2_{13}	6_{38}
6_{42}	$p^2qhphpq$	$2^5 \cdot 3^2$	3_3	2_7	6_{42}
6_{43}	$p^2hqphpq$	$2^5 \cdot 3^2$	3_3	2_{12}	6_{39}
6_{44}	pqh^2qhph	$2^5 \cdot 3^2$	3_3	2_8	6_{44}
6_{45}	p^2h	$2^6 \cdot 3$	3_3	2_{17}	6_{45}
6_{46}	$(pq^2)^2$	$2^6 \cdot 3$	3_3	2_{18}	6_{46}
6_{47}	p^2qp^2qh	$2^6 \cdot 3$	3_3	2_{19}	6_{47}
6_{48}	p^2qpqph	$2^6 \cdot 3$	3_3	2_{20}	6_{48}
6_{49}	p^2qpq	$2^4 \cdot 3^2$	3_3	2_{15}	6_{49}
6_{50}	p^2qhpq^2	$2^4 \cdot 3^2$	3_3	2_{16}	6_{50}
6_{51}	p^2q^3h	$2^5 \cdot 3$	3_3	2_{21}	6_{51}
6_{52}	p^2hph^2	$2^5 \cdot 3$	3_2	2_{24}	6_{52}
6_{53}	$pqpq^2h$	$2^5 \cdot 3$	3_3	2_{22}	6_{53}
8_1	$(p^2qph)^3$	$2^7 \cdot 3$	4_1	8_1	8_1
8_2	$(pqhqhq)^3$	$2^7 \cdot 3$	4_1	8_2	8_2
8_3	$(pq)^2$	2^7	4_4	8_3	8_3
8_4	$pqpqhq$	2^7	4_{20}	8_4	8_4
8_5	pqh^2qh	2^7	4_{20}	8_5	8_5
8_6	p^3hqph	2^7	4_{20}	8_6	8_6
8_7	$pqphphq$	2^7	4_{20}	8_7	8_7
8_8	pq^2hq	2^6	4_{11}	8_8	8_8
8_9	p^2qpq^2	2^6	4_{11}	8_9	8_9
8_{10}	p^2q^2ph	2^6	4_{16}	8_{10}	8_{10}
8_{11}	p^2hphq	2^6	4_{19}	8_{11}	8_{11}
8_{12}	p^2h^2qh	2^6	4_{27}	8_{12}	8_{12}
8_{13}	p^3qphq	2^6	4_{27}	8_{13}	8_{13}
8_{14}	$pqpqpqh$	2^6	4_{16}	8_{14}	8_{14}
9	$(pqpqh)^2$	$2^3 \cdot 3^2$	9	3_1	9
10_1	p^2q	$2^4 \cdot 5$	5	10_1	2_2
10_2	p^2qh	$2^4 \cdot 5$	5	10_2	2_1
10_3	p^2h^2q	$2^4 \cdot 5$	5	10_3	2_3
10_4	ph	$2^3 \cdot 5$	5	10_4	2_{15}

Conjugacy classes of $H_2 = \langle p_2, q_2, h_2 \rangle$, *with subscripts dropped (continued)*

Class	*Representative*	|*Centralizer*|	2P	3P	5P
10_5	p^2q^2	$2^3 \cdot 5$	5	10_5	2_4
10_6	$phqh$	$2^3 \cdot 5$	5	10_6	2_{16}
10_7	p^2q^2h	$2^3 \cdot 5$	5	10_7	2_5
12_1	$(p^2qph)^2$	$2^7 \cdot 3^2$	6_8	4_1	12_1
12_2	$(pqhqhq)^2$	$2^7 \cdot 3^2$	6_8	4_1	12_2
12_3	p^3hpq	$2^5 \cdot 3^3$	6_4	4_1	12_3
12_4	p^3hqh	$2^6 \cdot 3^2$	6_9	4_3	12_4
12_5	ph^2qhqh	$2^6 \cdot 3^2$	6_9	4_2	12_5
12_6	p^2h^2pqhpq	$2^6 \cdot 3^2$	6_8	4_1	12_6
12_7	pqh	$2^6 \cdot 3$	6_{30}	4_5	12_7
12_8	q^2h	$2^6 \cdot 3$	6_{30}	4_6	12_8
12_9	$pqphq$	$2^6 \cdot 3$	6_{30}	4_7	12_9
12_{10}	pq^4h	$2^6 \cdot 3$	6_{31}	4_8	12_{10}
12_{11}	$pqphpqh$	$2^6 \cdot 3$	6_{31}	4_9	12_{11}
12_{12}	$p^2qpq^2h^2$	$2^6 \cdot 3$	6_{30}	4_{10}	12_{12}
12_{13}	pq^2hq^2	$2^4 \cdot 3^2$	6_{21}	4_2	12_{13}
12_{14}	$pqpqhpq^2$	$2^4 \cdot 3^2$	6_{21}	4_3	12_{14}
12_{15}	qh	$2^5 \cdot 3$	6_{13}	4_{11}	12_{15}
12_{16}	qh^2	$2^5 \cdot 3$	6_{30}	4_{12}	12_{16}
12_{17}	$pqph$	$2^5 \cdot 3$	6_{14}	4_{23}	12_{24}
12_{18}	$pqhq$	$2^5 \cdot 3$	6_{31}	4_{13}	12_{18}
12_{19}	p^2qhq	$2^5 \cdot 3$	6_{14}	4_{25}	12_{23}
12_{20}	p^2hqh	$2^5 \cdot 3$	6_{15}	4_{14}	12_{20}
12_{21}	pq^2h^2	$2^5 \cdot 3$	6_{30}	4_{15}	12_{21}
12_{22}	$phphq$	$2^5 \cdot 3$	6_4	4_4	12_{22}
12_{23}	pq^2ph^2	$2^5 \cdot 3$	6_{16}	4_{25}	12_{19}
12_{24}	$p^2q^2p^2h$	$2^5 \cdot 3$	6_{16}	4_{23}	12_{17}
12_{25}	p^2hph^2q	$2^5 \cdot 3$	6_9	4_{17}	12_{25}
12_{26}	p^3qh^2pq	$2^5 \cdot 3$	6_{15}	4_{16}	12_{26}
12_{27}	$p^2hphphq$	$2^5 \cdot 3$	6_{13}	4_{18}	12_{27}
12_{28}	pq^2	$2^4 \cdot 3$	6_{46}	4_{21}	12_{28}
12_{29}	p^3q	$2^4 \cdot 3$	6_{45}	4_{22}	12_{29}
12_{30}	p^3qh	$2^4 \cdot 3$	6_{46}	4_{24}	12_{30}
12_{31}	p^2hph	$2^4 \cdot 3$	6_{45}	4_{26}	12_{31}
16_1	pq	2^5	8_3	16_1	16_2
16_2	p^3q^2	2^5	8_3	16_2	16_1
18_1	$pqpqh$	$2^3 \cdot 3^2$	9	6_2	18_3
18_2	$pqph^2$	$2^3 \cdot 3^2$	9	6_1	18_5
18_3	pq^2ph	$2^3 \cdot 3^2$	9	6_3	18_1
18_4	p^2qhqh	$2^3 \cdot 3^2$	9	6_6	18_4
18_5	pq^2pqh	$2^3 \cdot 3^2$	9	6_5	18_2
18_6	$pqhqph^2$	$2^3 \cdot 3^2$	9	6_7	18_6
18_7	p^3hqphq	$2^3 \cdot 3^2$	9	6_4	18_7
20_1	phq	$2^3 \cdot 5$	10_1	20_1	4_2
20_2	phq^2hq	$2^3 \cdot 5$	10_1	20_2	4_3
24_1	p^2qph	$2^4 \cdot 3$	12_1	8_1	24_1
24_2	$pqhqhq$	$2^4 \cdot 3$	12_2	8_2	24_2

A.4. *Conjugacy classes of $D_2 = \langle p_2, q_2 \rangle \cong \langle p_1, q_1 \rangle$, with subscripts dropped*

Class	*Representative*	$\lvert$*Centralizer*$\rvert$	2P	3P	5P
1	1	$2^{18} \cdot 3 \cdot 5$	1	1	1
2_1	$(pq)^8$	$2^{18} \cdot 3 \cdot 5$	1	2_1	2_1
2_2	$(p^2q)^5$	$2^{18} \cdot 3 \cdot 5$	1	2_2	2_2
2_3	$(p^2qpq^2pq^2)^5$	$2^{18} \cdot 3 \cdot 5$	1	2_3	2_3
2_4	$(q)^3$	$2^{17} \cdot 3 \cdot 5$	1	2_4	2_4
2_5	$(p^3q^2p^2qpq)^3$	$2^{17} \cdot 3 \cdot 5$	1	2_5	2_5
2_6	$(p^2qpq^2)^4$	$2^{16} \cdot 3$	1	2_6	2_6
2_7	$(p^3qpq^4)^3$	$2^{16} \cdot 3$	1	2_7	2_7
2_8	$(p^3qp^2qpqpq)^4$	$2^{16} \cdot 3$	1	2_8	2_8
2_9	$(p^3qpqp^2q^4)^3$	$2^{16} \cdot 3$	1	2_9	2_9
2_{10}	$(p^3qp^2q)^4$	2^{17}	1	2_{10}	2_{10}
2_{11}	$(p^2qpq^2pqp^2q^2)^2$	2^{17}	1	2_{11}	2_{11}
2_{12}	$(p^3qpq)^3$	$2^{15} \cdot 3$	1	2_{12}	2_{12}
2_{13}	$(p^3qpqp^2q)^3$	$2^{15} \cdot 3$	1	2_{13}	2_{13}
2_{14}	$(p^3qp^2q^2pq)^4$	2^{16}	1	2_{14}	2_{14}
2_{15}	$(pq^2)^6$	$2^{13} \cdot 3$	1	2_{15}	2_{15}
2_{16}	$(p^3q)^6$	$2^{13} \cdot 3$	1	2_{16}	2_{16}
2_{17}	$(pqpqpqpq^2)^3$	$2^{13} \cdot 3$	1	2_{17}	2_{17}
2_{18}	$(p^3qpq^2pqpq^4)^3$	$2^{13} \cdot 3$	1	2_{18}	2_{18}
2_{19}	$(p^3q^2pqpq)^4$	2^{14}	1	2_{19}	2_{19}
2_{20}	$(p^3q^2pqpq^2)^2$	2^{14}	1	2_{20}	2_{20}
2_{21}	$(pq^2pq^5)^3$	$2^{12} \cdot 3$	1	2_{21}	2_{21}
2_{22}	$(p^3qpq^2pqpq)^3$	$2^{12} \cdot 3$	1	2_{22}	2_{22}
2_{23}	$(p^2q^2pqpqpq^2pq^2)^2$	2^{13}	1	2_{23}	2_{23}
2_{24}	$p^3qpqp^2qpq^2pqpqpq^2$	2^{13}	1	2_{24}	2_{24}
2_{25}	$(p^2qpq)^3$	$2^{11} \cdot 3$	1	2_{25}	2_{25}
2_{26}	$(p^3q^2pq^2pq^2)^3$	$2^{11} \cdot 3$	1	2_{26}	2_{26}
2_{27}	$(p)^2$	2^{12}	1	2_{27}	2_{27}
2_{28}	$(pq^3)^2$	2^{12}	1	2_{28}	2_{28}
2_{29}	$p^3qp^2qpqpq^2pqp^2q$	2^{12}	1	2_{29}	2_{29}
2_{30}	$p^3qpqpqpqpqpqpq$	2^{12}	1	2_{30}	2_{30}
2_{31}	$p^3qpq^2pqp^2qpqp^2q^2p^2q$	2^{12}	1	2_{31}	2_{31}
2_{32}	p^2q^3	2^{11}	1	2_{32}	2_{32}
2_{33}	$p^3q^2pqpq^2pqp^2q$	2^{11}	1	2_{33}	2_{33}
2_{34}	$p^3qpqpqpq^2pqpqp^3q^2$	2^{10}	1	2_{34}	2_{34}
3	$(q)^2$	$2^6 \cdot 3$	3	1	3
4_1	$(p^2qpq^4)^3$	$2^{11} \cdot 3$	2_2	4_1	4_1
4_2	$(p^3qp^3qpq)^3$	$2^{11} \cdot 3$	2_2	4_2	4_2
4_3	$(pq)^4$	2^{12}	2_1	4_3	4_3
4_4	$(pqpq^2pq^2)^2$	2^{12}	2_1	4_4	4_4
4_5	$(p^3qp^2q)^2$	2^{11}	2_{10}	4_5	4_5
4_6	$(p^2q^2pqpqpq^2)^2$	2^{11}	2_{10}	4_6	4_6
4_7	$p^3q^2pq^2pq^2pqpq$	2^{11}	2_{10}	4_7	4_7
4_8	$p^3qpq^2pqp^2qp^2q^2$	2^{11}	2_{11}	4_8	4_8
4_9	$p^3qp^2qp^3qpqp^2q^2$	2^{11}	2_{10}	4_9	4_9
4_{10}	$p^3qp^2q^2p^3qpqp^2q$	2^{11}	2_{11}	4_{10}	4_{10}
4_{11}	$p^3qpqpq^2p^3q^2p^2q$	2^{11}	2_{11}	4_{11}	4_{11}
4_{12}	$p^3qp^2qpqp^2q^2p^3qpq$	2^{11}	2_{11}	4_{12}	4_{12}
4_{13}	$(pq^2)^3$	$2^9 \cdot 3$	2_{15}	4_{13}	4_{13}
4_{14}	$(p^3q)^3$	$2^9 \cdot 3$	2_{16}	4_{14}	4_{14}
4_{15}	$(p^3qpq^2pq)^3$	$2^9 \cdot 3$	2_{15}	4_{15}	4_{15}
4_{16}	$(p^3qp^3qpq^2)^3$	$2^9 \cdot 3$	2_{16}	4_{16}	4_{16}
4_{17}	$(p^2qpq^2)^2$	2^{10}	2_6	4_{17}	4_{17}
4_{18}	$(p^3qpqpq^2)^2$	2^{10}	2_{10}	4_{18}	4_{18}
4_{19}	$(p^3qp^2q^2pq)^2$	2^{10}	2_{14}	4_{19}	4_{19}
4_{20}	$(p^3qp^2qpqpq)^2$	2^{10}	2_8	4_{20}	4_{20}
4_{21}	$p^2qpqp^2qpq^2$	2^{10}	2_8	4_{21}	4_{21}
4_{22}	$p^3qp^2qpqpq^2pq^2$	2^{10}	2_{14}	4_{22}	4_{22}
4_{23}	$p^3qpqpqp^2qpq^4$	2^{10}	2_{10}	4_{23}	4_{23}
4_{24}	$p^3qpqpqpq^2pqp^2q$	2^{10}	2_{11}	4_{24}	4_{24}
4_{25}	$p^3qp^2q^2p^3q^2pqpq$	2^{10}	2_{11}	4_{25}	4_{25}
4_{26}	$p^3qpq^2pqp^3q^2pq^2$	2^{10}	2_{10}	4_{26}	4_{26}
4_{27}	$p^2qp^2q^2pq^2pq^2pqp^2q^2$	2^{10}	2_2	4_{27}	4_{27}
4_{28}	$p^3q^2pqp^2q^2p^2q^2pqpq^2$	2^{10}	2_6	4_{28}	4_{28}
4_{29}	$(p^3q^2pqpq)^2$	2^9	2_{19}	4_{29}	4_{29}
4_{30}	$(p^2qpq^2p^2q^2)^2$	2^9	2_{19}	4_{30}	4_{30}
4_{31}	$p^3q^2pq^2p^2q$	2^9	2_{16}	4_{31}	4_{31}
4_{32}	p^2qpqpq^5	2^9	2_{16}	4_{32}	4_{32}
4_{33}	$p^3qpqpqp^2q^2$	2^9	2_{15}	4_{33}	4_{33}
4_{34}	$p^2qpq^2pqp^2q^2$	2^9	2_{11}	4_{34}	4_{34}
4_{35}	$p^3q^2p^2q^2pqpq$	2^9	2_{16}	4_{35}	4_{35}

Conjugacy classes of $D_2 = \langle p_2, q_2 \rangle$, with subscripts dropped (continued)

Class	*Representative*	*Centralizer*	2P	3P	5P
4_{36}	$p^2q^2pqpqpq^2$	2^9	2_{14}	4_{36}	4_{36}
4_{37}	$p^3qp^3q^2pqp^2q$	2^9	2_{16}	4_{37}	4_{37}
4_{38}	$p^3qp^2q^2pq^2p^2q$	2^9	2_{13}	4_{38}	4_{38}
4_{39}	$p^3q^2pqpqpqpq$	2^9	2_{16}	4_{39}	4_{39}
4_{40}	$p^3qp^3qpq^2pqpq^2$	2^9	2_{15}	4_{40}	4_{40}
4_{41}	$p^3qp^2qpqp^2qp^2q^2$	2^9	2_{14}	4_{41}	4_{41}
4_{42}	$p^3q^2pqpqp^2qpqpq$	2^9	2_{11}	4_{42}	4_{42}
4_{43}	$p^3qp^2q^2pq^2pq^3pq$	2^9	2_{13}	4_{43}	4_{43}
4_{44}	$p^2q^2pqp^2q^2pqpq^2pq$	2^9	2_{16}	4_{44}	4_{44}
4_{45}	$p^3qp^2q^2p^2qpqpqp^2q$	2^9	2_{10}	4_{45}	4_{45}
4_{46}	p^2qpqpq^2	2^8	2_{16}	4_{46}	4_{46}
4_{47}	$p^3q^2pqpq^2$	2^8	2_{20}	4_{47}	4_{47}
4_{48}	$p^3qpqp^3q^2$	2^8	2_{19}	4_{48}	4_{48}
4_{49}	$p^2qpqpq^2p^2q^2$	2^8	2_{20}	4_{49}	4_{49}
4_{50}	$p^2qpqpqp^2qpq^2$	2^8	2_{20}	4_{50}	4_{50}
4_{51}	$p^2q^2pqpqpq^2pq^2$	2^8	2_{23}	4_{51}	4_{51}
4_{52}	$p^3qpqp^3qpqpqpq$	2^8	2_{16}	4_{52}	4_{52}
4_{53}	$p^2qp^2q^2pqpqp^2qpq$	2^8	2_{19}	4_{53}	4_{53}
4_{54}	$p^2qpqp^2q^2pqpqpq^2$	2^8	2_{23}	4_{54}	4_{54}
4_{55}	p	2^7	2_{27}	4_{55}	4_{55}
4_{56}	pq^3	2^7	2_{28}	4_{56}	4_{56}
4_{57}	$p^2qpqpqpq^2$	2^7	2_{27}	4_{57}	4_{57}
4_{58}	$p^2q^2p^2q^2pqpq$	2^7	2_{20}	4_{58}	4_{58}
4_{59}	$p^2qpqpqpq^5$	2^7	2_{28}	4_{59}	4_{59}
5	$(p^2q)^2$	$2^3 \cdot 5$	5	5	1
6_1	$(pq^2)^2$	$2^6 \cdot 3$	3	2_{15}	6_1
6_2	$(p^3q)^2$	$2^6 \cdot 3$	3	2_{16}	6_2
6_3	p^3qpq^3	$2^6 \cdot 3$	3	2_2	6_3
6_4	$pqpqpqpq^2$	$2^6 \cdot 3$	3	2_{17}	6_4
6_5	$p^3q^2p^2qpq^2$	$2^6 \cdot 3$	3	2_3	6_5
6_6	$p^3q^2p^2q^4pq$	$2^6 \cdot 3$	3	2_1	6_6
6_7	$p^3qpq^2pqpq^4$	$2^6 \cdot 3$	3	2_{18}	6_7
6_8	q	$2^5 \cdot 3$	3	2_4	6_8
6_9	p^3qpq	$2^5 \cdot 3$	3	2_{12}	6_{10}
6_{10}	p^3qp^2qpq	$2^5 \cdot 3$	3	2_{12}	6_9
6_{11}	p^3qpqp^2q	$2^5 \cdot 3$	3	2_{13}	6_{19}
6_{12}	p^3qpq^4	$2^5 \cdot 3$	3	2_7	6_{12}
6_{13}	p^3q^4pq	$2^5 \cdot 3$	3	2_6	6_{13}
6_{14}	pq^2pq^5	$2^5 \cdot 3$	3	2_{21}	6_{14}
6_{15}	$p^3q^2p^2qpq$	$2^5 \cdot 3$	3	2_5	6_{15}
6_{16}	p^3qpq^2pqpq	$2^5 \cdot 3$	3	2_{22}	6_{16}
6_{17}	$p^3qpqp^2q^4$	$2^5 \cdot 3$	3	2_9	6_{17}
6_{18}	p^3qpqpq^3pq	$2^5 \cdot 3$	3	2_8	6_{18}
6_{19}	$p^3q^2p^2qpqp^2q$	$2^5 \cdot 3$	3	2_{13}	6_{11}
6_{20}	p^2qpq	$2^4 \cdot 3$	3	2_{25}	6_{20}
6_{21}	$p^3q^2pq^2pq^2$	$2^4 \cdot 3$	3	2_{26}	6_{21}
8_1	$(pq)^2$	2^7	4_3	8_1	8_1
8_2	p^3qp^2q	2^7	4_5	8_2	8_2
8_3	p^2q^2pqpq	2^7	4_5	8_3	8_3
8_4	$pqpq^2pq^2$	2^7	4_4	8_4	8_4
8_5	$p^2qp^2qpq^2$	2^7	4_4	8_5	8_5
8_6	$p^2q^2pqpq^4$	2^7	4_5	8_6	8_6
8_7	$p^3qpqpq^2p^2q^2$	2^7	4_5	8_7	8_7
8_8	p^2qpq^2	2^6	4_{17}	8_8	8_8
8_9	p^3qpq^2	2^6	4_3	8_9	8_9
8_{10}	p^3qpqpq^2	2^6	4_{18}	8_{16}	8_{10}
8_{11}	$p^2qpqpqpq$	2^6	4_{17}	8_{11}	8_{11}
8_{12}	$p^3qp^2q^2pq$	2^6	4_{19}	8_{12}	8_{12}
8_{13}	p^3qp^2qpqpq	2^6	4_{20}	8_{13}	8_{13}
8_{14}	$p^3qp^2q^2pq^2$	2^6	4_{19}	8_{14}	8_{14}
8_{15}	$p^2q^2pqpqpq^2$	2^6	4_6	8_{15}	8_{15}
8_{16}	p^3qpqpq^5	2^6	4_{18}	8_{10}	8_{16}
8_{17}	$p^3qpq^2p^2qpq$	2^6	4_{20}	8_{17}	8_{17}
8_{18}	p^3q^2pqpq	2^5	4_{29}	8_{18}	8_{18}
8_{19}	$p^2qpq^2p^2q^2$	2^5	4_{30}	8_{19}	8_{19}
10_1	p^2q	$2^3 \cdot 5$	5	10_1	2_2
10_2	p^2q^2	$2^3 \cdot 5$	5	10_3	2_4
10_3	$pqpq^3$	$2^3 \cdot 5$	5	10_2	2_4
10_4	$p^2qpq^2pq^2$	$2^3 \cdot 5$	5	10_4	2_3
10_5	$p^2q^2pq^2pq^2$	$2^3 \cdot 5$	5	10_5	2_1
10_6	$p^3qp^3qp^2q$	$2^3 \cdot 5$	5	10_7	2_5
10_7	$p^3q^2pqpqpq$	$2^3 \cdot 5$	5	10_6	2_5

Conjugacy classes of $D_2 = \langle p_2, q_2 \rangle$, with subscripts dropped (continued)

Class	*Representative*	*Centralizer*	2P	3P	5P
12_1	pq^2	$2^4 \cdot 3$	6_1	4_{13}	12_1
12_2	p^3q	$2^4 \cdot 3$	6_2	4_{14}	12_2
12_3	p^2qpq^4	$2^4 \cdot 3$	6_3	4_1	12_3
12_4	p^3qpq^2pq	$2^4 \cdot 3$	6_1	4_{15}	12_4
12_5	p^3qp^3qpq	$2^4 \cdot 3$	6_3	4_2	12_5
12_6	$p^3qp^3qpq^2$	$2^4 \cdot 3$	6_2	4_{16}	12_6
16_1	pq	2^5	8_1	16_1	16_2
16_2	p^3q^2	2^5	8_1	16_2	16_1

A.5. *Conjugacy classes of* $H(\mathrm{Fi}_{23}) = H = \langle x_0, y_0, h_0\rangle$*, with subscripts dropped*

Class	*Representative*	*Centralizer*	2P	3P	5P	7P	11P	13P
1	1	$2^{18}\cdot 3^9\cdot 5^2\cdot 7\cdot 11\cdot 13$	1	1	1	1	1	1
2_1	$(xy^2)^7$	$2^{18}\cdot 3^9\cdot 5^2\cdot 7\cdot 11\cdot 13$	1	2_1	2_1	2_1	2_1	2_1
2_2	$(y)^7$	$2^{17}\cdot 3^6\cdot 5\cdot 7\cdot 11$	1	2_2	2_2	2_2	2_2	2_2
2_3	$(xyhy)^{11}$	$2^{17}\cdot 3^6\cdot 5\cdot 7\cdot 11$	1	2_3	2_3	2_3	2_3	2_3
2_4	x	$2^{18}\cdot 3^4\cdot 5$	1	2_4	2_4	2_4	2_4	2_4
2_5	$(yh)^{10}$	$2^{18}\cdot 3^4\cdot 5$	1	2_5	2_5	2_5	2_5	2_5
2_6	$(xh)^6$	$2^{16}\cdot 3^3$	1	2_6	2_6	2_6	2_6	2_6
3_1	$(xyxh)^{10}$	$2^9\cdot 3^7\cdot 5\cdot 7$	3_1	1	3_1	3_1	3_1	3_1
3_2	h	$2^8\cdot 3^9$	3_2	1	3_2	3_2	3_2	3_2
3_3	$(xyxy^3)^4$	$2^7\cdot 3^7$	3_3	1	3_3	3_3	3_3	3_3
3_4	$(xyhyh)^6$	$2^4\cdot 3^7$	3_4	1	3_4	3_4	3_4	3_4
4_1	$(xy^2xh)^3$	$2^{12}\cdot 3^3$	2_4	4_1	4_1	4_1	4_1	4_1
4_2	$(yh)^5$	$2^{11}\cdot 3^2\cdot 5$	2_5	4_2	4_2	4_2	4_2	4_2
4_3	$(xhy)^5$	$2^{11}\cdot 3^2\cdot 5$	2_5	4_3	4_3	4_3	4_3	4_3
4_4	$(y^2h)^4$	$2^{12}\cdot 3$	2_4	4_4	4_4	4_4	4_4	4_4
4_5	$(xh)^3$	$2^{10}\cdot 3^2$	2_6	4_5	4_5	4_5	4_5	4_5
4_6	$(xyh)^2$	$2^{10}\cdot 3^2$	2_6	4_6	4_6	4_6	4_6	4_6
4_7	$(xh^2y^3)^3$	$2^{10}\cdot 3$	2_5	4_7	4_7	4_7	4_7	4_7
5	$(yh)^4$	$2^4\cdot 3\cdot 5^2$	5	5	1	5	5	5
6_1	$(y^3h^2)^5$	$2^9\cdot 3^7\cdot 5\cdot 7$	3_1	2_1	6_1	6_1	6_1	6_1
6_2	$(xy^2h^2yhy)^3$	$2^8\cdot 3^9$	3_2	2_1	6_2	6_2	6_2	6_2
6_3	$(xyxh)^5$	$2^8\cdot 3^5\cdot 5$	3_1	2_2	6_3	6_3	6_3	6_3
6_4	$(xhy^3)^5$	$2^8\cdot 3^5\cdot 5$	3_1	2_3	6_4	6_4	6_4	6_4
6_5	$xyh^2yh^2xh^2xh$	$2^7\cdot 3^7$	3_3	2_1	6_5	6_5	6_5	6_5
6_6	$(xyxyh)^3$	$2^8\cdot 3^6$	3_2	2_3	6_6	6_6	6_6	6_6
6_7	$(xyxhy)^3$	$2^8\cdot 3^6$	3_2	2_2	6_7	6_7	6_7	6_7
6_8	$(xyhyh)^3$	$2^4\cdot 3^7$	3_4	2_1	6_8	6_8	6_8	6_8
6_9	$(xy^2xh)^2$	$2^8\cdot 3^4$	3_2	2_4	6_9	6_9	6_9	6_9
6_{10}	$(xy^2xyh)^3$	$2^8\cdot 3^4$	3_2	2_5	6_{10}	6_{10}	6_{10}	6_{10}
6_{11}	$(yhyh^2)^4$	$2^9\cdot 3^3$	3_1	2_4	6_{11}	6_{11}	6_{11}	6_{11}
6_{12}	$(xh^2y^3)^2$	$2^9\cdot 3^3$	3_1	2_5	6_{12}	6_{12}	6_{12}	6_{12}
6_{13}	y^7h	$2^5\cdot 3^5$	3_3	2_2	6_{13}	6_{13}	6_{13}	6_{13}
6_{14}	$xyxyxhyxhy^2$	$2^5\cdot 3^5$	3_3	2_3	6_{14}	6_{14}	6_{14}	6_{14}
6_{15}	$(xh)^2$	$2^8\cdot 3^3$	3_2	2_6	6_{15}	6_{15}	6_{15}	6_{15}
6_{16}	$(xhxhy^2h)^2$	$2^8\cdot 3^3$	3_2	2_6	6_{16}	6_{16}	6_{16}	6_{16}
6_{17}	$xyhxhxh$	$2^7\cdot 3^3$	3_3	2_6	6_{17}	6_{17}	6_{17}	6_{17}
6_{18}	$(xy^2xhyhy)^2$	$2^7\cdot 3^3$	3_3	2_6	6_{18}	6_{18}	6_{18}	6_{18}
6_{19}	$xyxy^2xyxyhy^2$	$2^7\cdot 3^3$	3_1	2_6	6_{19}	6_{19}	6_{19}	6_{19}
6_{20}	$(xyxy^3)^2$	$2^6\cdot 3^3$	3_3	2_5	6_{20}	6_{20}	6_{20}	6_{20}
6_{21}	xh^2yh^2y	$2^6\cdot 3^3$	3_3	2_4	6_{21}	6_{21}	6_{21}	6_{21}
6_{22}	$xyxhy^2xyh$	$2^5\cdot 3^3$	3_3	2_6	6_{22}	6_{22}	6_{22}	6_{22}
6_{23}	$xyxh^2yhyh$	$2^5\cdot 3^3$	3_3	2_6	6_{23}	6_{23}	6_{23}	6_{23}
6_{24}	$(xyxyhyh)^2$	$2^4\cdot 3^3$	3_4	2_6	6_{24}	6_{24}	6_{24}	6_{24}
6_{25}	xyh^2y^2xh	$2^4\cdot 3^3$	3_4	2_6	6_{25}	6_{25}	6_{25}	6_{25}
7	$(y)^2$	$2^2\cdot 3\cdot 7$	7	7	7	1	7	7
8_1	$(yhyh^2)^3$	$2^7\cdot 3$	4_1	8_1	8_1	8_1	8_1	8_1
8_2	$(xyhy^2h)^3$	$2^7\cdot 3$	4_1	8_2	8_2	8_2	8_2	8_2
8_3	$(y^2h)^2$	2^7	4_4	8_3	8_3	8_3	8_3	8_3
8_4	xyh	2^6	4_6	8_4	8_4	8_4	8_4	8_4
8_5	xyh^2yh	2^6	4_6	8_5	8_5	8_5	8_5	8_5
9_1	$(xyxyh)^2$	$2^3\cdot 3^4$	9_1	3_2	9_1	9_1	9_1	9_1
9_2	$(xyhy^3)^2$	$2^2\cdot 3^4$	9_2	3_2	9_2	9_2	9_2	9_2
9_3	$(xyhyh)^2$	$2\cdot 3^3$	9_3	3_4	9_3	9_3	9_3	9_3
10_1	$(y^3h^2)^3$	$2^4\cdot 3\cdot 5^2$	5	10_1	2_1	10_1	10_1	10_1
10_2	yh^2	$2^3\cdot 3\cdot 5$	5	10_2	2_2	10_2	10_2	10_2
10_3	$(xhy^3)^3$	$2^3\cdot 3\cdot 5$	5	10_3	2_3	10_3	10_3	10_3
10_4	$(yh)^2$	$2^4\cdot 5$	5	10_4	2_5	10_4	10_4	10_4
10_5	xh^2y	$2^4\cdot 5$	5	10_5	2_4	10_5	10_5	10_5
11_1	$(xy^3)^2$	$2^2\cdot 11$	11_2	11_1	11_1	11_2	1	11_2
11_2	$(xy^3)^4$	$2^2\cdot 11$	11_1	11_2	11_2	11_1	1	11_1
12_1	$(yhyh^2)^2$	$2^7\cdot 3^2$	6_{11}	4_1	12_1	12_1	12_1	12_1
12_2	$(xyhy^2h)^2$	$2^7\cdot 3^2$	6_{11}	4_1	12_2	12_2	12_2	12_2
12_3	xy^2xh	$2^5\cdot 3^3$	6_9	4_1	12_3	12_3	12_3	12_3
12_4	xy^2xy^3h	$2^6\cdot 3^2$	6_{12}	4_3	12_4	12_4	12_4	12_4
12_5	$xyxy^3xh^2$	$2^6\cdot 3^2$	6_{12}	4_2	12_5	12_5	12_5	12_5
12_6	$xy^2xhxhyhy$	$2^6\cdot 3^2$	6_{11}	4_1	12_6	12_6	12_6	12_6
12_7	xh	$2^5\cdot 3^2$	6_{15}	4_5	12_7	12_7	12_7	12_7
12_8	$xyxh^2y$	$2^5\cdot 3^2$	6_{15}	4_6	12_8	12_8	12_8	12_8
12_9	$xhxhy^2h$	$2^5\cdot 3^2$	6_{16}	4_6	12_9	12_9	12_9	12_9
12_{10}	xy^2xhyhy	$2^5\cdot 3^2$	6_{18}	4_5	12_{10}	12_{10}	12_{10}	12_{10}
12_{11}	$xhxh^2y^4$	$2^5\cdot 3^2$	6_{18}	4_6	12_{11}	12_{11}	12_{11}	12_{11}

Conjugacy classes of $H = \langle x_0, y_0, h_0 \rangle$, with subscripts dropped (continued)

Class	*Representative*	*Centralizer*	2P	3P	5P	7P	11P	13P
12_{12}	xhy^4hyh	$2^5 \cdot 3^2$	6_{16}	4_5	12_{12}	12_{12}	12_{12}	12_{12}
12_{13}	$xyxy^3$	$2^4 \cdot 3^2$	6_{20}	4_2	12_{13}	12_{13}	12_{13}	12_{13}
12_{14}	$xyxhy^3h$	$2^4 \cdot 3^2$	6_{20}	4_3	12_{14}	12_{14}	12_{14}	12_{14}
12_{15}	xh^2y^3	$2^5 \cdot 3$	6_{12}	4_7	12_{15}	12_{15}	12_{15}	12_{15}
12_{16}	xy^3hy^2h	$2^5 \cdot 3$	6_9	4_4	12_{16}	12_{16}	12_{16}	12_{16}
12_{17}	$xyxyhyh$	$2^3 \cdot 3^2$	6_{24}	4_6	12_{17}	12_{17}	12_{17}	12_{17}
12_{18}	xy^3h^2xh	$2^3 \cdot 3^2$	6_{24}	4_5	12_{18}	12_{18}	12_{18}	12_{18}
13_1	$(xy^3h)^2$	$2 \cdot 13$	13_2	13_1	13_2	13_2	13_2	1
13_2	$(xy^3h)^4$	$2 \cdot 13$	13_1	13_2	13_1	13_1	13_1	1
14_1	xy^2	$2^2 \cdot 3 \cdot 7$	7	14_1	14_1	2_1	14_1	14_1
14_2	y	$2^2 \cdot 7$	7	14_2	14_2	2_2	14_2	14_2
14_3	xh^2y^2	$2^2 \cdot 7$	7	14_3	14_3	2_3	14_3	14_3
15	$(xyxh)^2$	$2^2 \cdot 3 \cdot 5$	15	5	3_1	15	15	15
16_1	y^2h	2^5	8_3	16_1	16_2	16_2	16_1	16_2
16_2	$xyxyhy$	2^5	8_3	16_2	16_1	16_1	16_2	16_1
18_1	xhy^2hy^2h	$2^3 \cdot 3^4$	9_1	6_2	18_1	18_1	18_1	18_1
18_2	xy^2h^2yhy	$2^2 \cdot 3^4$	9_2	6_2	18_2	18_2	18_2	18_2
18_3	$xyxyh$	$2^3 \cdot 3^3$	9_1	6_6	18_5	18_3	18_5	18_3
18_4	$xyxhy$	$2^3 \cdot 3^3$	9_1	6_7	18_6	18_4	18_6	18_4
18_5	xy^2xhy	$2^3 \cdot 3^3$	9_1	6_6	18_3	18_5	18_3	18_5
18_6	xhy^3h^2	$2^3 \cdot 3^3$	9_1	6_7	18_4	18_6	18_4	18_6
18_7	$xyhy^3$	$2^2 \cdot 3^3$	9_2	6_6	18_7	18_7	18_7	18_7
18_8	$xyxh^2xh$	$2^2 \cdot 3^3$	9_2	6_7	18_8	18_8	18_8	18_8
18_9	xy^2xyh	$2^3 \cdot 3^2$	9_1	6_{10}	18_9	18_9	18_9	18_9
18_{10}	xh^2yhy	$2^3 \cdot 3^2$	9_1	6_9	18_{10}	18_{10}	18_{10}	18_{10}
18_{11}	$xyhyh$	$2 \cdot 3^3$	9_3	6_8	18_{11}	18_{11}	18_{11}	18_{11}
20_1	yh	$2^3 \cdot 5$	10_4	20_1	4_2	20_1	20_1	20_1
20_2	xhy	$2^3 \cdot 5$	10_4	20_2	4_3	20_2	20_2	20_2
21	$xhyhy$	$2 \cdot 3 \cdot 7$	21	7	21	3_1	21	21
22_1	xy^3	$2^2 \cdot 11$	11_1	22_1	22_1	22_3	2_1	22_3
22_2	$xyhy$	$2^2 \cdot 11$	11_1	22_2	22_2	22_5	2_3	22_5
22_3	$xyxh^2$	$2^2 \cdot 11$	11_2	22_3	22_3	22_1	2_1	22_1
22_4	$xyxhy^2$	$2^2 \cdot 11$	11_2	22_4	22_4	22_6	2_2	22_6
22_5	$xyhxh^2$	$2^2 \cdot 11$	11_2	22_5	22_5	22_2	2_3	22_2
22_6	xy^2xy^2h	$2^2 \cdot 11$	11_1	22_6	22_6	22_4	2_2	22_4
24_1	$yhyh^2$	$2^4 \cdot 3$	12_1	8_1	24_1	24_1	24_1	24_1
24_2	$xyhy^2h$	$2^4 \cdot 3$	12_2	8_2	24_2	24_2	24_2	24_2
26_1	xy^3h	$2 \cdot 13$	13_1	26_1	26_2	26_2	26_2	2_1
26_2	xhy^2h	$2 \cdot 13$	13_2	26_2	26_1	26_1	26_1	2_1
30_1	$xyxh$	$2^2 \cdot 3 \cdot 5$	15	10_2	6_3	30_1	30_1	30_1
30_2	xhy^3	$2^2 \cdot 3 \cdot 5$	15	10_3	6_4	30_2	30_2	30_2
30_3	y^3h^2	$2^2 \cdot 3 \cdot 5$	15	10_1	6_1	30_3	30_3	30_3
42	xhy^2hy	$2 \cdot 3 \cdot 7$	21	14_1	42	6_1	42	42

A.6. *Conjugacy classes of* $\mathfrak{G} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h}, \mathfrak{e} \rangle$

Class	*Representative*	*Centralizer*	2P	3P	5P	7P	11P	13P	17P	23P
1	1	$2^{18} \cdot 3^{13} \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 23$	1	1	1	1	1	1	1	1
2_1	$(y)^7$	$2^{18} \cdot 3^9 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	1	2_1	2_1	2_1	2_1	2_1	2_1	2_1
2_2	e	$2^{18} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$	1	2_2	2_2	2_2	2_2	2_2	2_2	2_2
2_3	x	$2^{18} \cdot 3^5 \cdot 5$	1	2_3	2_3	2_3	2_3	2_3	2_3	2_3
3_1	$(he)^4$	$2^9 \cdot 3^{10} \cdot 5 \cdot 7 \cdot 13$	3_1	1	3_1	3_1	3_1	3_1	3_1	3_1
3_2	h	$2^{10} \cdot 3^{13}$	3_2	1	3_2	3_2	3_2	3_2	3_2	3_2
3_3	$(xyeh)^4$	$2^7 \cdot 3^{10} \cdot 5$	3_3	1	3_3	3_3	3_3	3_3	3_3	3_3
3_4	$(xyhyh)^6$	$2^4 \cdot 3^{10}$	3_4	1	3_4	3_4	3_4	3_4	3_4	3_4
4_1	$(xhy)^5$	$2^{11} \cdot 3^4 \cdot 5 \cdot 7$	2_2	4_1	4_1	4_1	4_1	4_1	4_1	4_1
4_2	$(xyh)^2$	$2^{12} \cdot 3^4$	2_3	4_2	4_2	4_2	4_2	4_2	4_2	4_2
4_3	xe	$2^{11} \cdot 3^2 \cdot 5$	2_2	4_3	4_3	4_3	4_3	4_3	4_3	4_3
4_4	$(xh)^3$	$2^{12} \cdot 3^2$	2_3	4_4	4_4	4_4	4_4	4_4	4_4	4_4
5	$(yh)^4$	$2^4 \cdot 3^2 \cdot 5^2 \cdot 7$	5	5	1	5	5	5	5	5
6_1	$(xyxh)^5$	$2^9 \cdot 3^7 \cdot 5 \cdot 7$	3_1	2_1	6_1	6_1	6_1	6_1	6_1	6_1
6_2	$(xyxhy)^3$	$2^8 \cdot 3^9$	3_2	2_1	6_2	6_2	6_2	6_2	6_2	6_2
6_3	$(he)^2$	$2^9 \cdot 3^5 \cdot 5$	3_1	2_2	6_3	6_3	6_3	6_3	6_3	6_3
6_4	$(yehe)^3$	$2^9 \cdot 3^6$	3_2	2_2	6_4	6_4	6_4	6_4	6_4	6_4
6_5	$xheh^2e$	$2^7 \cdot 3^7$	3_3	2_1	6_5	6_5	6_5	6_5	6_5	6_5
6_6	$(xh)^2$	$2^{10} \cdot 3^5$	3_2	2_3	6_6	6_6	6_6	6_6	6_6	6_6
6_7	$(xehy)^5$	$2^7 \cdot 3^5 \cdot 5$	3_3	2_3	6_7	6_7	6_7	6_7	6_7	6_7
6_8	$(xhye)^3$	$2^8 \cdot 3^5$	3_2	2_3	6_8	6_8	6_8	6_8	6_8	6_8
6_9	$(xhe)^2$	$2^9 \cdot 3^4$	3_1	2_3	6_9	6_9	6_9	6_9	6_9	6_9
6_{10}	$(xyhyh)^3$	$2^4 \cdot 3^7$	3_4	2_1	6_{10}	6_{10}	6_{10}	6_{10}	6_{10}	6_{10}
6_{11}	$(xyeh)^2$	$2^6 \cdot 3^5$	3_3	2_2	6_{11}	6_{11}	6_{11}	6_{11}	6_{11}	6_{11}
6_{12}	$(xyexe)^2$	$2^7 \cdot 3^4$	3_3	2_3	6_{12}	6_{12}	6_{12}	6_{12}	6_{12}	6_{12}
6_{13}	xh^2ehe	$2^6 \cdot 3^4$	3_3	2_3	6_{13}	6_{13}	6_{13}	6_{13}	6_{13}	6_{13}
6_{14}	$xyhyehe$	$2^4 \cdot 3^5$	3_4	2_3	6_{14}	6_{14}	6_{14}	6_{14}	6_{14}	6_{14}
6_{15}	$(xh^2ey)^2$	$2^4 \cdot 3^4$	3_4	2_3	6_{15}	6_{15}	6_{15}	6_{15}	6_{15}	6_{15}
7	$(y)^2$	$2^3 \cdot 3 \cdot 5 \cdot 7$	7	7	7	1	7	7	7	7
8_1	xyh	$2^7 \cdot 3$	4_2	8_1	8_1	8_1	8_1	8_1	8_1	8_1
8_2	$(xye)^2$	$2^7 \cdot 3$	4_4	8_2	8_2	8_2	8_2	8_2	8_2	8_2
8_3	xey	$2^7 \cdot 3$	4_2	8_3	8_3	8_3	8_3	8_3	8_3	8_3
9_1	$(xyh^2e)^3$	$2^3 \cdot 3^7$	9_1	3_2	9_1	9_1	9_1	9_1	9_1	9_1
9_2	$(xhye)^2$	$2^3 \cdot 3^6$	9_2	3_2	9_2	9_2	9_2	9_2	9_2	9_2
9_3	$(xyhey)^4$	$2^3 \cdot 3^6$	9_3	3_2	9_3	9_3	9_3	9_3	9_3	9_3
9_4	$(yh^2eh^2)^2$	$2 \cdot 3^6$	9_4	3_2	9_4	9_4	9_4	9_4	9_4	9_4
9_5	$(xyhyh)^2$	$2 \cdot 3^4$	9_5	3_4	9_5	9_5	9_5	9_5	9_5	9_5
10_1	yh^2	$2^4 \cdot 3 \cdot 5^2$	5	10_1	2_1	10_1	10_1	10_1	10_1	10_1
10_2	$(yh)^2$	$2^4 \cdot 3 \cdot 5$	5	10_2	2_2	10_2	10_2	10_2	10_2	10_2
10_3	xh^2y	$2^4 \cdot 3 \cdot 5$	5	10_3	2_3	10_3	10_3	10_3	10_3	10_3
11	$(xy^3)^2$	$2^2 \cdot 11$	11	11	11	11	1	11	11	11
12_1	$(xeyh^2)^5$	$2^6 \cdot 3^3 \cdot 5$	6_3	4_1	12_1	12_1	12_1	12_1	12_1	12_1
12_2	$(xheh)^2$	$2^7 \cdot 3^3$	6_9	4_2	12_2	12_2	12_2	12_2	12_2	12_2
12_3	$xyxh^2y$	$2^7 \cdot 3^3$	6_6	4_2	12_3	12_3	12_3	12_3	12_3	12_3
12_4	$xehe$	$2^5 \cdot 3^4$	6_8	4_2	12_4	12_4	12_4	12_4	12_4	12_4
12_5	$(xyhey)^3$	$2^5 \cdot 3^4$	6_4	4_1	12_5	12_5	12_5	12_5	12_5	12_5
12_6	xh	$2^7 \cdot 3^2$	6_6	4_4	12_6	12_6	12_6	12_6	12_6	12_6
12_7	xhe	$2^7 \cdot 3^2$	6_9	4_2	12_7	12_7	12_7	12_7	12_7	12_7
12_8	$xyxhxhe$	$2^5 \cdot 3^3$	6_{12}	4_2	12_8	12_8	12_8	12_8	12_8	12_8
12_9	he	$2^6 \cdot 3^2$	6_3	4_3	12_9	12_9	12_9	12_9	12_9	12_9
12_{10}	$xyeh$	$2^4 \cdot 3^3$	6_{11}	4_1	12_{10}	12_{10}	12_{10}	12_{10}	12_{10}	12_{10}
12_{11}	$xyexe$	$2^5 \cdot 3^2$	6_{12}	4_4	12_{11}	12_{11}	12_{11}	12_{11}	12_{11}	12_{11}
12_{12}	$xhexeh$	$2^5 \cdot 3^2$	6_8	4_4	12_{12}	12_{12}	12_{12}	12_{12}	12_{12}	12_{12}
12_{13}	xh^2ey	$2^3 \cdot 3^3$	6_{15}	4_2	12_{13}	12_{13}	12_{13}	12_{13}	12_{13}	12_{13}
12_{14}	y^3e	$2^4 \cdot 3^2$	6_{11}	4_3	12_{14}	12_{14}	12_{14}	12_{14}	12_{14}	12_{14}
12_{15}	$xeyhey$	$2^3 \cdot 3^2$	6_{15}	4_4	12_{15}	12_{15}	12_{15}	12_{15}	12_{15}	12_{15}
13_1	yeh	$2 \cdot 3 \cdot 13$	13_2	13_1	13_2	13_2	13_2	1	13_1	13_1
13_2	$(xy^2eh)^2$	$2 \cdot 3 \cdot 13$	13_1	13_2	13_1	13_1	13_1	1	13_2	13_2
14_1	y	$2^2 \cdot 3 \cdot 7$	7	14_1	14_1	2_1	14_1	14_1	14_1	14_1
14_2	$xyxe$	$2^3 \cdot 7$	7	14_2	14_2	2_2	14_2	14_2	14_2	14_2
15_1	$(xyxh)^2$	$2^3 \cdot 3^2 \cdot 5$	15_1	5	3_1	15_1	15_1	15_1	15_1	15_1
15_2	$(xehy)^2$	$2 \cdot 3^2 \cdot 5$	15_2	5	3_3	15_2	15_2	15_2	15_2	15_2
16_1	xye	2^5	8_2	16_1	16_2	16_2	16_1	16_2	16_1	16_2
16_2	$yheh$	2^5	8_2	16_2	16_1	16_1	16_2	16_1	16_2	16_1
17	$xhey$	17	17	17	17	17	17	17	1	17
18_1	$xyxhy$	$2^3 \cdot 3^4$	9_2	6_2	18_1	18_1	18_1	18_1	18_1	18_1
18_2	$xeyxeh$	$2^2 \cdot 3^4$	9_3	6_2	18_2	18_2	18_2	18_2	18_2	18_2
18_3	$xhye$	$2^3 \cdot 3^3$	9_2	6_8	18_3	18_3	18_3	18_3	18_3	18_3
18_4	$yehe$	$2^3 \cdot 3^3$	9_2	6_4	18_4	18_4	18_4	18_4	18_4	18_4
18_5	$(xyhey)^2$	$2^3 \cdot 3^3$	9_3	6_4	18_5	18_5	18_5	18_5	18_5	18_5
18_6	$xhehy$	$2^3 \cdot 3^3$	9_1	6_8	18_6	18_6	18_6	18_6	18_6	18_6

Conjugacy classes of $\mathfrak{G} = \langle \mathfrak{x}, \mathfrak{y}, \mathfrak{h}, \mathfrak{e} \rangle$ *(continued)*

Class	*Representative*	*Centralizer*	2P	3P	5P	7P	11P	13P	17P	23P
18_7	$xyhyh$	$2 \cdot 3^3$	9_5	6_{10}	18_7	18_7	18_7	18_7	18_7	18_7
18_8	yh^2eh^2	$2 \cdot 3^3$	9_4	6_8	18_8	18_8	18_8	18_8	18_8	18_8
20_1	xhy	$2^3 \cdot 3 \cdot 5$	10_2	20_1	4_1	20_1	20_1	20_1	20_1	20_1
20_2	yh	$2^3 \cdot 5$	10_2	20_2	4_3	20_2	20_2	20_2	20_2	20_2
21	yhe	$2 \cdot 3 \cdot 7$	21	7	21	3_1	21	21	21	21
22_1	xy^3	$2^2 \cdot 11$	11	22_1	22_1	22_3	2_1	22_3	22_3	22_1
22_2	$xyhy$	$2^2 \cdot 11$	11	22_2	22_2	22_2	2_2	22_2	22_2	22_2
22_3	xey^2	$2^2 \cdot 11$	11	22_3	22_3	22_1	2_1	22_1	22_1	22_3
23_1	$xyeh^2$	23	23_1	23_1	23_2	23_2	23_2	23_1	23_2	1
23_2	yh^2ye	23	23_2	23_2	23_1	23_1	23_1	23_2	23_1	1
24_1	$xheh$	$2^4 \cdot 3$	12_2	8_3	24_1	24_1	24_1	24_1	24_1	24_1
24_2	$xyhyxe$	$2^4 \cdot 3$	12_6	8_2	24_2	24_2	24_2	24_2	24_2	24_2
24_3	$xyhy^2h$	$2^4 \cdot 3$	12_7	8_1	24_3	24_3	24_3	24_3	24_3	24_3
26_1	xy^3h	$2 \cdot 13$	13_1	26_1	26_2	26_2	26_2	2_1	26_1	26_1
26_2	xy^2eh	$2 \cdot 13$	13_2	26_2	26_1	26_1	26_1	2_1	26_2	26_2
27	xyh^2e	3^3	27	9_1	27	27	27	27	27	27
28	$xhyhxe$	$2^2 \cdot 7$	14_2	28	28	4_1	28	28	28	28
30_1	xhy^3	$2^3 \cdot 3 \cdot 5$	15_1	10_2	6_3	30_1	30_1	30_1	30_1	30_1
30_2	$xyxh$	$2^2 \cdot 3 \cdot 5$	15_1	10_1	6_1	30_2	30_2	30_2	30_2	30_2
30_3	$xehy$	$2 \cdot 3 \cdot 5$	15_2	10_3	6_7	30_3	30_3	30_3	30_3	30_3
35	y^2he	$5 \cdot 7$	35	35	7	5	35	35	35	35
36_1	$xyhey$	$2^2 \cdot 3^2$	18_5	12_5	36_1	36_1	36_1	36_1	36_1	36_1
36_2	xy^2eyh	$2^2 \cdot 3^2$	18_6	12_4	36_2	36_2	36_2	36_2	36_2	36_2
39_1	$xhyhe$	$3 \cdot 13$	39_2	13_2	39_2	39_2	39_2	3_1	39_1	39_1
39_2	xh^2xey	$3 \cdot 13$	39_1	13_1	39_1	39_1	39_1	3_1	39_2	39_2
42	$yeyeh$	$2 \cdot 3 \cdot 7$	21	14_1	42	6_1	42	42	42	42
60	$xeyh^2$	$2^2 \cdot 3 \cdot 5$	30_1	20_1	12_1	60	60	60	60	60

Appendix B. Character Tables of Local Subgroups of Fi_{23} and $2\,\mathrm{Fi}_{22}$

B.1. *Character table of* $E(\mathrm{Fi}_{23}) = E = \langle x_1, y_1, e_1 \rangle$

2	18	18	18	18	14	14	7	11	12	12	11	9	9	8	3	7	7	7	6	6	5	5	3	3
3	2	2	2	2	1	1	2	1	1	1	1	.	.	.	1	2	2	2	2	2	1	1	.	.
5	1	1	1	1	.	.	1	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.
7	1	1	1	.	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
11	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
23	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b
2P	1a	1a	1a	1a	1a	1a	3a	2b	2c	2c	2b	2e	2e	2d	5a	3a	3a	3a	3a	3a	3a	3a	7a	7b
3P	1a	2a	2b	2c	2d	2e	1a	4a	4b	4c	4d	4e	4f	4g	5a	2c	2c	2a	2b	2c	2d	2e	7b	7a
5P	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	1a	6a	6b	6c	6d	6e	6f	6g	7b	7a
7P	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	5a	6a	6b	6c	6d	6e	6f	6g	1a	1a
11P	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b
23P	1a	2a	2b	2c	2d	2e	3a	4a	4b	4c	4d	4e	4f	4g	5a	6a	6b	6c	6d	6e	6f	6g	7a	7b
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	22	22	22	22	6	6	4	6	6	6	6	2	2	2	2	4	4	4	4	4	.	.	1	1
$X.3$	45	45	45	45	−3	−3	.	−3	−3	−3	−3	1	1	1	.	.	.	.	.	.	.	.	A	$\bar{A}$
$X.4$	45	45	45	45	−3	−3	.	−3	−3	−3	−3	1	1	1	.	.	.	.	.	.	.	.	$\bar{A}$	A
$X.5$	230	230	230	230	22	22	5	22	22	22	22	2	2	2	.	5	5	5	5	5	1	1	−1	−1
$X.6$	231	231	231	231	7	7	6	7	7	7	7	−1	−1	−1	1	6	6	6	6	6	−2	−2	.	.
$X.7$	231	231	231	231	7	7	−3	7	7	7	7	−1	−1	−1	1	−3	−3	−3	−3	−3	1	1	.	.
$X.8$	231	231	231	231	7	7	−3	7	7	7	7	−1	−1	−1	1	−3	−3	−3	−3	−3	1	1	.	.
$X.9$	253	253	253	253	13	13	1	13	13	13	13	1	1	1	−2	1	1	1	1	1	1	1	1	1
$X.10$	253	−99	29	−3	29	−3	10	−15	−3	5	1	5	−3	1	3	.	−6	.	2	.	2	.	1	1
$X.11$	506	154	26	−6	42	10	11	14	−6	2	−2	6	−2	2	1	9	3	1	−1	−3	3	1	2	2
$X.12$	770	770	770	770	−14	−14	5	−14	−14	−14	−14	−2	−2	−2	.	5	5	5	5	5	1	1	.	.
$X.13$	770	770	770	770	−14	−14	5	−14	−14	−14	−14	−2	−2	−2	.	5	5	5	5	5	1	1	.	.
$X.14$	896	896	896	896	.	.	−4	.	.	.	.	.	.	.	1	−4	−4	−4	−4	−4	.	.	.	.
$X.15$	896	896	896	896	.	.	−4	.	.	.	.	.	.	.	1	−4	−4	−4	−4	−4	.	.	.	.
$X.16$	990	990	990	990	−18	−18	.	−18	−18	−18	−18	2	2	2	.	.	.	.	.	.	.	.	$\bar{A}$	A
$X.17$	990	990	990	990	−18	−18	.	−18	−18	−18	−18	2	2	2	.	.	.	.	.	.	.	.	A	$\bar{A}$
$X.18$	1035	1035	1035	1035	27	27	.	27	27	27	27	−1	−1	−1	.	.	.	.	.	.	.	.	−1	−1
$X.19$	1288	−56	−56	8	56	−8	10	.	8	−8	.	4	4	−4	3	−10	2	−2	−2	2	2	−2	.	.
$X.20$	1518	−594	174	−18	62	−2	15	−34	−2	14	−2	2	2	2	3	−15	−9	9	3	−3	−1	1	−1	−1
$X.21$	2024	2024	2024	2024	8	8	−1	8	8	8	8	.	.	.	−1	−1	−1	−1	−1	−1	−1	−1	1	1
$X.22$	2530	−990	290	−30	−46	18	10	18	18	2	−14	−2	−2	−2	.	.	−6	.	2	.	2	.	A	$\bar{A}$
$X.23$	2530	−990	290	−30	−46	18	10	18	18	2	−14	−2	−2	−2	.	.	−6	.	2	.	2	.	$\bar{A}$	A
$X.24$	3542	−1386	406	−42	70	6	5	−42	6	22	−10	2	2	2	−3	15	−3	−9	1	3	1	3	.	.
$X.25$	3542	1078	182	−42	70	−26	14	42	−10	14	−6	2	−6	−2	2	6	6	−2	2	−6	−2	−2	.	.
$X.26$	3542	−1386	406	−42	70	6	5	−42	6	22	−10	2	2	2	−3	−15	−3	9	1	−3	1	−3	.	.
$X.27$	3795	−1485	435	−45	83	−13	15	−41	−13	11	7	3	−5	−1	.	15	−9	−9	3	3	−1	−1	1	1
$X.28$	3795	−1485	435	−45	−13	19	15	−1	19	11	−17	−5	3	−1	.	−15	−9	9	3	−3	−1	1	1	1
$X.29$	5313	−2079	609	−63	49	17	−15	−35	17	25	−19	−3	5	1	3	15	9	−9	−3	3	1	−1	.	.
$X.30$	7084	2156	364	−84	140	76	19	28	−20	−4	−4	4	4	4	−1	21	3	5	−5	−3	−1	1	.	.
$X.31$	8855	−3465	1015	−105	7	39	−10	−21	39	31	−37	3	−5	−1	.	.	6	.	−2	.	−2	.	.	.
$X.32$	10120	3080	520	−120	168	40	−5	56	−24	8	−8	.	.	.	.	−15	3	−7	7	−3	3	1	−2	−2
$X.33$	10626	3234	546	−126	−14	−46	6	14	2	10	−2	−2	6	2	1	−6	6	−6	6	−6	−2	2	.	.
$X.34$	10626	3234	546	−126	−14	−46	−3	14	2	10	−2	−2	6	2	1	3	−3	3	−3	3	1	−1	.	.
$X.35$	10626	3234	546	−126	−14	−46	−3	14	2	10	−2	−2	6	2	1	3	−3	3	−3	3	1	−1	.	.
$X.36$	11385	−4455	1305	−135	−87	9	.	45	9	−15	−3	5	−3	1	.	.	.	.	.	.	.	.	A	$\bar{A}$
$X.37$	11385	−4455	1305	−135	−87	9	.	45	9	−15	−3	5	−3	1	.	.	.	.	.	.	.	.	$\bar{A}$	A
$X.38$	12880	−560	−560	80	112	−16	10	.	16	−16	.	8	8	−8	.	−10	2	−2	−2	2	−2	2	.	.
$X.39$	12880	−560	−560	80	−112	16	10	.	−16	16	.	.	.	.	.	−10	2	−2	−2	2	2	−2	.	.
$X.40$	12880	−560	−560	80	−112	16	10	.	−16	16	.	.	.	.	.	−10	2	−2	−2	2	2	−2	.	.
$X.41$	14168	−616	−616	88	168	−24	20	.	24	−24	.	−4	−4	4	3	−20	4	−4	−4	4	.	.	.	.
$X.42$	14168	4312	728	−168	56	−72	11	56	−8	24	−8	.	.	.	−2	9	3	1	−1	−3	−1	−3	.	.
$X.43$	17710	5390	910	−210	−98	62	25	−70	14	−26	10	−6	2	−2	.	15	9	−1	1	−9	1	−1	.	.
$X.44$	20608	−896	−896	128	.	.	−20	.	.	.	.	.	.	.	3	20	−4	4	4	−4	.	.	.	.
$X.45$	20608	−896	−896	128	.	.	−20	.	.	.	.	.	.	.	3	20	−4	4	4	−4	.	.	.	.
$X.46$	22770	−8910	2610	−270	162	−30	.	−78	−30	18	18	−2	−2	−2	.	.	.	.	.	.	.	.	−1	−1
$X.47$	22770	6930	1170	−270	−126	−30	.	−42	18	−6	6	6	−2	2	.	.	.	.	.	.	.	.	F	$\bar{F}$
$X.48$	22770	6930	1170	−270	−126	−30	.	−42	18	−6	6	6	−2	2	.	.	.	.	.	.	.	.	$\bar{F}$	F
$X.49$	26565	−10395	3045	−315	−91	5	15	49	5	−19	1	−3	5	1	.	15	−9	−9	3	3	−1	−1	.	.
$X.50$	28336	8624	1456	−336	112	112	−14	.	−16	−16	.	.	.	.	1	−6	−6	2	−2	6	−2	−2	.	.
$X.51$	30360	−11880	3480	−360	−8	−8	−15	8	−8	−8	8	.	.	.	.	−15	9	9	−3	−3	1	1	1	1
$X.52$	32384	9856	1664	−384	.	.	−16	.	.	.	.	.	.	.	−1	−24	.	−8	8	.	.	.	2	2
$X.53$	35420	10780	1820	−420	28	−36	5	28	−4	12	−4	−4	−4	−4	.	15	−3	7	−7	3	1	3	.	.
$X.54$	56672	−2464	−2464	352	224	−32	−10	.	32	−32	.	.	.	.	−3	10	−2	2	2	−2	2	−2	.	.
$X.55$	57960	−2520	−2520	360	−168	24	.	.	−24	24	.	4	4	−4	.	.	.	.	.	.	.	.	.	.
$X.56$	70840	−3080	−3080	440	−56	8	10	.	−8	8	.	−4	−4	4	.	−10	2	−2	−2	2	−2	2	.	.

Character table of $E(\mathrm{Fi}_{23})$ (continued)

2	7	7	7	5	5	3	3	3	1	1	5	5	4	4	3	3	2	2	2	2	1	1	5	5	1	1
3	.	.	.	.	.	1	.	.	.	.	1	1	1	1	.	.	.	.	.	.	1	1	.	.	.	.
5	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.
7	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	1	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
23	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	8a	8b	8c	8d	8e	10a	10b	10c	11a	11b	12a	12b	12c	12d	14a	14b	14c	14d	14e	14f	15a	15b	16a	16b	22a	22b
2P	4b	4c	4c	4e	4f	5a	5a	5a	11b	11a	6b	6b	6d	6d	7b	7a	7a	7b	7b	7a	15a	15b	8a	8a	11a	11b
3P	8a	8b	8c	8d	8e	10a	10b	10c	11a	11b	4b	4c	4d	4a	14b	14a	14d	14c	14f	14e	5a	5a	16a	16b	22a	22b
5P	8a	8b	8c	8d	8e	2c	2b	2a	11a	11b	12a	12b	12c	12d	14b	14a	14d	14c	14f	14e	3a	3a	16b	16a	22a	22b
7P	8a	8b	8c	8d	8e	10a	10b	10c	11b	11a	12a	12b	12c	12d	2b	2b	2a	2a	2d	2d	15b	15a	16b	16a	22b	22a
11P	8a	8b	8c	8d	8e	10a	10b	10c	1a	1a	12a	12b	12c	12d	14a	14b	14c	14d	14e	14f	15b	15a	16a	16b	2a	2a
23P	8a	8b	8c	8d	8e	10a	10b	10c	11a	11b	12a	12b	12c	12d	14a	14b	14c	14d	14e	14f	15a	15b	16b	16a	22a	22b
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	2	2	2	.	.	2	2	2	.	.	.	.	.	.	1	1	1	1	−1	−1	−1	−1	.	.	.	.
X.3	1	1	1	−1	−1	.	.	.	1	1	.	.	.	.	$\bar{A}$	A	A	$\bar{A}$	$-\bar{A}$	$-A$	.	.	−1	−1	1	1
X.4	1	1	1	−1	−1	.	.	.	1	1	.	.	.	.	A	$\bar{A}$	$\bar{A}$	A	$-A$	$-\bar{A}$	.	.	−1	−1	1	1
X.5	2	2	2	.	.	.	.	.	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1	.	.	.	.	−1	−1
X.6	−1	−1	−1	−1	−1	1	1	1	.	.	−2	−2	−2	−2	.	.	.	.	.	.	1	1	−1	−1	.	.
X.7	−1	−1	−1	−1	−1	1	1	1	.	.	1	1	1	1	.	.	.	.	.	.	B	$\bar{B}$	−1	−1	.	.
X.8	−1	−1	−1	−1	−1	1	1	1	.	.	1	1	1	1	.	.	.	.	.	.	$\bar{B}$	B	−1	−1	.	.
X.9	1	1	1	−1	−1	−2	−2	−2	.	.	1	1	1	1	1	1	1	1	−1	−1	1	1	−1	−1	.	.
X.10	1	−3	1	1	1	−3	−1	1	.	.	.	2	−2	.	1	1	−1	−1	1	1	.	.	−1	−1	.	.
X.11	−2	2	−2	.	.	−1	1	−1	.	.	−3	−1	1	−1	−2	−2	.	.	.	.	1	1	.	.	.	.
X.12	−2	−2	−2	.	.	.	.	.	.	.	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.
X.13	−2	−2	−2	.	.	.	.	.	.	.	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.
X.14	.	.	.	.	.	1	1	1	D	$\bar{D}$	.	.	.	.	.	.	.	.	.	.	1	1	.	.	$\bar{D}$	D
X.15	.	.	.	.	.	1	1	1	$\bar{D}$	D	.	.	.	.	.	.	.	.	.	.	1	1	.	.	D	$\bar{D}$
X.16	2	2	2	.	.	.	.	.	.	.	.	.	.	.	A	$\bar{A}$	$\bar{A}$	A	A	$\bar{A}$	.	.	.	.	.	.
X.17	2	2	2	.	.	.	.	.	.	.	.	.	.	.	$\bar{A}$	A	A	$\bar{A}$	$\bar{A}$	A	.	.	.	.	.	.
X.18	−1	−1	−1	1	1	.	.	.	1	1	.	.	.	.	−1	−1	−1	−1	−1	−1	.	.	1	1	1	1
X.19	.	.	.	−2	2	3	−1	−1	1	1	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
X.20	2	−2	−2	.	.	−3	−1	1	.	.	1	−1	1	−1	−1	−1	1	1	−1	−1	.	.	.	.	.	.
X.21	.	.	.	.	.	−1	−1	−1	.	.	−1	−1	−1	−1	1	1	1	1	1	1	−1	−1	.	.	.	.
X.22	−2	2	2	.	.	.	.	.	.	.	.	2	−2	.	$\bar{A}$	A	$-A$	$-\bar{A}$	$\bar{A}$	A	.	.	.	.	.	.
X.23	−2	2	2	.	.	.	.	.	.	.	.	2	−2	.	A	$\bar{A}$	$-\bar{A}$	$-A$	A	$\bar{A}$	.	.	.	.	.	.
X.24	2	−2	−2	.	.	3	1	−1	.	.	3	1	−1	−3	.	.	.	.	.	.	.	.	.	.	.	.
X.25	2	2	−2	.	.	−2	2	−2	.	.	2	2	.	.	.	.	.	.	.	.	−1	−1	.	.	.	.
X.26	2	−2	−2	.	.	3	1	−1	.	.	−3	1	−1	3	.	.	.	.	.	.	.	.	.	.	.	.
X.27	−1	−1	3	−1	−1	.	.	.	.	.	−1	−1	1	1	1	1	−1	−1	−1	−1	.	.	1	1	.	.
X.28	−1	3	−1	−1	−1	.	.	.	.	.	1	−1	1	−1	1	1	−1	−1	1	1	.	.	1	1	.	.
X.29	1	1	−3	−1	−1	−3	−1	1	.	.	−1	1	−1	1	.	.	.	.	.	.	.	.	1	1	.	.
X.30	−4	.	.	.	.	1	−1	1	.	.	1	−1	−1	1	.	.	.	.	.	.	−1	−1	.	.	.	.
X.31	−1	−1	3	1	1	.	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	−1	−1	.	.
X.32	.	.	.	.	.	.	.	.	.	.	−3	−1	1	−1	2	2	.	.	.	.	.	.	.	.	.	.
X.33	−2	−2	2	.	.	−1	1	−1	.	.	2	−2	−2	2	.	.	.	.	.	.	1	1	.	.	.	.
X.34	−2	−2	2	.	.	−1	1	−1	.	.	−1	1	1	−1	.	.	.	.	.	.	B	$\bar{B}$	.	.	.	.
X.35	−2	−2	2	.	.	−1	1	−1	.	.	−1	1	1	−1	.	.	.	.	.	.	$\bar{B}$	B	.	.	.	.
X.36	1	−3	1	−1	−1	.	.	.	.	.	.	.	.	.	$\bar{A}$	A	$-A$	$-\bar{A}$	$-\bar{A}$	$-A$	.	.	1	1	.	.
X.37	1	−3	1	−1	−1	.	.	.	.	.	.	.	.	.	A	$\bar{A}$	$-\bar{A}$	$-A$	$-A$	$-\bar{A}$	.	.	1	1	.	.
X.38	.	.	.	.	.	.	.	.	−1	−1	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	1	1
X.39	.	.	.	.	.	.	.	.	−1	−1	2	−2	.	.	.	.	.	.	.	.	.	.	E	$\bar{E}$	1	1
X.40	.	.	.	.	.	.	.	.	−1	−1	2	−2	.	.	.	.	.	.	.	.	.	.	$\bar{E}$	E	1	1
X.41	.	.	.	2	−2	3	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.42	.	.	.	.	.	2	−2	2	.	.	1	3	1	−1	.	.	.	.	.	.	1	1	.	.	.	.
X.43	2	−2	2	.	.	.	.	.	.	.	−1	1	1	−1	.	.	.	.	.	.	.	.	.	.	.	.
X.44	.	.	.	.	.	3	−1	−1	D	$\bar{D}$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$-\bar{D}$	$-D$
X.45	.	.	.	.	.	3	−1	−1	$\bar{D}$	D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$-D$	$-\bar{D}$
X.46	−2	2	2	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	1	1	1	.	.	.	.	.	.
X.47	−2	2	−2	.	.	.	.	.	.	.	.	.	.	.	$-\bar{F}$	$-F$	.	.	.	.	.	.	.	.	.	.
X.48	−2	2	−2	.	.	.	.	.	.	.	.	.	.	.	$-F$	$-\bar{F}$	.	.	.	.	.	.	.	.	.	.
X.49	1	1	−3	1	1	.	.	.	.	.	−1	−1	1	1	.	.	.	.	.	.	.	.	−1	−1	.	.
X.50	.	.	.	.	.	−1	1	−1	.	.	2	2	.	.	.	.	.	.	.	.	1	1	.	.	.	.
X.51	.	.	.	.	.	.	.	.	.	.	1	1	−1	−1	1	1	−1	−1	−1	−1	.	.	.	.	.	.
X.52	.	.	.	.	.	1	−1	1	.	.	.	.	.	.	−2	−2	.	.	.	.	−1	−1	.	.	.	.
X.53	4	.	.	.	.	.	.	.	.	.	−1	−3	−1	1	.	.	.	.	.	.	.	.	.	.	.	.
X.54	.	.	.	.	.	−3	1	1	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.55	.	.	.	2	−2	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
X.56	.	.	.	−2	2	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $E(\mathrm{Fi}_{23})$ (continued)

2	.	.	2	2	1	1
3	.	.	.	.	1	1
5	.	.	.	.	1	1
7	.	.	1	1	.	.
11	.	.	.	.	.	.
23	1	1	.	.	.	.
	23a	23b	28a	28b	30a	30b
2P	23a	23b	14a	14b	15a	15b
3P	23a	23b	28b	28a	10a	10a
5P	23b	23a	28b	28a	6a	6a
7P	23b	23a	4a	4a	30b	30a
11P	23b	23a	28a	28b	30b	30a
23P	1a	1a	28a	28b	30a	30b
X.1	1	1	1	1	1	1
X.2	−1	−1	−1	−1	−1	−1
X.3	−1	−1	$-\bar{A}$	$-A$	.	.
X.4	−1	−1	$-A$	$-\bar{A}$	.	.
X.5	.	.	1	1	.	.
X.6	1	1	.	.	1	1
X.7	1	1	.	.	B	$\bar{B}$
X.8	1	1	.	.	$\bar{B}$	B
X.9	.	.	−1	−1	1	1
X.10	.	.	−1	−1	.	.
X.11	.	.	.	.	−1	−1
X.12	C	$\bar{C}$	.	.	.	.
X.13	$\bar{C}$	C	.	.	.	.
X.14	−1	−1	.	.	1	1
X.15	−1	−1	.	.	1	1
X.16	1	1	A	$\bar{A}$	.	.
X.17	1	1	$\bar{A}$	A	.	.
X.18	.	.	−1	−1	.	.
X.19	.	.	.	.	.	.
X.20	.	.	1	1	.	.
X.21	.	.	1	1	−1	−1
X.22	.	.	$-\bar{A}$	$-A$	.	.
X.23	.	.	$-A$	$-\bar{A}$	.	.
X.24	.	.	.	.	.	.
X.25	.	.	.	.	1	1
X.26	.	.	.	.	.	.
X.27	.	.	1	1	.	.
X.28	.	.	−1	−1	.	.
X.29	.	.	.	.	.	.
X.30	.	.	.	.	1	1
X.31	.	.	.	.	.	.
X.32	.	.	.	.	.	.
X.33	.	.	.	.	−1	−1
X.34	.	.	.	.	$-B$	$-\bar{B}$
X.35	.	.	.	.	$-\bar{B}$	$-B$
X.36	.	.	$\bar{A}$	A	.	.
X.37	.	.	A	$\bar{A}$	.	.
X.38	.	.	.	.	.	.
X.39	.	.	.	.	.	.
X.40	.	.	.	.	.	.
X.41	.	.	.	.	.	.
X.42	.	.	.	.	−1	−1
X.43	.	.	.	.	.	.
X.44	.	.	.	.	.	.
X.45	.	.	.	.	.	.
X.46	.	.	−1	−1	.	.
X.47	.	.	.	.	.	.
X.48	.	.	.	.	.	.
X.49	.	.	.	.	.	.
X.50	.	.	.	.	−1	−1
X.51	.	.	1	1	.	.
X.52	.	.	.	.	1	1
X.53	.	.	.	.	.	.
X.54	.	.	.	.	.	.
X.55	.	.	.	.	.	.
X.56	.	.	.	.	.	.

$A = \zeta(7)_7^4+\zeta(7)_7^2+\zeta(7)_7, B = -2\zeta(15)_3\zeta(15)_5^3-2\zeta(15)_3\zeta(15)_5^2-\zeta(15)_3-\zeta(15)_5^3-\zeta(15)_5^2-1, C = \zeta(23)_{23}^{18}+\zeta(23)_{23}^{16}+\zeta(23)_{23}^{13}+\zeta(23)_{23}^{12}+\zeta(23)_{23}^{9}+\zeta(23)_{23}^{8}+\zeta(23)_{23}^{6}+\zeta(23)_{23}^{4}+\zeta(23)_{23}^{3}+\zeta(23)_{23}^{2}+\zeta(23)_{23}, D = \zeta(11)_{11}^9+\zeta(11)_{11}^5+\zeta(11)_{11}^4+\zeta(11)_{11}^3+\zeta(11)_{11}, E = -2\zeta(8)_8^3-2\zeta(8)_8, F = 2\zeta(7)_7^4+2\zeta(7)_7^2+2\zeta(7)_7.$

B.2. *Character table of* $D(\mathrm{Fi}_{23}) = D = \langle x_1, y_1\rangle \cong \langle x_0, y_0\rangle$

2	18	18	17	17	18	18	16	14	14	13	7	11	11	12	12	10	10	10	9	9	8	8
3	2	2	2	2	1	1	2	1	1	.	2	1	1	.	.	1	1	.	.	.	.	.
5	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
7	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
11	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	4a	4b	4c	4d	4e	4f	4g	4h	4i	4j	4k
2P	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	3a	2e	2e	2d	2d	2f	2f	2e	2g	2g	2h	2i
3P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	1a	4a	4b	4c	4d	4e	4f	4g	4h	4i	4j	4k
5P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	4a	4b	4c	4d	4e	4f	4g	4h	4i	4j	4k
7P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	4a	4b	4c	4d	4e	4f	4g	4h	4i	4j	4k
11P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	4a	4b	4c	4d	4e	4f	4g	4h	4i	4j	4k
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	21	21	21	21	21	21	21	5	5	5	3	5	5	5	5	5	5	5	1	1	1	1
$X.3$	45	45	45	45	45	45	45	−3	−3	−3	.	−3	−3	−3	−3	−3	−3	−3	1	1	1	1
$X.4$	45	45	45	45	45	45	45	−3	−3	−3	.	−3	−3	−3	−3	−3	−3	−3	1	1	1	1
$X.5$	55	55	55	55	55	55	55	7	7	7	1	7	7	7	7	7	7	7	3	3	3	−1
$X.6$	77	77	−35	−35	13	13	−3	13	13	−3	5	−7	−7	−3	5	1	1	1	−3	5	1	1
$X.7$	99	99	99	99	99	99	99	3	3	3	.	3	3	3	3	3	3	3	3	3	3	−1
$X.8$	154	154	154	154	154	154	154	10	10	10	1	10	10	10	10	10	10	10	−2	−2	−2	2
$X.9$	176	−176	−64	64	−16	16	.	−16	16	.	5	8	−8	.	.	4	−4	.	.	.	.	4
$X.10$	176	−176	64	−64	−16	16	.	−16	16	.	5	−8	8	.	.	4	−4	.	.	.	.	4
$X.11$	210	210	210	210	210	210	210	2	2	2	3	2	2	2	2	2	2	2	−2	−2	−2	−2
$X.12$	231	231	231	231	231	231	231	7	7	7	−3	7	7	7	7	7	7	7	−1	−1	−1	−1
$X.13$	280	280	280	280	280	280	280	−8	−8	−8	1	−8	−8	−8	−8	−8	−8	−8	.	.	.	.
$X.14$	280	280	280	280	280	280	280	−8	−8	−8	1	−8	−8	−8	−8	−8	−8	−8	.	.	.	.
$X.15$	330	330	90	90	10	10	−6	26	26	10	6	6	6	−6	2	−2	−2	−2	−2	6	2	2
$X.16$	385	385	−175	−175	65	65	−15	17	17	1	−2	−11	−11	1	9	5	5	−3	5	−3	1	1
$X.17$	385	385	385	385	385	385	385	1	1	1	−2	1	1	1	1	1	1	1	1	1	1	1
$X.18$	385	385	−175	−175	65	65	−15	17	17	1	7	−11	−11	1	9	5	5	−3	5	−3	1	1
$X.19$	616	616	−56	−56	−24	−24	8	24	24	−8	4	.	.	8	−8	.	.	.	4	4	−4	4
$X.20$	616	616	−56	−56	−24	−24	8	24	24	−8	4	.	.	8	−8	.	.	.	4	4	−4	−4
$X.21$	616	616	−280	−280	104	104	−24	8	8	8	−5	−8	−8	8	8	8	8	−8	.	.	.	.
$X.22$	616	616	−280	−280	104	104	−24	8	8	8	−5	−8	−8	8	8	8	8	−8	.	.	.	.
$X.23$	672	−672	.	.	32	−32	.	−32	32	.	6	.	.	.	.	−8	8	.	.	.	.	.
$X.24$	693	693	−315	−315	117	117	−27	21	21	5	.	−15	−15	5	13	9	9	−7	−3	5	1	1
$X.25$	770	770	−350	−350	130	130	−30	−14	−14	18	5	2	2	18	2	10	10	−14	−2	−2	−2	−2
$X.26$	990	990	270	270	30	30	−18	−18	−18	−2	.	−6	−6	14	6	−6	−6	2	−6	2	−2	2
$X.27$	990	990	270	270	30	30	−18	−18	−18	−2	.	−6	−6	14	6	−6	−6	2	−6	2	−2	2
$X.28$	1056	−1056	−384	384	−96	96	.	−32	32	.	3	16	−16	.	.	8	−8	.	.	.	.	.
$X.29$	1056	−1056	384	−384	−96	96	.	−32	32	.	3	−16	16	.	.	8	−8	.	.	.	.	.
$X.30$	1155	1155	−525	−525	195	195	−45	35	35	−13	3	−17	−17	−13	11	−1	−1	7	−5	3	−1	−1
$X.31$	1155	1155	−525	−525	195	195	−45	−13	−13	3	3	7	7	3	−5	−1	−1	−1	7	−1	3	−1
$X.32$	1760	−1760	640	−640	−160	160	.	32	−32	.	5	16	−16	.	.	−8	8	.	.	.	.	.
$X.33$	1760	−1760	−640	640	−160	160	.	32	−32	.	5	−16	16	.	.	−8	8	.	.	.	.	.
$X.34$	1760	−1760	640	−640	−160	160	.	32	−32	.	5	16	−16	.	.	−8	8	.	.	.	.	.
$X.35$	1760	−1760	−640	640	−160	160	.	32	−32	.	5	−16	16	.	.	−8	8	.	.	.	.	.
$X.36$	1980	1980	540	540	60	60	−36	60	60	28	.	12	12	−4	12	−12	−12	−4	4	4	4	.
$X.37$	2310	2310	630	630	70	70	−42	22	22	−26	6	18	18	−10	14	2	2	−6	−6	2	−2	−2
$X.38$	2310	2310	−1050	−1050	390	390	−90	22	22	−10	−3	−10	−10	−10	6	−2	−2	6	2	2	2	−2
$X.39$	2310	2310	630	630	70	70	−42	22	22	38	6	−6	−6	−10	−18	2	2	2	2	−6	−2	2
$X.40$	2310	2310	630	630	70	70	−42	−10	−10	6	6	−6	−6	22	14	−14	−14	2	2	−6	−2	−2
$X.41$	2464	−2464	−896	896	−224	224	.	−32	32	.	7	16	−16	.	.	8	−8	.	.	.	.	.
$X.42$	2464	−2464	896	−896	−224	224	.	−32	32	.	−2	−16	16	.	.	8	−8	.	.	.	.	.
$X.43$	2464	−2464	−896	896	−224	224	.	−32	32	.	−2	16	−16	.	.	8	−8	.	.	.	.	.
$X.44$	2464	−2464	896	−896	−224	224	.	−32	32	.	7	−16	16	.	.	8	−8	.	.	.	.	.
$X.45$	2640	−2640	960	−960	−240	240	.	16	−16	.	3	8	−8	.	.	−4	4	.	.	.	.	−4
$X.46$	2640	2640	720	720	80	80	−48	16	16	16	−6	.	.	16	16	−16	−16	.	.	.	.	.
$X.47$	2640	−2640	−960	960	−240	240	.	16	−16	.	3	−8	8	.	.	−4	4	.	.	.	.	−4
$X.48$	3360	−3360	.	.	160	−160	.	−32	32	.	−6	.	.	.	.	−8	8	.	.	.	.	.
$X.49$	3360	−3360	.	.	160	−160	.	−32	32	.	−6	.	.	.	.	−8	8	.	.	.	.	.
$X.50$	3465	3465	−1575	−1575	585	585	−135	9	9	−7	.	−3	−3	−7	1	−3	−3	5	1	−7	−3	1
$X.51$	3465	3465	−1575	−1575	585	585	−135	−39	−39	9	.	21	21	9	−15	−3	−3	−3	−3	5	1	1
$X.52$	3696	−3696	−1344	1344	−336	336	.	−16	16	.	−3	8	−8	.	.	4	−4	.	.	.	.	−4
$X.53$	3696	−3696	1344	−1344	−336	336	.	−16	16	.	−3	−8	8	.	.	4	−4	.	.	.	.	−4
$X.54$	4620	4620	1260	1260	140	140	−84	44	44	12	−6	12	12	−20	−4	4	4	−4	−4	−4	−4	.
$X.55$	5544	5544	−504	−504	−216	−216	72	24	24	−8	.	.	.	8	−8	.	.	.	4	4	−4	−4
$X.56$	5544	5544	−504	−504	−216	−216	72	24	24	−8	.	.	.	8	−8	.	.	.	4	4	−4	4
$X.57$	6160	−6160	−2240	2240	−560	560	.	16	−16	.	−5	−8	8	.	.	−4	4	.	.	.	.	4
$X.58$	6160	6160	−560	−560	−240	−240	80	48	48	−16	4	.	.	16	−16	.	.	.	−8	−8	8	.
$X.59$	6160	−6160	2240	−2240	−560	560	.	16	−16	.	−5	8	−8	.	.	−4	4	.	.	.	.	4
$X.60$	6160	6160	−560	−560	−240	−240	80	−48	−48	16	4	.	.	−16	16	.	.	.	.	.	.	.
$X.61$	6160	6160	−560	−560	−240	−240	80	−48	−48	16	4	.	.	−16	16	.	.	.	.	.	.	.
$X.62$	6720	−6720	.	.	320	−320	.	−64	64	.	6	.	.	.	.	−16	16	.	.	.	.	.
$X.63$	6720	−6720	.	.	320	−320	.	64	−64	.	6	.	.	.	.	16	−16	.	.	.	.	.
$X.64$	6930	6930	1890	1890	210	210	−126	−30	−30	−46	.	6	6	2	10	6	6	−2	6	−2	2	2
$X.65$	6930	6930	1890	1890	210	210	−126	−30	−30	18	.	−18	−18	2	−22	6	6	6	−2	6	2	−2
$X.66$	7392	−7392	.	.	352	−352	.	32	−32	.	−6	.	.	.	.	8	−8	.	.	.	.	.
$X.67$	8064	−8064	.	.	384	−384	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.68$	8064	−8064	.	.	384	−384	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.69$	9856	9856	−896	−896	−384	−384	128	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	.

Character table of $D(\mathrm{Fi}_{23})$ *(continued)*

2	8	7	3	7	7	7	5	5	5	5	6	6	5	5	2	2	7	7	7	6	6	6	5	5	3	3	3	3	3	3
3	.	.	.	2	2	2	2	2	2	2	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	1
7	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	4l	4m	5a	6a	6b	6c	6d	6e	6f	6g	6h	6i	6j	6k	7a	7b	8a	8b	8c	8d	8e	8f	8g	8h	10a	10b	10c	10d	10e	10f
2P	2i	2h	5a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	7a	7b	4c	4d	4d	4e	4c	4e	4h	4i	5a	5a	5a	5a	5a	5a
3P	4l	4m	5a	2f	2f	2a	2f	2f	2c	2b	2e	2d	2h	2g	7b	7a	8a	8b	8c	8d	8e	8f	8g	8h	10b	10a	10c	10f	10e	10d
5P	4l	4m	1a	6a	6b	6c	6d	6e	6f	6g	6h	6i	6j	6k	7b	7a	8a	8b	8c	8d	8e	8f	8g	8h	2c	2c	2e	2b	2d	2b
7P	4l	4m	5a	6a	6b	6c	6d	6e	6f	6g	6h	6i	6j	6k	1a	1a	8a	8b	8c	8d	8e	8f	8g	8h	10b	10a	10c	10f	10e	10d
11P	4l	4m	5a	6a	6b	6c	6d	6e	6f	6g	6h	6i	6j	6k	7a	7b	8a	8b	8c	8d	8e	8f	8g	8h	10a	10b	10c	10d	10e	10f
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	3	3	3	3	3	3	3	3	3	−1	−1	.	.	1	1	1	1	1	1	−1	−1	1	1	1	1	1	1
X.3	1	1	.	.	.	.	.	.	.	.	.	.	.	.	A	$\bar{A}$	1	1	1	1	1	1	−1	−1	.	.	.	.	.	.
X.4	1	1	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{A}$	A	1	1	1	1	1	1	−1	−1	.	.	.	.	.	.
X.5	−1	−1	.	1	1	1	1	1	1	1	1	1	1	1	−1	−1	3	3	3	−1	−1	−1	1	1	.	.	.	.	.	.
X.6	1	1	2	−3	−3	5	−3	−3	1	1	1	1	1	1	.	.	1	−3	1	−1	1	−1	1	1	.	.	−2	.	−2	.
X.7	−1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	1	1	3	3	3	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
X.8	2	2	−1	1	1	1	1	1	1	1	1	1	1	1	.	.	−2	−2	−2	2	2	2	.	.	−1	−1	−1	−1	−1	−1
X.9	−4	.	1	−3	3	−5	3	−3	1	−1	1	−1	1	−1	1	1	.	.	.	−2	.	2	.	.	−1	−1	1	1	−1	1
X.10	−4	.	1	−3	3	−5	−3	3	−1	1	1	−1	1	−1	1	1	.	.	.	2	.	−2	.	.	1	1	1	−1	−1	−1
X.11	−2	−2	.	3	3	3	3	3	3	3	3	3	−1	−1	.	.	−2	−2	−2	−2	−2	−2	.	.	.	.	.	.	.	.
X.12	−1	−1	1	−3	−3	−3	−3	−3	−3	−3	−3	−3	1	1	.	.	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1
X.13	.	.	.	1	1	1	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.14	.	.	.	1	1	1	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.15	2	2	.	6	6	6	.	.	.	.	−2	−2	2	2	1	1	−2	2	−2	.	−2	.	.	.	.	.	.	.	.	.
X.16	1	1	.	6	6	−2	.	.	−4	−4	2	2	2	2	.	.	1	1	−3	−1	1	−1	−1	−1	.	.	.	.	.	.
X.17	1	1	.	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	.	.	1	1	1	1	1	1	1	1	.	.	.	.	.	.
X.18	1	1	.	−9	−9	7	−3	−3	5	5	−1	−1	−1	−1	.	.	1	1	−3	−1	1	−1	−1	−1	.	.	.	.	.	.
X.19	4	−4	1	−4	−4	4	2	2	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	−1	−1	1	−1	1	−1
X.20	−4	4	1	−4	−4	4	2	2	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.	−2	2	−1	−1	1	−1	1	−1
X.21	.	.	1	3	3	−5	3	3	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.	.	.	C	$-C$	−1	$-C$	−1	C
X.22	.	.	1	3	3	−5	3	3	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.	.	.	$-C$	C	−1	C	−1	$-C$
X.23	.	.	2	6	−6	−6	.	.	.	.	−2	2	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	2	.
X.24	1	1	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−3	1	−1	1	−1	1	1	.	.	2	.	2	.
X.25	−2	−2	.	−3	−3	5	−3	−3	1	1	1	1	1	1	.	.	−2	2	2	2	−2	2	.	.	.	.	.	.	.	.
X.26	2	2	.	.	.	.	.	.	.	.	.	.	.	.	A	$\bar{A}$	2	2	−2	.	−2	.	.	.	.	.	.	.	.	.
X.27	2	2	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{A}$	A	2	2	−2	.	−2	.	.	.	.	.	.	.	.	.
X.28	.	.	1	3	−3	−3	3	−3	−3	3	3	−3	−1	1	−1	−1	.	.	.	.	.	.	.	.	−1	−1	1	1	−1	1
X.29	.	.	1	3	−3	−3	−3	3	3	−3	3	−3	−1	1	−1	−1	.	.	.	.	.	.	.	.	1	1	1	−1	−1	−1
X.30	−1	−1	.	3	3	3	−3	−3	−3	−3	3	3	−1	−1	.	.	−1	−1	3	1	−1	1	−1	−1	.	.	.	.	.	.
X.31	−1	−1	.	3	3	3	−3	−3	−3	−3	3	3	−1	−1	.	.	3	−1	−5	1	−1	1	1	1	.	.	.	.	.	.
X.32	.	.	.	−3	3	−5	−3	3	−1	1	1	−1	1	−1	A	$\bar{A}$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.33	.	.	.	−3	3	−5	3	−3	1	−1	1	−1	1	−1	$\bar{A}$	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.34	.	.	.	−3	3	−5	−3	3	−1	1	1	−1	1	−1	$\bar{A}$	A	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.35	.	.	.	−3	3	−5	3	−3	1	−1	1	−1	1	−1	A	$\bar{A}$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.36	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−4	.	.	.	.	.	.	.	.	.	.	.	.	.
X.37	−2	−2	.	6	6	6	.	.	.	.	−2	−2	−2	−2	.	.	2	2	−2	.	2	.	.	.	.	.	.	.	.	.
X.38	−2	−2	.	−3	−3	−3	3	3	3	3	−3	−3	1	1	.	.	2	−2	−2	2	−2	2	.	.	.	.	.	.	.	.
X.39	2	2	.	6	6	6	.	.	.	.	−2	−2	−2	−2	.	.	2	−2	2	.	−2	.	.	.	.	.	.	.	.	.
X.40	−2	−2	.	6	6	6	.	.	.	.	−2	−2	2	2	.	.	2	−2	2	.	2	.	.	.	.	.	.	.	.	.
X.41	.	.	−1	−9	9	−7	3	−3	5	−5	−1	1	−1	1	.	.	.	.	.	.	.	.	.	.	1	1	−1	−1	1	−1
X.42	.	.	−1	6	−6	2	.	.	4	−4	2	−2	2	−2	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	1	1	1
X.43	.	.	−1	6	−6	2	.	.	−4	4	2	−2	2	−2	.	.	.	.	.	.	.	.	.	.	1	1	−1	−1	1	−1
X.44	.	.	−1	−9	9	−7	−3	3	−5	5	−1	1	−1	1	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	1	1	1
X.45	4	.	.	3	−3	−3	−3	3	3	−3	3	−3	−1	1	1	1	.	.	.	−2	.	2	.	.	.	.	.	.	.	.
X.46	.	.	.	−6	−6	−6	.	.	.	.	2	2	−2	−2	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.47	4	.	.	3	−3	−3	3	−3	−3	3	3	−3	−1	1	1	1	.	.	.	2	.	−2	.	.	.	.	.	.	.	.
X.48	.	.	.	−6	6	6	.	.	.	.	2	−2	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.49	.	.	.	−6	6	6	.	.	.	.	2	−2	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.50	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−3	5	1	−1	1	−1	1	1	.	.	.	.	.	.
X.51	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−3	1	−1	1	−1	−1	−1	.	.	.	.	.	.
X.52	4	.	1	−3	3	3	−3	3	3	−3	−3	3	1	−1	.	.	.	.	.	2	.	−2	.	.	−1	−1	1	1	−1	1
X.53	4	.	1	−3	3	3	3	−3	−3	3	−3	3	1	−1	.	.	.	.	.	−2	.	2	.	.	1	1	1	−1	−1	−1
X.54	.	.	.	−6	−6	−6	.	.	.	.	2	2	2	2	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.
X.55	−4	4	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	1	1	−1	1	−1	1
X.56	4	−4	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2	1	1	−1	1	−1	1
X.57	−4	.	.	3	−3	5	−3	3	−1	1	−1	1	−1	1	.	.	.	.	.	−2	.	2	.	.	.	.	.	.	.	.
X.58	.	.	.	−4	−4	4	2	2	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.59	−4	.	.	3	−3	5	3	−3	1	−1	−1	1	−1	1	.	.	.	.	.	2	.	−2	.	.	.	.	.	.	.	.
X.60	.	.	.	−4	−4	4	2	2	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.61	.	.	.	−4	−4	4	2	2	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.62	.	.	.	6	−6	−6	.	.	.	.	−2	2	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.63	.	.	.	6	−6	−6	.	.	.	.	−2	2	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.64	2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2	2	.	−2	.	.	.	.	.	.	.	.	.
X.65	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2	−2	.	2	.	.	.	.	.	.	.	.	.
X.66	.	.	2	−6	6	6	.	.	.	.	2	−2	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	2	.
X.67	.	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	C	$-C$	1	C	−1	$-C$
X.68	.	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$-C$	C	1	$-C$	−1	C
X.69	.	.	1	8	8	−8	−4	−4	4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	−1	1	−1

Character table of $D(\mathrm{Fi}_{23})$ *(continued)*

2	3	1	1	5	5	4	4	2	2	2	2	2	2	5	5	1	1
3	.	.	.	1	1	1	1	.	.	.	.	.	.	.	.	.	.
5	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
7	.	.	.	.	.	.	.	1	1	1	1	1	1	.	.	.	.
11	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	1	1
	10g	11a	11b	12a	12b	12c	12d	14a	14b	14c	14d	14e	14f	16a	16b	22a	22b
2P	5a	11b	11a	6a	6a	6h	6h	7a	7b	7a	7b	7a	7b	8a	8a	11a	11b
3P	10g	11a	11b	4f	4e	4a	4b	14f	14e	14d	14c	14b	14a	16a	16b	22a	22b
5P	2a	11a	11b	12a	12b	12c	12d	14f	14e	14d	14c	14b	14a	16b	16a	22a	22b
7P	10g	11b	11a	12a	12b	12c	12d	2b	2a	2c	2c	2a	2b	16b	16a	22b	22a
11P	10g	1a	1a	12a	12b	12c	12d	14a	14b	14c	14d	14e	14f	16a	16b	2a	2a
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	−1	−1	−1	−1
X.3	.	1	1	.	.	.	.	A	$\bar{A}$	A	$\bar{A}$	A	$\bar{A}$	−1	−1	1	1
X.4	.	1	1	.	.	.	.	$\bar{A}$	A	$\bar{A}$	A	$\bar{A}$	A	−1	−1	1	1
X.5	.	.	.	1	1	1	1	−1	−1	−1	−1	−1	−1	1	1	.	.
X.6	2	.	.	1	1	−1	−1	.	.	.	.	.	.	−1	−1	.	.
X.7	−1	.	.	.	.	.	.	1	1	1	1	1	1	−1	−1	.	.
X.8	−1	.	.	1	1	1	1	.	.	.	.	.	.	.	.	.	.
X.9	−1	.	.	−1	1	−1	1	−1	−1	1	1	−1	−1	.	.	.	.
X.10	−1	.	.	−1	1	1	−1	1	−1	−1	−1	−1	1	.	.	.	.
X.11	.	1	1	−1	−1	−1	−1	.	.	.	.	.	.	.	.	1	1
X.12	1	.	.	1	1	1	1	.	.	.	.	.	.	−1	−1	.	.
X.13	.	B	$\bar{B}$	1	1	1	1	.	.	.	.	.	.	.	.	$\bar{B}$	B
X.14	.	$\bar{B}$	B	1	1	1	1	.	.	.	.	.	.	.	.	B	$\bar{B}$
X.15	.	.	.	−2	−2	.	.	−1	1	−1	−1	1	−1	.	.	.	.
X.16	.	.	.	2	2	−2	−2	.	.	.	.	.	.	1	1	.	.
X.17	.	.	.	−2	−2	−2	−2	.	.	.	.	.	.	1	1	.	.
X.18	.	.	.	−1	−1	1	1	.	.	.	.	.	.	1	1	.	.
X.19	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.20	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.21	1	.	.	−1	−1	1	1	.	.	.	.	.	.	.	.	.	.
X.22	1	.	.	−1	−1	1	1	.	.	.	.	.	.	.	.	.	.
X.23	−2	1	1	2	−2	.	.	.	.	.	.	.	.	.	.	−1	−1
X.24	−2	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	.	.
X.25	.	.	.	1	1	−1	−1	.	.	.	.	.	.	.	.	.	.
X.26	.	.	.	.	.	.	.	$-A$	$\bar{A}$	$-A$	$-\bar{A}$	A	$-\bar{A}$	.	.	.	.
X.27	.	.	.	.	.	.	.	$-\bar{A}$	A	$-\bar{A}$	$-A$	$\bar{A}$	$-A$	.	.	.	.
X.28	−1	.	.	1	−1	1	−1	1	1	−1	−1	1	1	.	.	.	.
X.29	−1	.	.	1	−1	−1	1	−1	1	1	1	1	−1	.	.	.	.
X.30	.	.	.	−1	−1	1	1	.	.	.	.	.	.	1	1	.	.
X.31	.	.	.	−1	−1	1	1	.	.	.	.	.	.	−1	−1	.	.
X.32	.	.	.	−1	1	1	−1	A	$-\bar{A}$	$-A$	$-\bar{A}$	$-A$	$\bar{A}$	.	.	.	.
X.33	.	.	.	−1	1	−1	1	$-\bar{A}$	$-A$	$\bar{A}$	A	$-\bar{A}$	$-A$	.	.	.	.
X.34	.	.	.	−1	1	1	−1	$\bar{A}$	$-A$	$-\bar{A}$	$-A$	$-\bar{A}$	A	.	.	.	.
X.35	.	.	.	−1	1	−1	1	$-A$	$-\bar{A}$	A	$\bar{A}$	$-A$	$-\bar{A}$	.	.	.	.
X.36	.	.	.	.	.	.	.	1	−1	1	1	−1	1	.	.	.	.
X.37	.	.	.	2	2	.	.	.	.	.	.	.	.	.	.	.	.
X.38	.	.	.	1	1	−1	−1	.	.	.	.	.	.	.	.	.	.
X.39	.	.	.	2	2	.	.	.	.	.	.	.	.	.	.	.	.
X.40	.	.	.	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.
X.41	1	.	.	1	−1	1	−1	.	.	.	.	.	.	.	.	.	.
X.42	1	.	.	−2	2	2	−2	.	.	.	.	.	.	.	.	.	.
X.43	1	.	.	−2	2	−2	2	.	.	.	.	.	.	.	.	.	.
X.44	1	.	.	1	−1	−1	1	.	.	.	.	.	.	.	.	.	.
X.45	.	.	.	1	−1	−1	1	1	−1	−1	−1	−1	1	.	.	.	.
X.46	.	.	.	2	2	.	.	−1	1	−1	−1	1	−1	.	.	.	.
X.47	.	.	.	1	−1	1	−1	−1	−1	1	1	−1	−1	.	.	.	.
X.48	.	B	$\bar{B}$	2	−2	.	.	.	.	.	.	.	.	.	.	$-\bar{B}$	$-B$
X.49	.	$\bar{B}$	B	2	−2	.	.	.	.	.	.	.	.	.	.	$-B$	$-\bar{B}$
X.50	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	.	.
X.51	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.
X.52	−1	.	.	−1	1	−1	1	.	.	.	.	.	.	.	.	.	.
X.53	−1	.	.	−1	1	1	−1	.	.	.	.	.	.	.	.	.	.
X.54	.	.	.	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.
X.55	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.56	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.57	.	.	.	1	−1	1	−1	.	.	.	.	.	.	.	.	.	.
X.58	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.59	.	.	.	1	−1	−1	1	.	.	.	.	.	.	.	.	.	.
X.60	.	.	.	.	.	.	.	.	.	.	.	.	.	D	$\bar{D}$	.	.
X.61	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{D}$	D	.	.
X.62	.	−1	−1	−2	2	.	.	.	.	.	.	.	.	.	.	1	1
X.63	.	−1	−1	2	−2	.	.	.	.	.	.	.	.	.	.	1	1
X.64	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.65	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.66	−2	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.
X.67	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
X.68	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
X.69	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

$A = -\zeta(7)_7^4 - \zeta(7)_7^2 - \zeta(7)_7 - 1, B = -\zeta(11)_{11}^9 - \zeta(11)_{11}^5 - \zeta(11)_{11}^4 - \zeta(11)_{11}^3 - \zeta(11)_{11} - 1, C = 2\zeta(5)_5^3 + 2\zeta(5)_5^2 + 1, D = -2\zeta(8)_8^3 - 2\zeta(8)_8.$

B.3. *Character table of* $H(\mathrm{Fi}_{23}) = H = \langle x_0, y_0, h_0 \rangle$

2	18	18	17	17	18	18	16	9	8	7	4	12	11	11	12	10	10	10
3	9	9	6	6	4	4	3	7	9	7	7	3	2	2	1	2	2	1
5	2	2	1	1	1	1	.	1	.	.	.	.	1	1	.	.	.	.
7	1	1	1	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.
11	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
13	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$3a$	$3b$	$3c$	$3d$	$4a$	$4b$	$4c$	$4d$	$4e$	$4f$	$4g$
$2P$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$3a$	$3b$	$3c$	$3d$	$2d$	$2e$	$2e$	$2d$	$2f$	$2f$	$2e$
$3P$	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$1a$	$1a$	$1a$	$1a$	$4a$	$4b$	$4c$	$4d$	$4e$	$4f$	$4g$
$5P$	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$3a$	$3b$	$3c$	$3d$	$4a$	$4b$	$4c$	$4d$	$4e$	$4f$	$4g$
$7P$	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$3a$	$3b$	$3c$	$3d$	$4a$	$4b$	$4c$	$4d$	$4e$	$4f$	$4g$
$11P$	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$3a$	$3b$	$3c$	$3d$	$4a$	$4b$	$4c$	$4d$	$4e$	$4f$	$4g$
$13P$	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$3a$	$3b$	$3c$	$3d$	$4a$	$4b$	$4c$	$4d$	$4e$	$4f$	$4g$
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	78	78	−34	−34	14	14	−2	15	−3	6	−3	6	−6	−6	−2	2	2	2
$X.3$	352	−352	.	.	−32	32	.	−8	28	10	1	.	.	.	.	−8	8	.
$X.4$	429	429	77	77	45	45	13	6	24	15	−3	13	5	5	−3	5	5	5
$X.5$	1001	1001	−231	−231	41	41	−7	56	29	2	2	1	−11	−11	9	5	5	−3
$X.6$	1430	1430	−154	−154	86	86	6	−1	−28	26	−1	14	−6	−6	6	10	10	2
$X.7$	2080	−2080	−384	384	−96	96	.	64	−26	19	1	.	16	−16	.	−8	8	.
$X.8$	2080	−2080	384	−384	−96	96	.	64	−26	19	1	.	−16	16	.	−8	8	.
$X.9$	3003	3003	539	539	59	59	−37	105	6	15	6	11	11	11	−5	3	3	−5
$X.10$	3080	3080	616	616	136	136	40	119	2	20	2	24	24	24	8	.	.	8
$X.11$	5824	−5824	896	−896	−64	64	.	154	−8	−8	−8	.	−16	16	.	.	.	.
$X.12$	5824	−5824	−896	896	−64	64	.	154	−8	−8	−8	.	16	−16	.	.	.	.
$X.13$	5824	−5824	−896	896	−64	64	.	154	−8	−8	−8	.	16	−16	.	.	.	.
$X.14$	5824	−5824	896	−896	−64	64	.	154	−8	−8	−8	.	−16	16	.	.	.	.
$X.15$	10725	10725	−715	−715	165	165	−43	15	114	24	6	29	−15	−15	−11	9	9	−7
$X.16$	13650	13650	1330	1330	210	210	114	105	123	33	15	−14	10	10	2	10	10	10
$X.17$	13728	−13728	−1408	1408	−224	224	.	120	120	39	12	.	16	−16	.	−8	8	.
$X.18$	13728	−13728	1408	−1408	−224	224	.	120	120	39	12	.	−16	16	.	−8	8	.
$X.19$	27456	−27456	.	.	−448	448	.	−120	−84	60	−3	.	.	.	.	−16	16	.
$X.20$	30030	30030	1694	1694	526	526	62	−21	−102	60	6	38	26	26	−2	18	18	2
$X.21$	32032	32032	−2464	−2464	544	544	−32	91	−44	64	10	64	−24	−24	.	.	.	8
$X.22$	43680	43680	−4256	−4256	416	416	−32	399	−60	48	−6	.	−24	−24	.	.	.	8
$X.23$	45045	45045	4389	4389	309	309	133	441	90	−18	9	29	41	41	5	1	1	1
$X.24$	48048	−48048	4928	−4928	−272	272	.	546	96	−3	15	.	−40	40	.	−4	4	.
$X.25$	48048	−48048	−4928	4928	−272	272	.	546	96	−3	15	.	40	−40	.	−4	4	.
$X.26$	48048	48048	−1232	−1232	432	432	48	−84	258	6	−12	−16	.	.	−16	16	16	.
$X.27$	50050	50050	770	770	130	130	−126	−35	235	19	−8	−14	10	10	18	18	18	−6
$X.28$	50050	50050	−5390	−5390	450	450	−46	595	73	10	19	10	−50	−50	−14	−2	−2	6
$X.29$	75075	75075	1155	1155	835	835	131	−210	150	51	−12	83	−5	−5	19	3	3	11
$X.30$	75075	75075	7315	7315	515	515	51	735	−93	−3	−12	−5	55	55	−13	7	7	−1
$X.31$	75075	75075	−5005	−5005	−125	−125	83	420	−12	42	15	−5	15	15	3	−1	−1	−9
$X.32$	81081	81081	3465	3465	633	633	−87	.	162	81	.	33	5	5	−7	−3	−3	−3
$X.33$	105600	−105600	.	.	640	−640	.	120	−24	48	−24	.	.	.	.	.	.	.
$X.34$	105600	−105600	.	.	640	−640	.	120	−24	48	−24	.	.	.	.	.	.	.
$X.35$	114400	114400	−8800	−8800	480	480	32	685	28	−8	−26	64	−40	−40	.	.	.	−8
$X.36$	123200	−123200	.	.	−960	960	.	−280	404	62	−28	.	.	.	.	−16	16	.
$X.37$	133056	−133056	.	.	192	−192	.	.	−108	−54	54	.	.	.	.	−16	16	.
$X.38$	138600	138600	−9240	−9240	360	360	−24	630	−153	−45	9	8	−40	−40	8	8	8	−8
$X.39$	138600	138600	−9240	−9240	360	360	−24	630	−153	−45	9	8	−40	−40	8	8	8	−8
$X.40$	146432	−146432	11264	−11264	−1024	1024	.	776	−16	56	−16	.	−64	64	.	.	.	.
$X.41$	146432	−146432	−11264	11264	−1024	1024	.	776	−16	56	−16	.	64	−64	.	.	.	.
$X.42$	150150	150150	8470	8470	70	70	54	525	57	48	−24	−34	−30	−30	22	−6	−6	10
$X.43$	205920	205920	1056	1056	864	864	160	−279	−144	72	18	64	24	24	.	.	.	−8
$X.44$	228800	−228800	.	.	−320	320	.	−160	380	−52	29	.	.	.	.	−16	16	.
$X.45$	235872	−235872	−8064	8064	−1056	1056	.	.	−324	81	.	.	16	−16	.	−24	24	.
$X.46$	235872	−235872	8064	−8064	−1056	1056	.	.	−324	81	.	.	−16	16	.	−24	24	.
$X.47$	289575	289575	12375	12375	615	615	183	405	162	−81	.	15	35	35	23	3	3	−5
$X.48$	300300	300300	−7700	−7700	1420	1420	−84	−210	−291	114	−21	−4	−20	−20	−20	20	20	−4
$X.49$	320320	320320	14784	14784	1344	1344	192	406	−116	64	−8	.	16	16	.	.	.	16
$X.50$	320320	−320320	9856	−9856	576	−576	.	406	−116	64	−8	.	16	−16	.	.	.	.
$X.51$	320320	−320320	−9856	9856	576	−576	.	406	−116	64	−8	.	−16	16	.	.	.	.
$X.52$	360855	360855	18711	18711	1431	1431	279	729	.	.	.	−9	39	39	−9	−9	−9	7
$X.53$	370656	370656	−15840	−15840	1248	1248	−96	405	324	.	.	.	−40	−40	.	.	.	−8
$X.54$	400400	−400400	−24640	24640	−560	560	.	1610	−64	−37	17	.	40	−40	.	4	−4	.
$X.55$	400400	400400	6160	6160	1040	1040	16	−280	−307	−10	17	−48	.	.	16	16	16	.
$X.56$	400400	−400400	24640	−24640	−560	560	.	1610	−64	−37	17	.	−40	40	.	4	−4	.
$X.57$	400400	400400	6160	6160	1040	1040	16	−280	−307	−10	17	−48	.	.	16	16	16	.
$X.58$	400400	400400	−18480	−18480	−240	−240	80	980	98	−28	−10	16	.	.	−16	.	.	.
$X.59$	400400	400400	−18480	−18480	−240	−240	80	980	98	−28	−10	16	.	.	−16	.	.	.
$X.60$	436800	−436800	17920	−17920	320	−320	.	840	210	−42	−33	.	.	.	.	−16	16	.
$X.61$	436800	−436800	−17920	17920	320	−320	.	840	210	−42	−33	.	.	.	.	−16	16	.
$X.62$	450450	450450	25410	25410	210	210	−350	1575	171	−18	9	−6	30	30	2	−2	−2	−10
$X.63$	450450	450450	6930	6930	1170	1170	−110	−315	−72	9	9	130	10	10	−30	−14	−14	−6
$X.64$	480480	−480480	−9856	9856	−1696	1696	.	−336	−174	69	15	.	−16	16	.	8	−8	.
$X.65$	480480	−480480	9856	−9856	−1696	1696	.	−336	−174	69	15	.	16	−16	.	8	−8	.
$X.66$	576576	576576	14784	14784	576	576	−320	126	180	72	18	.	−16	−16	.	.	.	−16
$X.67$	577368	577368	−21384	−21384	216	216	−72	729	.	.	.	72	−24	−24	24	.	.	−8
$X.68$	579150	579150	−6930	−6930	1230	1230	174	−405	324	81	.	−18	−10	−10	30	6	6	−10
$X.69$	582400	582400	−8960	−8960	−1280	−1280	256	280	172	64	10	.	.	.	.	.	.	.
$X.70$	582400	582400	−8960	−8960	−1280	−1280	256	280	172	64	10	.	.	.	.	.	.	.
$X.71$	600600	600600	21560	21560	920	920	−136	525	−96	12	−42	72	40	40	24	.	.	−8
$X.72$	600600	600600	9240	9240	−1000	−1000	280	210	−15	57	−15	−8	−40	−40	−8	−8	−8	−8
$X.73$	600600	600600	9240	9240	−1000	−1000	280	210	−15	57	−15	−8	−40	−40	−8	−8	−8	−8
$X.74$	675675	675675	10395	10395	−165	−165	411	.	135	−54	54	75	−45	−45	11	−5	−5	3
$X.75$	686400	−686400	28160	−28160	−960	960	.	960	6	78	−21	.	.	.	.	16	−16	.
$X.76$	686400	−686400	−28160	28160	−960	960	.	960	6	78	−21	.	.	.	.	16	−16	.
$X.77$	720720	720720	−33264	−33264	1104	1104	−112	1386	−18	36	−18	−80	−16	−16	16	.	.	16
$X.78$	800800	800800	−12320	−12320	800	800	−416	−245	520	16	34	.	40	40	.	.	.	8
$X.79$	852930	852930	−29646	−29646	1026	1026	18	729	.	.	.	−54	6	6	18	−18	−18	−2
$X.80$	915200	−915200	28160	−28160	−1280	1280	.	440	548	8	8	.	.	.	.	.	.	.
$X.81$	915200	−915200	−28160	28160	−1280	1280	.	440	548	8	8	.	.	.	.	.	.	.
$X.82$	938223	938223	−2673	−2673	1647	1647	207	−729	.	.	.	63	15	15	15	−9	−9	−1
$X.83$	972972	972972	−24948	−24948	1836	1836	−180	.	−243	.	.	60	−36	−36	−20	−12	−12	12
$X.84$	982800	982800	15120	15120	−240	−240	−240	.	−135	54	27	−48	.	.	16	−16	−16	.
$X.85$	982800	982800	15120	15120	−240	−240	−240	.	−135	54	27	−48	.	.	16	−16	−16	.
$X.86$	1029600	−1029600	.	.	−1440	1440	.	−720	252	90	9	.	.	.	.	−8	8	.
$X.87$	1201200	−1201200	24640	−24640	−1680	1680	.	−210	132	−39	24	.	−40	40	.	12	−12	.

Character table of $H(Fi_{23})$ (continued)

2	4	9	8	8	8	7	8	8	4	8	8	9	9	5	5	8	8	7	7	7	6	6
3	1	7	9	5	5	7	6	6	7	4	4	3	3	5	5	3	3	3	3	3	3	3
5	2	1	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
7	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
13	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	5a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}
2P	5a	3a	3b	3a	3a	3c	3b	3b	3d	3b	3b	3a	3a	3c	3c	3b	3b	3c	3c	3a	3c	3c
3P	5a	2a	2a	2b	2c	2a	2c	2b	2a	2d	2e	2d	2e	2b	2c	2f	2f	2f	2f	2f	2e	2d
5P	1a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}
7P	5a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}
11P	5a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}
13P	5a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	3	15	−3	−7	−7	6	−7	−7	−3	5	5	−1	−1	2	2	1	1	−2	−2	1	2	2
X.3	2	8	−28	.	.	−10	.	.	−1	4	−4	−8	8	.	.	.	.	6	−6	.	2	−2
X.4	4	6	24	14	14	15	−4	−4	−3	.	.	6	6	5	5	4	4	7	7	−2	3	3
X.5	1	56	29	−6	−6	2	−15	−15	2	5	5	8	8	−6	−6	−7	−7	2	2	2	2	2
X.6	5	−1	−28	−19	−19	26	8	8	−1	−4	−4	−1	−1	8	8	.	.	−6	−6	−3	2	2
X.7	5	−64	26	−24	24	−19	6	−6	−1	−6	6	.	.	3	−3	6	−6	−3	3	.	3	−3
X.8	5	−64	26	24	−24	−19	−6	6	−1	−6	6	.	.	−3	3	−6	6	−3	3	.	3	−3
X.9	3	105	6	17	17	15	26	26	6	14	14	−7	−7	−1	−1	2	2	−1	−1	−7	−1	−1
X.10	5	119	2	31	31	20	22	22	2	10	10	7	7	4	4	−2	−2	4	4	7	4	4
X.11	−1	−154	8	−4	4	8	−32	32	8	8	−8	2	−2	−4	4	.	.	.	.	.	4	−4
X.12	−1	−154	8	4	−4	8	32	−32	8	8	−8	2	−2	4	−4	.	.	.	.	.	4	−4
X.13	−1	−154	8	4	−4	8	32	−32	8	8	−8	2	−2	4	−4	.	.	.	.	.	4	−4
X.14	−1	−154	8	−4	4	8	−32	32	8	8	−8	2	−2	−4	4	.	.	.	.	.	4	−4
X.15	.	15	114	5	5	24	14	14	6	−6	−6	15	15	−4	−4	−10	−10	8	8	5	.	.
X.16	.	105	123	25	25	33	7	7	15	3	3	9	9	7	7	15	15	9	9	9	−3	−3
X.17	3	−120	−120	−40	40	−39	4	−4	−12	−8	8	−8	8	−13	13	−12	12	9	−9	.	−1	1
X.18	3	−120	−120	40	−40	−39	−4	4	−12	−8	8	−8	8	13	−13	12	−12	9	−9	.	−1	1
X.19	6	120	84	.	.	−60	.	.	3	20	−20	8	−8	.	.	.	.	−12	12	.	4	−4
X.20	5	−21	−102	29	29	60	−34	−34	6	−14	−14	−5	−5	2	2	−10	−10	−4	−4	5	4	4
X.21	7	91	−44	−79	−79	64	20	20	10	4	4	−5	−5	2	2	4	4	−8	−8	1	4	4
X.22	5	399	−60	−71	−71	48	−44	−44	−6	20	20	−1	−1	10	10	4	4	−8	−8	1	−4	−4
X.23	−5	441	90	−21	−21	−18	42	42	9	−6	−6	9	9	6	6	10	10	−2	−2	19	6	6
X.24	−2	−546	−96	−4	4	3	−68	68	−15	16	−16	10	−10	5	−5	12	−12	3	−3	.	5	−5
X.25	−2	−546	−96	4	−4	3	68	−68	−15	16	−16	10	−10	−5	5	−12	12	3	−3	.	5	−5
X.26	−2	−84	258	−44	−44	6	10	10	−12	18	18	12	12	−8	−8	−6	−6	6	6	−12	6	6
X.27	.	−35	235	5	5	19	−13	−13	−8	−5	−5	13	13	5	5	3	3	3	3	−3	−5	−5
X.28	.	595	73	−35	−35	10	−71	−71	19	9	9	3	3	−8	−8	−7	−7	2	2	5	6	6
X.29	.	−210	150	30	30	51	−6	−6	−12	−2	−2	−2	−2	3	3	2	2	11	11	−10	7	7
X.30	.	735	−93	25	25	−3	79	79	−12	11	11	−1	−1	−11	−11	−9	−9	−3	−3	9	5	5
X.31	.	420	−12	−10	−10	42	−64	−64	15	28	28	4	4	−10	−10	8	8	2	2	−10	−2	−2
X.32	6	.	162	90	90	81	−18	−18	.	−6	−6	.	.	9	9	6	6	9	9	−6	−3	−3
X.33	.	−120	24	.	.	−48	.	.	24	−8	8	−8	8	.	.	.	.	.	.	.	8	−8
X.34	.	−120	24	.	.	−48	.	.	24	−8	8	−8	8	.	.	.	.	.	.	.	8	−8
X.35	.	685	28	−25	−25	−8	−52	−52	−26	12	12	−3	−3	2	2	−4	−4	8	8	−1	.	.
X.36	.	280	−404	.	.	−62	.	.	28	12	−12	−24	24	.	.	.	.	18	−18	.	6	−6
X.37	6	.	108	.	.	54	.	.	−54	12	−12	.	.	.	.	.	.	6	−6	.	−6	6
X.38	.	630	−153	30	30	−45	−33	−33	9	−9	−9	6	6	3	3	15	15	3	3	6	3	3
X.39	.	630	−153	30	30	−45	−33	−33	9	−9	−9	6	6	3	3	15	15	3	3	6	3	3
X.40	7	−776	16	104	−104	−56	−32	32	16	−16	16	8	−8	−4	4	.	.	.	.	.	4	−4
X.41	7	−776	16	−104	104	−56	32	−32	16	−16	16	8	−8	4	−4	.	.	.	.	.	4	−4
X.42	.	525	57	55	55	48	73	73	−24	25	25	13	13	−8	−8	9	9	.	.	−9	−8	−8
X.43	−5	−279	−144	−69	−69	72	−24	−24	18	.	.	9	9	12	12	−8	−8	−8	−8	−5	.	.
X.44	.	160	−380	.	.	52	.	.	−29	4	−4	−32	32	.	.	.	.	−12	12	.	−4	4
X.45	−3	.	324	−72	72	−81	−36	36	.	−12	12	.	.	9	−9	12	−12	−9	9	.	−3	3
X.46	−3	.	324	72	−72	−81	36	−36	.	−12	12	.	.	−9	9	−12	12	−9	9	.	−3	3
X.47	.	405	162	−45	−45	−81	−18	−18	.	−6	−6	21	21	9	9	6	6	−9	−9	3	3	3
X.48	.	−210	−291	−50	−50	114	49	49	−21	−11	−11	−2	−2	4	4	9	9	−6	−6	6	−2	−2
X.49	−5	406	−116	114	114	64	−12	−12	−8	12	12	6	6	6	6	−12	−12	.	.	−6	.	.
X.50	−5	−406	116	−44	44	−64	−28	28	8	36	−36	−18	18	10	−10	−12	12	.	.	.	.	.
X.51	−5	−406	116	44	−44	−64	28	−28	8	36	−36	−18	18	−10	10	12	−12	.	.	.	.	.
X.52	5	729	.	81	81	.	.	.	.	.	.	9	9	.	.	.	.	.	.	9	.	.
X.53	6	405	324	−45	−45	.	36	36	.	−12	−12	21	21	−18	−18	−12	−12	.	.	3	.	.
X.54	.	−1610	64	20	−20	37	124	−124	−17	16	−16	−14	14	−7	7	12	−12	−3	3	.	−1	1
X.55	.	−280	−307	40	40	−10	−23	−23	17	5	5	8	8	−14	−14	1	1	−2	−2	−8	2	2
X.56	.	−1610	64	−20	20	37	−124	124	−17	16	−16	−14	14	7	−7	−12	12	−3	3	.	−1	1
X.57	.	−280	−307	40	40	−10	−23	−23	17	5	5	8	8	−14	−14	1	1	−2	−2	−8	2	2
X.58	.	980	98	60	60	−28	−66	−66	−10	−6	−6	−12	−12	6	6	−10	−10	−4	−4	−4	.	.
X.59	.	980	98	60	60	−28	−66	−66	−10	−6	−6	−12	−12	6	6	−10	−10	−4	−4	−4	.	.
X.60	.	−840	−210	−80	80	42	−46	46	33	14	−14	8	−8	−8	8	18	−18	−6	6	.	−2	2
X.61	.	−840	−210	80	−80	42	46	−46	33	14	−14	8	−8	8	−8	−18	18	−6	6	.	−2	2
X.62	.	1575	171	−15	−15	−18	111	111	9	3	3	−9	−9	12	12	7	7	−2	−2	−23	6	6
X.63	.	−315	−72	45	45	9	−36	−36	9	.	.	21	21	−9	−9	4	4	1	1	−11	−3	−3
X.64	5	336	174	−136	136	−69	−26	26	−15	14	−14	−16	16	−1	1	6	−6	3	−3	.	1	−1
X.65	5	336	174	136	−136	−69	26	−26	−15	14	−14	−16	16	1	−1	−6	6	3	−3	.	1	−1
X.66	1	126	180	114	114	72	−12	−12	18	36	36	−18	−18	6	6	4	4	−8	−8	10	.	.
X.67	−7	729	.	81	81	.	.	.	.	.	.	9	9	.	.	.	.	.	.	9	.	.
X.68	.	−405	324	−45	−45	81	36	36	.	−12	−12	−21	−21	9	9	−12	−12	9	9	3	−3	−3
X.69	.	280	172	40	40	64	4	4	10	−20	−20	−8	−8	−14	−14	4	4	−8	−8	−8	4	4
X.70	.	280	172	40	40	64	4	4	10	−20	−20	−8	−8	−14	−14	4	4	−8	−8	−8	4	4
X.71	.	525	−96	5	5	12	−40	−40	−42	−16	−16	−19	−19	−22	−22	8	8	−4	−4	5	−4	−4
X.72	.	210	−15	−30	−30	57	33	33	−15	17	17	2	2	−3	−3	1	1	1	1	10	5	5
X.73	.	210	−15	−30	−30	57	33	33	−15	17	17	2	2	−3	−3	1	1	1	1	10	5	5
X.74	.	.	135	.	.	−54	27	27	54	15	15	.	.	.	.	3	3	−6	−6	.	−6	−6
X.75	.	−960	−6	80	−80	−78	−26	26	21	−6	6	.	.	−10	10	6	−6	−6	6	.	−6	6
X.76	.	−960	−6	−80	80	−78	26	−26	21	−6	6	.	.	10	−10	−6	6	−6	6	.	−6	6
X.77	−5	1386	−18	−54	−54	36	−54	−54	−18	6	6	−6	−6	.	.	2	2	−4	−4	2	.	.
X.78	.	−245	520	−35	−35	16	−8	−8	34	8	8	11	11	−8	−8	−8	−8	−8	−8	13	−4	−4
X.79	5	729	.	−81	−81	.	.	.	.	.	.	9	9	.	.	.	.	.	.	−9	.	.
X.80	.	−440	−548	80	−80	−8	28	−28	−8	−20	20	−8	8	26	−26	12	−12	.	.	.	−4	4
X.81	.	−440	−548	−80	80	−8	−28	28	−8	−20	20	−8	8	−26	26	−12	12	.	.	.	−4	4
X.82	−2	−729	.	−81	−81	.	.	.	.	.	.	−9	−9	.	.	.	.	.	.	−9	.	.
X.83	−3	.	−243	.	.	.	81	81	.	−27	−27	.	.	.	.	9	9	.	.	.	.	.
X.84	.	.	−135	.	.	54	−27	−27	27	−15	−15	.	.	.	.	−3	−3	6	6	.	6	6
X.85	.	.	−135	.	.	54	−27	−27	27	−15	−15	.	.	.	.	−3	−3	6	6	.	6	6
X.86	.	720	−252	.	.	−90	.	.	−9	36	−36	48	−48	.	.	.	.	6	−6	.	−6	6
X.87	.	210	−132	−20	20	39	92	−92	−24	12	−12	6	−6	−11	11	12	−12	−9	9	.	9	−9

Character table of $H(\mathrm{Fi}_{23})$ (continued)

2	5	5	4	4	2	7	7	7	6	6	3	2	1	4	3	3	4	4	2	2	7	7	5	6	6	6	5	5
3	3	3	3	3	1	1	1	.	.	.	4	4	3	1	1	1	.	.	.	.	2	2	3	2	2	2	2	2
5	.	.	.	.	.	.	.	.	.	.	.	.	.	2	1	1	1	1	.	.	.	.	.	.	.	.	.	.
7	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.
13	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	6_{22}	6_{23}	6_{24}	6_{25}	$7a$	$8a$	$8b$	$8c$	$8d$	$8e$	$9a$	$9b$	$9c$	$10a$	$10b$	$10c$	$10d$	$10e$	$11a$	$11b$	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$12g$	$12h$
$2P$	$3c$	$3c$	$3d$	$3d$	$7a$	$4a$	$4a$	$4d$	$4f$	$4f$	$9a$	$9b$	$9c$	$5a$	$5a$	$5a$	$5a$	$5a$	$11b$	$11a$	6_{11}	6_{11}	6_9	6_{12}	6_{12}	6_{11}	6_{15}	6_{15}
$3P$	$2f$	$2f$	$2f$	$2f$	$7a$	$8a$	$8b$	$8c$	$8d$	$8e$	$3b$	$3b$	$3d$	$10a$	$10b$	$10c$	$10d$	$10e$	$11a$	$11b$	$4a$	$4a$	$4a$	$4c$	$4b$	$4a$	$4e$	$4f$
$5P$	6_{22}	6_{23}	6_{24}	6_{25}	$7a$	$8a$	$8b$	$8c$	$8d$	$8e$	$9a$	$9b$	$9c$	$2a$	$2b$	$2c$	$2e$	$2d$	$11a$	$11b$	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$12g$	$12h$
$7P$	6_{22}	6_{23}	6_{24}	6_{25}	$1a$	$8a$	$8b$	$8c$	$8d$	$8e$	$9a$	$9b$	$9c$	$10a$	$10b$	$10c$	$10d$	$10e$	$11b$	$11a$	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$12g$	$12h$
$11P$	6_{22}	6_{23}	6_{24}	6_{25}	$7a$	$8a$	$8b$	$8c$	$8d$	$8e$	$9a$	$9b$	$9c$	$10a$	$10b$	$10c$	$10d$	$10e$	$1a$	$1a$	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$12g$	$12h$
$13P$	6_{22}	6_{23}	6_{24}	6_{25}	$7a$	$8a$	$8b$	$8c$	$8d$	$8e$	$9a$	$9b$	$9c$	$10a$	$10b$	$10c$	$10d$	$10e$	$11b$	$11a$	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$12g$	$12h$
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	−2	−2	1	1	1	2	−2	2	.	.	3	.	.	3	1	1	−1	−1	1	1	3	3	−3	−3	−3	−3	−1	−1
$X.3$	.	.	−3	3	2	.	.	.	.	.	−2	4	1	−2	.	.	2	−2	.	.	.	.	.	.	.	.	−2	2
$X.4$	1	1	1	1	2	1	1	1	1	1	.	3	.	4	2	2	.	.	.	.	−2	−2	4	2	2	−2	2	2
$X.5$	2	2	2	2	.	−3	1	1	−1	−1	2	2	−1	1	−1	−1	1	1	.	.	4	4	1	−2	−2	−2	−1	−1
$X.6$	.	.	3	3	2	−2	2	2	.	.	−1	−1	2	5	1	1	1	1	.	.	−1	−1	−4	−3	−3	5	−2	−2
$X.7$	−3	3	−3	3	1	.	.	.	.	.	4	−2	1	−5	1	−1	1	−1	1	1	.	.	.	−4	4	.	−2	2
$X.8$	3	−3	−3	3	1	.	.	.	.	.	4	−2	1	−5	−1	1	1	−1	1	1	.	.	.	4	−4	.	4	−4
$X.9$	−1	−1	2	2	.	−1	−1	3	−1	−1	3	.	.	3	−1	−1	−1	−1	.	.	5	5	2	5	5	5	.	.
$X.10$	4	4	−2	−2	.	4	4	.	.	.	5	−1	−1	5	1	1	1	1	.	.	3	3	6	3	3	3	.	.
$X.11$	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	−1	−1	1	A	$\bar{A}$	6	−6	.	−2	2	.	.	.
$X.12$	.	.	.	.	.	.	.	.	.	.	1	1	1	1	−1	1	−1	1	$\bar{A}$	A	6	−6	.	2	−2	.	.	.
$X.13$	.	.	.	.	.	.	.	.	.	.	1	1	1	1	−1	1	−1	1	A	$\bar{A}$	6	−6	.	2	−2	.	.	.
$X.14$	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	−1	−1	1	$\bar{A}$	A	6	−6	.	−2	2	.	.	.
$X.15$	−4	−4	2	2	1	1	−3	1	−1	−1	.	3	.	.	.	.	.	.	.	.	−1	−1	2	−3	−3	5	.	.
$X.16$	3	3	3	3	.	−2	−2	−2	2	2	.	3	.	.	.	.	.	.	−1	−1	1	1	−5	1	1	1	1	1
$X.17$	−3	3	.	.	1	.	.	.	.	.	3	3	.	−3	−3	3	−1	1	.	.	.	.	.	−4	4	.	−2	2
$X.18$	3	−3	.	.	1	.	.	.	.	.	3	3	.	−3	3	−3	−1	1	.	.	.	.	.	4	−4	.	−2	2
$X.19$	.	.	−3	3	2	.	.	.	.	.	−6	.	.	−6	.	.	−2	2	.	.	.	.	.	.	.	.	2	−2
$X.20$	2	2	2	2	.	2	−2	2	.	.	−3	−3	.	5	−1	−1	1	1	.	.	−1	−1	2	5	5	−7	.	.
$X.21$	−2	−2	−2	−2	.	4	−4	.	.	.	1	1	−2	7	1	1	−1	−1	.	.	−5	−5	−8	−3	−3	1	.	.
$X.22$	−2	−2	−2	−2	.	−4	4	.	.	.	3	3	.	5	−1	−1	1	1	−1	−1	3	3	.	−3	−3	−3	.	.
$X.23$	−2	−2	1	1	.	1	5	1	−1	−1	.	.	.	−5	−1	−1	−1	−1	.	.	5	5	2	−1	−1	−1	−2	−2
$X.24$	3	−3	3	−3	.	.	.	.	2	−2	3	.	.	2	−2	2	2	−2	.	.	6	−6	.	−2	2	.	2	−2
$X.25$	−3	3	3	−3	.	.	.	.	−2	2	3	.	.	2	2	−2	2	−2	.	.	6	−6	.	2	−2	.	2	−2
$X.26$	.	.	.	.	.	.	.	.	.	.	.	3	.	−2	−2	−2	2	2	.	.	−4	−4	2	.	.	−4	−2	−2
$X.27$	−3	−3	.	.	.	2	2	−2	−2	−2	1	1	1	.	.	.	.	.	.	.	1	1	−5	1	1	1	3	3
$X.28$	−4	−4	−1	−1	.	2	−2	−2	.	.	4	−2	1	.	.	.	.	.	.	.	7	7	1	1	1	1	1	1
$X.29$	−1	−1	−4	−4	.	3	3	−1	−1	−1	−6	.	.	.	.	.	.	.	.	.	2	2	2	−2	−2	2	.	.
$X.30$	−3	−3	.	.	.	−5	−1	−1	1	1	3	.	.	.	.	.	.	.	.	.	7	7	−5	1	1	1	1	1
$X.31$	2	2	−1	−1	.	−1	3	3	1	1	−3	−3	.	.	.	.	.	.	.	.	4	4	4	−6	−6	−2	2	2
$X.32$	−3	−3	.	.	.	−7	−3	1	−1	−1	.	.	.	6	.	.	−2	−2	.	.	.	.	6	2	2	−6	.	.
$X.33$	.	.	.	.	−2	.	.	.	.	.	−3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.34$	.	.	.	.	−2	.	.	.	.	.	−3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.35$	2	2	2	2	−1	4	−4	.	.	.	4	−2	1	.	.	.	.	.	.	.	1	1	−8	−1	−1	−5	.	.
$X.36$	.	.	.	.	.	.	.	.	.	.	−4	2	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2
$X.37$	.	.	6	−6	.	.	.	.	.	.	.	.	.	−6	.	.	−2	2	.	.	.	.	.	.	.	.	2	−2
$X.38$	3	3	−3	−3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	−1	2	2	2	−1	−1
$X.39$	3	3	−3	−3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	−1	2	2	2	−1	−1
$X.40$	.	.	.	.	−1	.	.	.	.	.	5	−1	−1	−7	−1	1	−1	1	.	.	.	.	.	4	−4	.	.	.
$X.41$	.	.	.	.	−1	.	.	.	.	.	5	−1	−1	−7	1	−1	−1	1	.	.	.	.	.	−4	4	.	.	.
$X.42$	.	.	.	.	.	2	−2	2	.	.	−3	.	.	.	.	.	.	.	.	.	5	5	−7	3	3	−1	−3	−3
$X.43$	4	4	−2	−2	1	−4	4	.	.	.	.	.	.	−5	1	1	−1	−1	.	.	1	1	−8	3	3	−5	.	.
$X.44$	.	.	−3	3	−2	.	.	.	.	.	2	2	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2
$X.45$	3	−3	.	.	.	.	.	.	.	.	.	.	.	3	1	−1	1	−1	−1	−1	.	.	.	−4	4	.	.	.
$X.46$	−3	3	.	.	.	.	.	.	.	.	.	.	.	3	−1	1	1	−1	−1	−1	.	.	.	4	−4	.	.	.
$X.47$	−3	−3	.	.	−1	−1	−5	3	1	1	.	.	.	.	.	.	.	.	.	.	−3	−3	6	−1	−1	3	.	.
$X.48$	.	.	3	3	.	.	.	−4	.	.	−3	−3	.	.	.	.	.	.	.	.	2	2	5	−2	−2	2	−1	−1
$X.49$	6	6	.	.	.	.	.	.	.	.	4	−2	1	−5	−1	−1	−1	−1	.	.	−6	−6	.	−2	−2	6	.	.
$X.50$	6	−6	.	.	.	.	.	.	.	.	4	−2	1	5	1	−1	−1	1	.	.	−6	6	.	2	−2	.	.	.
$X.51$	−6	6	.	.	.	.	.	.	.	.	4	−2	1	5	−1	1	−1	1	.	.	−6	6	.	−2	2	.	.	.
$X.52$	.	.	.	.	−2	−1	−1	−1	−1	−1	.	.	.	5	1	1	1	1	.	.	−3	−3	.	−3	−3	−3	.	.
$X.53$	6	6	.	.	−1	−4	4	.	.	.	.	.	.	6	.	.	−2	−2	.	.	−3	−3	.	−1	−1	3	.	.
$X.54$	3	−3	−3	3	.	.	.	.	2	−2	−1	−1	−1	.	.	.	.	.	.	.	6	−6	.	2	−2	.	−2	2
$X.55$	−2	−2	1	1	.	.	.	.	.	.	−1	−1	−1	.	.	.	.	.	.	.	.	.	−3	.	.	.	−5	−5
$X.56$	−3	3	−3	3	.	.	.	.	−2	2	−1	−1	−1	.	.	.	.	.	.	.	6	−6	.	−2	2	.	−2	2
$X.57$	−2	−2	1	1	.	.	.	.	.	.	−1	−1	−1	.	.	.	.	.	.	.	.	.	−3	.	.	.	7	7
$X.58$	2	2	2	2	.	.	.	.	.	.	−4	2	−1	.	.	.	.	.	.	.	4	4	−2	.	.	4	.	.
$X.59$	2	2	2	2	.	.	.	.	.	.	−4	2	−1	.	.	.	.	.	.	.	4	4	−2	.	.	4	.	.
$X.60$	.	.	3	−3	.	.	.	.	.	.	−3	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	2	−2
$X.61$	.	.	3	−3	.	.	.	.	.	.	−3	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	−4	4
$X.62$	4	4	1	1	.	2	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.	3	3	3	−3	−3	−3	1	1
$X.63$	−5	−5	1	1	.	2	2	−2	2	2	.	.	.	.	.	.	.	.	.	.	1	1	4	1	1	1	−2	−2
$X.64$	−3	3	3	−3	.	.	.	.	.	.	−3	−3	.	−5	−1	1	1	−1	.	.	.	.	.	4	−4	.	−4	4
$X.65$	3	−3	3	−3	.	.	.	.	.	.	−3	−3	.	−5	1	−1	1	−1	.	.	.	.	.	−4	4	.	2	−2
$X.66$	−2	−2	−2	−2	.	.	.	.	.	.	.	.	.	1	−1	−1	1	1	.	.	−6	−6	.	2	2	6	.	.
$X.67$	.	.	.	.	1	−4	−4	.	.	.	.	.	.	−7	1	1	1	1	.	.	−3	−3	.	−3	−3	−3	.	.
$X.68$	−3	−3	.	.	−2	2	2	2	2	2	.	.	.	.	.	.	.	.	.	.	3	3	.	−1	−1	3	.	.
$X.69$	−2	−2	−2	−2	.	.	.	.	.	.	1	1	1	.	.	.	.	.	$\bar{A}$	A	.	.	.	.	.	.	.	.
$X.70$	−2	−2	−2	−2	.	.	.	.	.	.	1	1	1	.	.	.	.	.	A	$\bar{A}$	.	.	.	.	.	.	.	.
$X.71$	2	2	2	2	.	4	4	.	.	.	3	3	.	.	.	.	.	.	.	.	−3	−3	.	1	1	−3	.	.
$X.72$	1	1	1	1	.	.	.	.	.	.	−3	.	.	.	.	.	.	.	.	.	−2	−2	1	2	2	−2	1	1
$X.73$	1	1	1	1	.	.	.	.	.	.	−3	.	.	.	.	.	.	.	.	.	−2	−2	1	2	2	−2	1	1
$X.74$	.	.	6	6	.	3	3	−1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	3	.	.	.	1	1
$X.75$	6	−6	3	−3	1	.	.	.	.	.	.	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.76$	−6	6	3	−3	1	.	.	.	.	.	.	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4
$X.77$	−4	−4	2	2	.	.	.	.	.	.	.	.	.	−5	1	1	−1	−1	.	.	−2	−2	10	2	2	−2	.	.
$X.78$	4	4	−2	−2	.	4	−4	.	.	.	−5	1	1	.	.	.	.	.	.	.	3	3	.	1	1	−3	.	.
$X.79$	.	.	.	.	1	−2	2	−2	.	.	.	.	.	5	−1	−1	1	1	1	1	−3	−3	.	3	3	3	.	.
$X.80$	−6	6	.	.	−1	.	.	.	.	.	−1	−1	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.81$	6	−6	.	.	−1	.	.	.	.	.	−1	−1	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.82$	.	.	.	.	−1	−5	−5	−1	−1	−1	.	.	.	−2	2	2	2	2	.	.	3	3	.	3	3	3	.	.
$X.83$	.	.	.	.	.	.	.	4	.	.	.	.	.	−3	−3	−3	1	1	.	.	.	.	−3	.	.	.	3	3
$X.84$	.	.	3	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{A}$	A	.	.	−3	.	.	.	−1	−1
$X.85$	.	.	3	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	A	$\bar{A}$	.	.	−3	.	.	.	−1	−1
$X.86$	.	.	−3	3	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.87$	3	−3	.	.	.	.	.	.	−2	2	.	3	.	.	.	.	.	.	.	.	−6	6	.	−2	2	.	.	.

Character table of $H(\mathrm{Fi}_{23})$ (continued)

2	5	5	5	5	4	4	5	5	3	3	1	1	2	2	2	2	5	5	3	2	3	3	3	3	2	2	3
3	2	2	2	2	2	2	1	1	2	2	.	.	1	.	.	1	.	.	4	4	3	3	3	3	3	3	2
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
7	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
13	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	13a	13b	14a	14b	14c	15a	16a	16b	18a	18b	18c	18d	18e	18f	18g	18h	18i
2P	6_{16}	6_{18}	6_{18}	6_{16}	6_{20}	6_{20}	6_{12}	6_{9}	6_{24}	6_{24}	13b	13a	7a	7a	7a	15a	8c	8c	9a	9b	9a	9a	9a	9a	9b	9b	9a
3P	4f	4e	4f	4e	4b	4c	4g	4d	4f	4e	13a	13b	14a	14b	14c	5a	16a	16b	6_2	6_2	6_6	6_7	6_6	6_7	6_6	6_7	6_{10}
5P	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	13b	13a	14a	14b	14c	3a	16b	16a	18a	18b	18e	18f	18c	18d	18g	18h	18i
7P	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	13b	13a	2a	2b	2c	15a	16b	16a	18a	18b	18c	18d	18e	18f	18g	18h	18i
11P	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	13b	13a	14a	14b	14c	15a	16a	16b	18a	18b	18e	18f	18c	18d	18g	18h	18i
13P	12i	12j	12k	12l	12m	12n	12o	12p	12q	12r	1a	1a	14a	14b	14c	15a	16b	16a	18a	18b	18c	18d	18e	18f	18g	18h	18i
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	-1	2	2	-1	.	.	-1	1	-1	-1	.	.	1	1	1	.	.	.	3	.	-1	-1	-1	-1	2	2	-1
X.3	2	-2	2	-2	.	.	.	.	-1	1	1	1	-2	.	.	2	.	.	2	-4	.	.	.	.	.	.	2
X.4	2	-1	-1	2	-1	-1	2	.	-1	-1	.	.	2	.	.	1	-1	-1	.	3	2	2	2	2	-1	-1	.
X.5	-1	2	2	-1	-2	-2	.	-3	2	2	.	.	.	.	.	1	1	1	2	2	.	.	.	.	.	.	2
X.6	-2	-2	-2	-2	.	.	-1	.	1	1	.	.	2	.	.	-1	.	.	-1	-1	-1	-1	-1	-1	-1	-1	-1
X.7	-4	1	-1	4	1	-1	.	.	-1	1	.	.	-1	1	-1	-1	.	.	-4	2	.	.	.	.	.	.	.
X.8	2	1	-1	-2	-1	1	.	.	-1	1	.	.	-1	-1	1	-1	.	.	-4	2	.	.	.	.	.	.	.
X.9	.	3	3	.	-1	-1	1	-2	.	.	.	.	.	.	.	.	-1	-1	3	.	-1	-1	-1	-1	2	2	-1
X.10	.	.	.	.	.	.	-1	2	.	.	-1	-1	.	.	.	-1	.	.	5	-1	1	1	1	1	1	1	1
X.11	.	.	.	.	2	-2	.	.	.	.	.	.	.	.	.	-1	.	.	-1	-1	1	-1	1	-1	1	-1	1
X.12	.	.	.	.	-2	2	.	.	.	.	.	.	.	.	.	-1	.	.	-1	-1	-1	1	-1	1	-1	1	1
X.13	.	.	.	.	-2	2	.	.	.	.	.	.	.	.	.	-1	.	.	-1	-1	-1	1	-1	1	-1	1	1
X.14	.	.	.	.	2	-2	.	.	.	.	.	.	.	.	.	-1	.	.	-1	-1	1	-1	1	-1	1	-1	1
X.15	.	.	.	.	.	.	-1	-2	.	.	.	.	1	-1	-1	.	-1	-1	.	3	2	2	2	2	-1	-1	.
X.16	1	1	1	1	1	1	1	-1	1	1	.	.	.	.	.	.	.	.	.	3	-2	-2	-2	-2	1	1	.
X.17	2	1	-1	-2	1	-1	.	.	2	-2	.	.	-1	-1	1	.	.	.	-3	-3	1	-1	1	-1	1	-1	-1
X.18	2	1	-1	-2	-1	1	.	.	2	-2	.	.	-1	1	-1	.	.	.	-3	-3	-1	1	-1	1	-1	1	-1
X.19	-2	-4	4	2	.	.	.	.	1	-1	.	.	-2	.	.	.	.	.	6	.	.	.	.	.	.	.	-2
X.20	.	.	.	.	2	2	-1	-2	.	.	.	.	.	.	.	-1	.	.	-3	-3	-1	-1	-1	-1	-1	-1	1
X.21	.	.	.	.	.	.	-1	.	.	.	.	.	.	.	.	1	.	.	1	1	-1	-1	-1	-1	-1	-1	1
X.22	.	.	.	.	.	.	-1	.	.	.	.	.	.	.	.	-1	.	.	3	3	1	1	1	1	1	1	-1
X.23	-2	-2	-2	-2	2	2	1	2	1	1	.	.	.	.	.	1	-1	-1	.	.	.	.	.	.	.	.	.
X.24	-2	-1	1	2	-1	1	.	.	1	-1	.	.	.	.	.	1	.	.	-3	.	1	-1	1	-1	-2	2	-1
X.25	-2	-1	1	2	1	-1	.	.	1	-1	.	.	.	.	.	1	.	.	-3	.	-1	1	-1	1	2	-2	-1
X.26	-2	-2	-2	-2	.	.	.	2	-2	-2	.	.	.	.	.	1	.	.	.	3	-2	-2	-2	-2	1	1	.
X.27	3	3	3	3	1	1	-3	3	.	.	.	.	.	.	.	.	.	.	1	1	-1	-1	-1	-1	-1	-1	1
X.28	1	-2	-2	1	-2	-2	3	1	1	1	.	.	.	.	.	.	.	.	4	-2	-2	-2	-2	-2	-2	-2	.
X.29	.	3	3	.	1	1	2	-2	.	.	.	.	.	.	.	.	1	1	-6	.	.	.	.	.	.	.	-2
X.30	1	1	1	1	1	1	-1	-1	-2	-2	.	.	.	.	.	.	1	1	3	.	1	1	1	1	-2	-2	-1
X.31	2	2	2	2	.	.	.	.	-1	-1	.	.	.	.	.	.	-1	-1	-3	-3	-1	-1	-1	-1	-1	-1	1
X.32	.	-3	-3	.	-1	-1	.	2	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.
X.33	.	.	.	.	.	.	.	.	.	.	1	1	2	.	.	.	.	.	3	.	B	$\bar{B}$	$\bar{B}$	B	.	.	-1
X.34	.	.	.	.	.	.	.	.	.	.	1	1	2	.	.	.	.	.	3	.	$\bar{B}$	B	B	$\bar{B}$	.	.	-1
X.35	.	.	.	.	2	2	1	.	.	.	.	.	-1	-1	-1	.	.	.	4	-2	2	2	2	2	2	2	.
X.36	-2	2	-2	2	.	.	.	.	-2	2	-1	-1	.	.	.	.	.	.	4	-2	.	.	.	.	.	.	.
X.37	-2	2	-2	2	.	.	.	.	-2	2	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.38	-1	-1	-1	-1	-1	-1	-2	-1	-1	-1	C	D	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.39	-1	-1	-1	-1	-1	-1	-2	-1	-1	-1	D	C	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.40	.	.	.	.	2	-2	.	.	.	.	.	.	1	1	-1	1	.	.	-5	1	1	-1	1	-1	1	-1	1
X.41	.	.	.	.	-2	2	.	.	.	.	.	.	1	-1	1	1	.	.	-5	1	-1	1	-1	1	-1	1	1
X.42	-3	.	.	-3	.	.	1	1	.	.	.	.	.	.	.	.	.	.	-3	.	1	1	1	1	-2	-2	1
X.43	.	.	.	.	.	.	1	.	.	.	.	.	1	-1	-1	1	.	.	.	.	.	.	.	.	.	.	.
X.44	-2	-4	4	2	.	.	.	.	1	-1	.	.	2	.	.	.	.	.	-2	-2	.	.	.	.	.	.	2
X.45	.	3	-3	.	1	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.46	.	3	-3	.	-1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.47	.	3	3	.	-1	-1	1	2	.	.	.	.	-1	-1	-1	.	1	1	.	.	.	.	.	.	.	.	.
X.48	-1	2	2	-1	-2	-2	2	1	-1	-1	.	.	.	.	.	.	.	.	-3	-3	1	1	1	1	1	1	1
X.49	.	.	.	.	-2	-2	-2	.	.	.	.	.	.	.	.	1	.	.	4	-2	.	.	.	.	.	.	.
X.50	.	.	.	.	-2	2	.	.	.	.	.	.	.	.	.	1	.	.	-4	2	2	-2	2	-2	2	-2	.
X.51	.	.	.	.	2	-2	.	.	.	.	.	.	.	.	.	1	.	.	-4	2	-2	2	-2	2	-2	2	.
X.52	.	.	.	.	.	.	1	.	.	.	1	1	-2	.	.	-1	-1	-1	.	.	.	.	.	.	.	.	.
X.53	.	.	.	.	2	2	1	.	.	.	.	.	-1	1	1	.	.	.	.	.	.	.	.	.	.	.	.
X.54	2	1	-1	-2	1	-1	.	.	-1	1	.	.	.	.	.	.	.	.	1	1	1	-1	1	-1	1	-1	-1
X.55	7	-2	-2	7	.	.	.	1	1	1	.	.	.	.	.	.	.	.	-1	-1	1	1	1	1	1	1	-1
X.56	2	1	-1	-2	-1	1	.	.	-1	1	.	.	.	.	.	.	.	.	1	1	-1	1	-1	1	-1	1	-1
X.57	-5	-2	-2	-5	.	.	.	1	1	1	.	.	.	.	.	.	.	.	-1	-1	1	1	1	1	1	1	-1
X.58	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	E	$\bar{E}$	-4	2	.	.	.	.	.	.	.
X.59	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	$\bar{E}$	E	-4	2	.	.	.	.	.	.	.
X.60	4	2	-2	-4	.	.	.	.	1	-1	.	.	.	.	.	.	.	.	3	.	-1	1	-1	1	2	-2	1
X.61	-2	2	-2	2	.	.	.	.	1	-1	.	.	.	.	.	.	.	.	3	.	1	-1	1	-1	-2	2	1
X.62	1	-2	-2	1	.	.	-1	-1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.63	-2	1	1	-2	1	1	-3	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.64	-2	-1	1	2	-1	1	.	.	1	-1	.	.	.	.	.	-1	.	.	3	3	1	-1	1	-1	1	-1	1
X.65	4	-1	1	-4	1	-1	.	.	1	-1	.	.	.	.	.	-1	.	.	3	3	-1	1	-1	1	-1	1	1
X.66	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
X.67	.	.	.	.	.	.	1	.	.	.	-1	-1	1	1	1	-1	.	.	.	.	.	.	.	.	.	.	.
X.68	.	-3	-3	.	-1	-1	-1	.	.	.	.	.	-2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.69	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	1	1	1	1
X.70	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	1	1	1	1
X.71	.	.	.	.	-2	-2	1	.	.	.	.	.	.	.	.	.	.	.	3	3	-1	-1	-1	-1	-1	-1	-1
X.72	1	1	1	1	-1	-1	-2	1	1	1	.	.	.	.	.	.	.	.	-3	.	B	B	$\bar{B}$	$\bar{B}$	.	.	-1
X.73	1	1	1	1	-1	-1	-2	1	1	1	.	.	.	.	.	.	.	.	-3	.	$\bar{B}$	$\bar{B}$	B	B	.	.	-1
X.74	1	-2	-2	1	.	.	.	-1	-2	-2	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.
X.75	-4	-2	2	4	.	.	.	.	-1	1	.	.	-1	-1	1	.	.	.	.	-3	-2	2	-2	2	1	-1	.
X.76	2	-2	2	-2	.	.	.	.	-1	1	.	.	-1	1	-1	.	.	.	.	-3	2	-2	2	-2	-1	1	.
X.77	.	.	.	.	2	2	-2	-2	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
X.78	.	.	.	.	-2	-2	-1	.	.	.	.	.	.	.	.	.	.	.	-5	1	1	1	1	1	1	1	-1
X.79	.	.	.	.	.	.	1	.	.	.	.	.	1	-1	-1	-1	.	.	.	.	.	.	.	.	.	.	.
X.80	.	.	.	.	.	.	.	.	.	.	.	.	1	-1	1	.	.	.	1	1	1	-1	1	-1	1	-1	-1
X.81	.	.	.	.	.	.	.	.	.	.	.	.	1	1	-1	.	.	.	1	1	-1	1	-1	1	-1	1	-1
X.82	.	.	.	.	.	.	-1	.	.	.	.	.	-1	1	1	1	-1	-1	.	.	.	.	.	.	.	.	.
X.83	3	.	.	3	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.84	-1	2	2	-1	.	.	.	1	-1	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.85	-1	2	2	-1	.	.	.	1	-1	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.86	2	-2	2	-2	.	.	.	.	-1	1	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.87	.	3	-3	.	-1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	-3	2	-2	2	-2	-1	1	.

Character table of $H(\mathrm{Fi}_{23})$ *(continued)*

2	3	1	3	3	1	2	2	2	2	2	2	4	4	1	1	2	2	2	1
3	2	3	.	.	1	.	.	.	.	.	.	1	1	.	.	1	1	1	1
5	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	1	1	1	.
7	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
11	.	.	.	.	.	1	1	1	1	1	1	.	.	.	.	.	.	.	.
13	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.
	18*j*	18*k*	20*a*	20*b*	21*a*	22*a*	22*b*	22*c*	22*d*	22*e*	22*f*	24*a*	24*b*	26*a*	26*b*	30*a*	30*b*	30*c*	42*a*
2*P*	9*a*	9*c*	10*d*	10*d*	21*a*	11*a*	11*a*	11*b*	11*b*	11*b*	11*a*	12*a*	12*b*	13*a*	13*b*	15*a*	15*a*	15*a*	21*a*
3*P*	6_9	6_8	20*a*	20*b*	7*a*	22*a*	22*b*	22*c*	22*d*	22*e*	22*f*	8*a*	8*b*	26*a*	26*b*	10*b*	10*c*	10*a*	14*a*
5*P*	18*j*	18*k*	4*b*	4*c*	21*a*	22*a*	22*b*	22*c*	22*d*	22*e*	22*f*	24*a*	24*b*	26*b*	26*a*	6_3	6_4	6_1	42*a*
7*P*	18*j*	18*k*	20*a*	20*b*	3*a*	22*c*	22*e*	22*a*	22*f*	22*b*	22*d*	24*a*	24*b*	26*b*	26*a*	30*a*	30*b*	30*c*	6_1
11*P*	18*j*	18*k*	20*a*	20*b*	21*a*	2*a*	2*c*	2*a*	2*b*	2*c*	2*b*	24*a*	24*b*	26*b*	26*a*	30*a*	30*b*	30*c*	42*a*
13*P*	18*j*	18*k*	20*a*	20*b*	21*a*	22*c*	22*e*	22*a*	22*f*	22*b*	22*d*	24*a*	24*b*	2*a*	2*a*	30*a*	30*b*	30*c*	42*a*
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	−1	.	−1	−1	1	1	−1	1	−1	−1	−1	−1	1	.	.	−2	−2	.	1
X.3	−2	−1	.	.	−1	.	.	.	.	.	.	.	.	−1	−1	.	.	−2	1
X.4	.	.	.	.	−1	.	.	.	.	.	.	−2	−2	.	.	−1	−1	1	−1
X.5	2	−1	−1	−1	.	.	.	.	.	.	.	.	−2	.	.	−1	−1	1	.
X.6	−1	2	−1	−1	−1	.	.	.	.	.	.	1	−1	.	.	1	1	−1	−1
X.7	.	−1	1	−1	1	−1	−1	−1	1	−1	1	.	.	.	.	1	−1	1	−1
X.8	.	−1	−1	1	1	−1	1	−1	−1	1	−1	.	.	.	.	−1	1	1	−1
X.9	−1	.	1	1	.	.	.	.	.	.	.	−1	−1	.	.	2	2	.	.
X.10	1	−1	−1	−1	.	.	.	.	.	.	.	1	1	−1	−1	1	1	−1	.
X.11	−1	−1	−1	1	.	$-\bar{A}$	$-\bar{A}$	$-A$	A	$-A$	$\bar{A}$	.	.	.	.	1	−1	1	.
X.12	−1	−1	1	−1	.	$-A$	A	$-\bar{A}$	$-\bar{A}$	$\bar{A}$	$-A$	.	.	.	.	−1	1	1	.
X.13	−1	−1	1	−1	.	$-\bar{A}$	$\bar{A}$	$-A$	$-A$	A	$-\bar{A}$	.	.	.	.	−1	1	1	.
X.14	−1	−1	−1	1	.	$-A$	$-A$	$-\bar{A}$	$\bar{A}$	$-\bar{A}$	A	.	.	.	.	1	−1	1	.
X.15	.	.	.	.	1	.	.	.	.	.	.	1	3	.	.	.	.	.	1
X.16	.	.	.	.	.	−1	−1	−1	−1	−1	−1	1	1	.	.	.	.	.	.
X.17	1	.	1	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1
X.18	1	.	−1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1
X.19	2	.	.	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
X.20	1	.	1	1	.	.	.	.	.	.	.	−1	1	.	.	−1	−1	−1	.
X.21	1	−2	1	1	.	.	.	.	.	.	.	1	−1	.	.	1	1	1	.
X.22	−1	.	1	1	.	−1	1	−1	1	1	1	−1	1	.	.	−1	−1	−1	.
X.23	.	.	1	1	.	.	.	.	.	.	.	1	−1	.	.	−1	−1	1	.
X.24	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1	.
X.25	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	.
X.26	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	.
X.27	1	1	.	.	.	.	.	.	.	.	.	−1	−1	.	.	.	.	.	.
X.28	.	1	.	.	.	.	.	.	.	.	.	−1	1	.	.	.	.	.	.
X.29	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.30	−1	.	.	.	.	.	.	.	.	.	.	1	−1	.	.	.	.	.	.
X.31	1	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.
X.32	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.
X.33	1	.	.	.	1	.	.	.	.	.	.	.	.	−1	−1	.	.	.	−1
X.34	1	.	.	.	1	.	.	.	.	.	.	.	.	−1	−1	.	.	.	−1
X.35	.	1	.	.	−1	.	.	.	.	.	.	1	−1	.	.	.	.	.	−1
X.36	.	1	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.
X.37	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	.	.	.	.
X.38	.	.	.	.	.	.	.	.	.	.	.	.	.	D	C	.	.	.	.
X.39	.	.	.	.	.	.	.	.	.	.	.	.	.	C	D	.	.	.	.
X.40	−1	1	1	−1	−1	.	.	.	.	.	.	.	.	.	.	−1	1	−1	1
X.41	−1	1	−1	1	−1	.	.	.	.	.	.	.	.	.	.	1	−1	−1	1
X.42	1	.	.	.	.	.	.	.	.	.	.	−1	1	.	.	.	.	.	.
X.43	.	.	−1	−1	1	.	.	.	.	.	.	−1	1	.	.	1	1	1	1
X.44	−2	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1
X.45	.	.	1	−1	.	1	1	1	−1	1	−1	.	.	.	.	−2	2	.	.
X.46	.	.	−1	1	.	1	−1	1	1	−1	1	.	.	.	.	2	−2	.	.
X.47	.	.	.	.	−1	.	.	.	.	.	.	−1	1	.	.	.	.	.	−1
X.48	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.49	.	1	1	1	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	.
X.50	.	−1	1	−1	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1	.
X.51	.	−1	−1	1	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	.
X.52	.	.	−1	−1	1	.	.	.	.	.	.	−1	−1	1	1	1	1	−1	1
X.53	.	.	.	.	−1	.	.	.	.	.	.	−1	1	.	.	.	.	.	−1
X.54	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.55	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.56	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.57	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.58	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.59	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.60	−1	.	.	.	.	−1	−1	−1	1	−1	1	.	.	.	.	.	.	.	.
X.61	−1	.	.	.	.	−1	1	−1	−1	1	−1	.	.	.	.	.	.	.	.
X.62	.	.	.	.	.	.	.	.	.	.	.	−1	1	.	.	.	.	.	.
X.63	.	.	.	.	.	.	.	.	.	.	.	−1	−1	.	.	.	.	.	.
X.64	−1	.	−1	1	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	.
X.65	−1	.	1	−1	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	.
X.66	.	.	−1	−1	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	.
X.67	.	.	1	1	1	.	.	.	.	.	.	−1	−1	−1	−1	1	1	−1	1
X.68	.	.	.	.	1	.	.	.	.	.	.	−1	−1	.	.	.	.	.	1
X.69	1	1	.	.	.	A	A	$\bar{A}$	$\bar{A}$	$\bar{A}$	A	.	.	.	.	.	.	.	.
X.70	1	1	.	.	.	$\bar{A}$	$\bar{A}$	A	A	A	$\bar{A}$	.	.	.	.	.	.	.	.
X.71	−1	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.
X.72	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.73	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.74	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.75	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1
X.76	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1
X.77	.	.	−1	−1	.	.	.	.	.	.	.	.	.	.	.	1	1	1	.
X.78	−1	1	.	.	.	.	.	.	.	.	.	1	−1	.	.	.	.	.	.
X.79	.	.	1	1	1	1	−1	1	−1	−1	−1	1	−1	.	.	−1	−1	−1	1
X.80	1	−2	.	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
X.81	1	−2	.	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
X.82	.	.	.	.	−1	.	.	.	.	.	.	1	1	.	.	−1	−1	1	−1
X.83	.	.	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.84	.	.	.	.	.	A	$-A$	$\bar{A}$	$-\bar{A}$	$-\bar{A}$	$-A$	.	.	.	.	.	.	.	.
X.85	.	.	.	.	.	$\bar{A}$	$-\bar{A}$	A	$-A$	$-A$	$-\bar{A}$	.	.	.	.	.	.	.	.
X.86	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1
X.87	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(\mathrm{Fi}_{23})$ *(continued)*

	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g
2	18	18	17	17	18	18	16	9	8	7	4	12	11	11	12	10	10	10
3	9	9	6	6	4	4	3	7	9	7	7	3	2	2	1	2	2	1
5	2	2	1	1	1	1	.	1	.	.	.	.	1	1	.	.	.	.
7	1	1	1	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.
11	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
13	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2P	1a	1a	1a	1a	1a	1a	1a	3a	3b	3c	3d	2d	2e	2e	2d	2f	2f	2e
3P	1a	2a	2b	2c	2d	2e	2f	1a	1a	1a	1a	4a	4b	4c	4d	4e	4f	4g
5P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g
7P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g
11P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g
13P	1a	2a	2b	2c	2d	2e	2f	3a	3b	3c	3d	4a	4b	4c	4d	4e	4f	4g
X.88	1201200	1201200	−30800	−30800	560	560	176	420	−30	−48	51	−16	.	.	−16	16	16	.
X.89	1201200	−1201200	−24640	24640	−1680	1680	.	−210	132	−39	24	.	40	−40	.	12	−12	.
X.90	1297296	−1297296	44352	−44352	−1200	1200	.	1134	−324	−81	.	.	−40	40	.	−12	12	.
X.91	1297296	−1297296	−44352	44352	−1200	1200	.	1134	−324	−81	.	.	40	−40	.	−12	12	.
X.92	1360800	1360800	30240	30240	1440	1440	288	.	486	.	.	−96	.	.	−32	.	.	.
X.93	1372800	1372800	49280	49280	640	640	128	1560	−312	−24	12	.	.	.	.	.	.	.
X.94	1441792	1441792	.	.	.	.	.	−512	640	64	−8	.	.	.	.	.	.	.
X.95	1441792	−1441792	.	.	.	.	.	−512	640	64	−8	.	.	.	.	.	.	.
X.96	1663200	−1663200	.	.	2400	−2400	.	.	108	54	27	.	.	.	.	−8	8	.
X.97	1663200	−1663200	.	.	2400	−2400	.	.	108	54	27	.	.	.	.	−8	8	.
X.98	1791153	1791153	−5103	−5103	−2511	−2511	81	.	.	.	.	81	9	9	33	9	9	9
X.99	1876446	1876446	37422	37422	−1890	−1890	−306	729	.	.	.	54	−30	−30	−18	18	18	10
X.100	2027025	2027025	−24255	−24255	−1455	−1455	33	.	−324	81	.	9	85	85	−15	−3	−3	−3
X.101	2050048	2050048	.	.	2048	2048	.	−1232	−224	−80	−8	.	.	.	.	.	.	.
X.102	2196480	−2196480	11264	−11264	1024	−1024	.	−456	−240	−24	−24	.	64	−64	.	.	.	.
X.103	2196480	−2196480	−11264	11264	1024	−1024	.	−456	−240	−24	−24	.	−64	64	.	.	.	.
X.104	2316600	2316600	−43560	−43560	−840	−840	24	405	−162	.	.	−24	40	40	−8	.	.	−8
X.105	2358720	−2358720	8064	−8064	−1344	1344	.	−1134	−324	.	.	.	−16	16	.	.	.	.
X.106	2358720	−2358720	−8064	8064	−1344	1344	.	−1134	−324	.	.	.	16	−16	.	.	.	.
X.107	2402400	2402400	12320	12320	−160	−160	160	−735	−384	48	−6	−64	−40	−40	.	.	.	−8
X.108	2402400	−2402400	49280	−49280	1760	−1760	.	840	−60	66	21	.	80	−80	.	8	−8	.
X.109	2402400	−2402400	−49280	49280	1760	−1760	.	840	−60	66	21	.	−80	80	.	8	−8	.
X.110	2555904	2555904	−32768	−32768	.	.	.	−384	192	−96	−24	.	.	.	.	.	.	.
X.111	2555904	−2555904	32768	−32768	.	.	.	−384	192	−96	−24	.	.	.	.	.	.	.
X.112	2555904	−2555904	−32768	32768	.	.	.	−384	192	−96	−24	.	.	.	.	.	.	.
X.113	2555904	2555904	32768	32768	.	.	.	−384	192	−96	−24	.	.	.	.	.	.	.
X.114	2729376	2729376	7776	7776	−864	−864	−288	−729	.	.	.	.	−24	−24	.	.	.	8

Character table of $H(\mathrm{Fi}_{23})$ *(continued)*

	5a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}	6_{22}
2	4	9	8	8	8	7	8	8	4	8	8	9	9	5	5	8	8	7	7	7	6	6	5
3	1	7	9	5	5	7	6	6	7	4	4	3	3	5	5	3	3	3	3	3	3	3	3
5	2	1	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
7	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
13	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2P	5a	3a	3b	3a	3a	3c	3b	3b	3d	3b	3b	3a	3a	3c	3c	3b	3b	3c	3c	3a	3c	3c	3c
3P	5a	2a	2a	2b	2c	2a	2c	2b	2a	2d	2e	2d	2e	2b	2c	2f	2f	2f	2f	2f	2e	2d	2f
5P	1a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}	6_{22}
7P	5a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}	6_{22}
11P	5a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}	6_{22}
13P	5a	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}	6_{22}
X.88	.	420	−30	−20	−20	−48	34	34	51	2	2	−28	−28	−2	−2	2	2	8	8	−4	−4	−4	2
X.89	.	210	−132	20	−20	39	−92	92	−24	12	−12	6	−6	11	−11	−12	12	−9	9	.	9	−9	−3
X.90	−4	−1134	324	−36	36	81	36	−36	.	−12	12	6	−6	−9	9	−12	12	9	−9	.	3	−3	−3
X.91	−4	−1134	324	36	−36	81	−36	36	.	−12	12	6	−6	9	−9	12	−12	9	−9	.	3	−3	3
X.92	.	.	486	.	.	.	−54	−54	.	−18	−18	.	.	.	.	18	18	.	.	.	.	.	.
X.93	.	1560	−312	−40	−40	−24	32	32	12	−8	−8	−8	−8	−4	−4	−16	−16	8	8	8	−8	−8	−4
X.94	−8	−512	640	.	.	64	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.95	−8	512	−640	.	.	−64	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.96	.	.	−108	.	.	−54	.	.	−27	−12	12	.	.	.	.	.	.	−6	6	.	6	−6	.
X.97	.	.	−108	.	.	−54	.	.	−27	−12	12	.	.	.	.	.	.	−6	6	.	6	−6	.
X.98	3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.99	−4	729	.	−81	−81	.	.	.	.	.	.	9	9	.	.	.	.	.	.	−9	.	.	.
X.100	.	.	−324	−90	−90	81	−36	−36	.	12	12	.	.	−9	−9	12	12	9	9	6	−3	−3	3
X.101	−2	−1232	−224	.	.	−80	.	.	−8	32	32	−16	−16	.	.	.	.	.	.	.	8	8	.
X.102	5	456	240	104	−104	24	−32	32	24	16	−16	−8	8	−4	4	.	.	.	.	.	−4	4	.
X.103	5	456	240	−104	104	24	32	−32	24	16	−16	−8	8	4	−4	.	.	.	.	.	−4	4	.
X.104	.	405	−162	45	45	.	18	18	.	6	6	21	21	18	18	−6	−6	.	.	−3	.	.	−6
X.105	−5	1134	324	−36	36	.	36	−36	.	−12	12	−6	6	18	−18	−12	12	.	.	.	.	.	6
X.106	−5	1134	324	36	−36	.	−36	36	.	−12	12	−6	6	−18	18	12	−12	.	.	.	.	.	−6
X.107	.	−735	−384	35	35	48	8	8	−6	−16	−16	17	17	8	8	−8	−8	−8	−8	−5	−4	−4	4
X.108	.	−840	60	−40	40	−66	−32	32	−21	−4	4	8	−8	−4	4	.	.	6	−6	.	−2	2	.
X.109	.	−840	60	40	−40	−66	32	−32	−21	−4	4	8	−8	4	−4	.	.	6	−6	.	−2	2	.
X.110	4	−384	192	64	64	−96	64	64	−24	.	.	.	.	−8	−8	.	.	.	.	.	.	.	.
X.111	4	384	−192	−64	64	96	64	−64	24	.	.	.	.	8	−8	.	.	.	.	.	.	.	.
X.112	4	384	−192	64	−64	96	−64	64	24	.	.	.	.	−8	8	.	.	.	.	.	.	.	.
X.113	4	−384	192	−64	−64	−96	−64	−64	−24	.	.	.	.	8	8	.	.	.	.	.	.	.	.
X.114	1	−729	.	81	81	.	.	.	.	.	.	−9	−9	.	.	.	.	.	.	9	.	.	.

Character table of $H(\mathrm{Fi}_{23})$ (continued)

2	5	4	4	2	7	7	7	6	6	3	2	1	4	3	3	4	4	2	2	7	7	5	6	6	6	5	5	5
3	3	3	3	1	1	1	.	.	.	4	4	3	1	1	1	.	.	.	.	2	2	3	2	2	2	2	2	2
5	.	.	.	.	.	.	.	.	.	.	.	.	2	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.
7	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.
13	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	6_{23}	6_{24}	6_{25}	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11a	11b	12a	12b	12c	12d	12e	12f	12g	12h	12i
2P	3c	3d	3d	7a	4a	4a	4d	4f	4f	9a	9b	9c	5a	5a	5a	5a	5a	11b	11a	6_{11}	6_{11}	6_{9}	6_{12}	6_{12}	6_{11}	6_{15}	6_{15}	6_{16}
3P	2f	2f	2f	7a	8a	8b	8c	8d	8e	3b	3b	3d	10a	10b	10c	10d	10e	11a	11b	4a	4a	4a	4c	4b	4a	4e	4f	4f
5P	6_{23}	6_{24}	6_{25}	7a	8a	8b	8c	8d	8e	9a	9b	9c	2a	2b	2c	2e	2d	11a	11b	12a	12b	12c	12d	12e	12f	12g	12h	12i
7P	6_{23}	6_{24}	6_{25}	1a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11b	11a	12a	12b	12c	12d	12e	12f	12g	12h	12i
11P	6_{23}	6_{24}	6_{25}	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	1a	1a	12a	12b	12c	12d	12e	12f	12g	12h	12i
13P	6_{23}	6_{24}	6_{25}	7a	8a	8b	8c	8d	8e	9a	9b	9c	10a	10b	10c	10d	10e	11b	11a	12a	12b	12c	12d	12e	12f	12g	12h	12i
X.88	2	-1	-1	.	.	.	.	.	.	3	.	.	.	.	.	.	.	.	.	-4	-4	2	.	.	-4	-2	-2	-2
X.89	3	.	.	.	.	.	.	2	-2	.	3	.	.	.	.	.	.	.	.	-6	6	.	2	-2	.	.	.	.
X.90	3	.	.	.	.	.	.	2	-2	.	.	.	4	2	-2	.	.	.	.	-6	6	.	-2	2	.	.	.	.
X.91	-3	.	.	.	.	.	.	-2	2	.	.	.	4	-2	2	.	.	.	.	-6	6	.	2	-2	.	.	.	.
X.92	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	-6	.	.	.	.	.	.
X.93	-4	-4	-4	2	.	.	.	.	.	-3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.94	.	.	.	2	.	.	.	.	.	4	-2	-2	-8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.95	.	.	.	2	.	.	.	.	.	4	-2	-2	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.96	.	3	-3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-2	2	2
X.97	.	3	-3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-2	2	2
X.98	.	.	.	.	-3	-3	-3	1	1	.	.	.	3	-3	-3	-1	-1	1	1	.	.	.	.	.	.	.	.	.
X.99	.	.	.	-2	-2	2	2	.	.	.	.	.	-4	2	2	.	.	.	.	-3	-3	.	3	3	3	.	.	.
X.100	3	.	.	.	1	-3	1	-1	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	-2	-2	6	.	.	.
X.101	.	.	.	.	.	.	.	.	.	-2	4	1	-2	.	.	-2	-2	.	.	.	.	.	.	.	.	.	.	.
X.102	.	.	.	-1	.	.	.	.	.	3	3	.	-5	-1	1	1	-1	.	.	.	.	.	-4	4	.	.	.	.
X.103	.	.	.	-1	.	.	.	.	.	3	3	.	-5	1	-1	1	-1	.	.	.	.	.	4	-4	.	.	.	.
X.104	-6	.	.	-1	4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	-3	-3	-6	1	1	-3	.	.	.
X.105	-6	.	.	.	.	.	.	.	.	.	.	.	5	-1	1	-1	1	1	1	6	-6	.	-2	2	.	.	.	.
X.106	6	.	.	.	.	.	.	.	.	.	.	.	5	1	-1	-1	1	1	1	6	-6	.	2	-2	.	.	.	.
X.107	4	-2	-2	.	4	-4	.	.	.	3	3	.	.	.	.	.	.	.	.	5	5	8	-1	-1	-1	.	.	.
X.108	.	-3	3	.	.	.	.	.	.	-3	.	.	.	.	.	.	.	.	.	.	.	.	4	-4	.	2	-2	-2
X.109	.	-3	3	.	.	.	.	.	.	-3	.	.	.	.	.	.	.	.	.	.	.	.	-4	4	.	2	-2	-2
X.110	.	.	.	1	.	.	.	.	.	.	-3	.	4	2	2	.	.	-1	-1	.	.	.	.	.	.	.	.	.
X.111	.	.	.	1	.	.	.	.	.	.	-3	.	-4	-2	2	.	.	-1	-1	.	.	.	.	.	.	.	.	.
X.112	.	.	.	1	.	.	.	.	.	.	-3	.	-4	2	-2	.	.	-1	-1	.	.	.	.	.	.	.	.	.
X.113	.	.	.	1	.	.	.	.	.	.	-3	.	4	-2	-2	.	.	-1	-1	.	.	.	.	.	.	.	.	.
X.114	.	.	.	-1	-4	4	.	.	.	.	.	.	1	1	1	1	1	1	1	3	3	.	-3	-3	-3	.	.	.

Character table of $H(\mathrm{Fi}_{23})$ (continued)

2	5	5	5	4	4	5	5	3	3	1	1	2	2	2	2	5	5	3	2	3	3	3	3	2	2	3
3	2	2	2	2	2	1	1	2	2	.	.	1	.	.	1	.	.	4	4	3	3	3	3	3	3	2
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
7	.	.	.	.	.	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
13	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	12j	12k	12l	12m	12n	12o	12p	12q	12r	13a	13b	14a	14b	14c	15a	16a	16b	18a	18b	18c	18d	18e	18f	18g	18h	18i
2P	6_{18}	6_{18}	6_{16}	6_{20}	6_{20}	6_{12}	6_{9}	6_{24}	6_{24}	13b	13a	7a	7a	7a	15a	8c	8c	9a	9b	9a	9a	9a	9a	9b	9b	9a
3P	4e	4f	4e	4b	4c	4g	4d	4f	4e	13a	13b	14a	14b	14c	5a	16a	16b	6_{2}	6_{2}	6_{6}	6_{7}	6_{6}	6_{7}	6_{6}	6_{7}	6_{10}
5P	12j	12k	12l	12m	12n	12o	12p	12q	12r	13b	13a	14a	14b	14c	3a	16b	16a	18a	18b	18e	18f	18c	18d	18g	18h	18i
7P	12j	12k	12l	12m	12n	12o	12p	12q	12r	13b	13a	2a	2b	2c	15a	16b	16a	18a	18b	18c	18d	18e	18f	18g	18h	18i
11P	12j	12k	12l	12m	12n	12o	12p	12q	12r	13b	13a	14a	14b	14c	15a	16a	16b	18a	18b	18e	18f	18c	18d	18g	18h	18i
13P	12j	12k	12l	12m	12n	12o	12p	12q	12r	1a	1a	14a	14b	14c	15a	16b	16a	18a	18b	18c	18d	18e	18f	18g	18h	18i
X.88	4	4	-2	.	.	.	2	1	1	.	.	.	.	.	.	.	.	3	.	1	1	1	1	-2	-2	-1
X.89	3	-3	.	1	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	-3	-2	2	-2	2	1	-1	.
X.90	-3	3	.	-1	1	.	.	.	.	.	.	.	.	.	-1	.	.	.	.	.	.	.	.	.	.	.
X.91	-3	3	.	1	-1	.	.	.	.	.	.	.	.	.	-1	.	.	.	.	.	.	.	.	.	.	.
X.92	.	.	.	.	.	.	-2	.	.	-1	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.93	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	-3	.	-1	-1	-1	-1	2	2	1
X.94	.	.	.	.	.	.	.	.	.	1	1	2	.	.	-2	.	.	4	-2	.	.	.	.	.	.	.
X.95	.	.	.	.	.	.	.	.	.	1	1	-2	.	.	-2	.	.	-4	2	.	.	.	.	.	.	.
X.96	-2	2	-2	.	.	.	.	-1	1	$-C$	$-D$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.97	-2	2	-2	.	.	.	.	-1	1	$-D$	$-C$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.98	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-1	-1	.	.	.	.	.	.	.	.	.
X.99	.	.	.	.	.	1	.	.	.	.	.	-2	.	.	-1	.	.	.	.	.	.	.	.	.	.	.
X.100	-3	-3	.	1	1	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.
X.101	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-2	.	.	-2	4	.	.	.	.	.	.	2
X.102	.	.	.	-2	2	.	.	.	.	.	.	1	1	-1	-1	.	.	-3	-3	1	-1	1	-1	1	-1	-1
X.103	.	.	.	2	-2	.	.	.	.	.	.	1	-1	1	-1	.	.	-3	-3	-1	1	-1	1	-1	1	-1
X.104	.	.	.	-2	-2	1	-2	.	.	.	.	-1	1	1	.	.	.	.	.	.	.	.	.	.	.	.
X.105	.	.	.	2	-2	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
X.106	.	.	.	-2	2	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
X.107	.	.	.	2	2	1	.	.	.	.	.	.	.	.	.	.	.	3	3	-1	-1	-1	-1	-1	-1	-1
X.108	2	-2	2	2	-2	.	.	1	-1	.	.	.	.	.	.	.	.	3	.	1	-1	1	-1	-2	2	1
X.109	2	-2	2	-2	2	.	.	1	-1	.	.	.	.	.	.	.	.	3	.	-1	1	-1	1	2	-2	1
X.110	.	.	.	.	.	.	.	.	.	.	.	1	-1	-1	1	.	.	.	-3	-2	-2	-2	-2	1	1	.
X.111	.	.	.	.	.	.	.	.	.	.	.	-1	1	-1	1	.	.	.	3	-2	2	-2	2	1	-1	.
X.112	.	.	.	.	.	.	.	.	.	.	.	-1	-1	1	1	.	.	.	3	2	-2	2	-2	-1	1	.
X.113	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	.	.	.	-3	2	2	2	2	-1	-1	.
X.114	.	.	.	.	.	-1	.	.	.	.	.	-1	-1	-1	1	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(\mathrm{Fi}_{23})$ *(continued)*

2	3	1	3	3	1	2	2	2	2	2	2	4	4	1	1	2	2	2	1
3	2	3	.	.	1	.	.	.	.	.	.	1	1	.	.	1	1	1	1
5	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	1	1	1	.
7	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1
11	.	.	.	.	.	1	1	1	1	1	1	.	.	.	.	.	.	.	.
13	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.
	$18j$	$18k$	$20a$	$20b$	$21a$	$22a$	$22b$	$22c$	$22d$	$22e$	$22f$	$24a$	$24b$	$26a$	$26b$	$30a$	$30b$	$30c$	$42a$
$2P$	$9a$	$9c$	$10d$	$10d$	$21a$	$11a$	$11a$	$11b$	$11b$	$11b$	$11a$	$12a$	$12b$	$13a$	$13b$	$15a$	$15a$	$15a$	$21a$
$3P$	6_9	6_8	$20a$	$20b$	$7a$	$22a$	$22b$	$22c$	$22d$	$22e$	$22f$	$8a$	$8b$	$26a$	$26b$	$10b$	$10c$	$10a$	$14a$
$5P$	$18j$	$18k$	$4b$	$4c$	$21a$	$22a$	$22b$	$22c$	$22d$	$22e$	$22f$	$24a$	$24b$	$26b$	$26a$	6_3	6_4	6_1	$42a$
$7P$	$18j$	$18k$	$20a$	$20b$	$3a$	$22c$	$22e$	$22a$	$22f$	$22b$	$22d$	$24a$	$24b$	$26b$	$26a$	$30a$	$30b$	$30c$	6_1
$11P$	$18j$	$18k$	$20a$	$20b$	$21a$	$2a$	$2c$	$2a$	$2b$	$2c$	$2b$	$24a$	$24b$	$26b$	$26a$	$30a$	$30b$	$30c$	$42a$
$13P$	$18j$	$18k$	$20a$	$20b$	$21a$	$22c$	$22e$	$22a$	$22f$	$22b$	$22d$	$24a$	$24b$	$2a$	$2a$	$30a$	$30b$	$30c$	$42a$
$X.88$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.89$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.90$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	.
$X.91$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	.
$X.92$	.	.	.	.	.	1	1	1	1	1	1	.	.	−1	−1	.	.	.	.
$X.93$	1	.	.	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1
$X.94$	.	−2	.	.	−1	.	.	.	.	.	.	.	.	1	1	.	.	−2	−1
$X.95$	.	2	.	.	−1	.	.	.	.	.	.	.	.	−1	−1	.	.	2	1
$X.96$	.	.	.	.	.	.	.	.	.	.	.	.	.	D	C	.	.	.	.
$X.97$	.	.	.	.	.	.	.	.	.	.	.	.	.	C	D	.	.	.	.
$X.98$	.	.	−1	−1	.	1	1	1	1	1	1	.	.	.	.	.	.	.	.
$X.99$	.	.	.	.	1	.	.	.	.	.	.	1	−1	.	.	−1	−1	−1	1
$X.100$	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	.	.
$X.101$	2	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.
$X.102$	1	.	−1	1	−1	.	.	.	.	.	.	.	.	.	.	−1	1	1	1
$X.103$	1	.	1	−1	−1	.	.	.	.	.	.	.	.	.	.	1	−1	1	1
$X.104$	.	.	.	.	−1	.	.	.	.	.	.	1	1	.	.	.	.	.	−1
$X.105$	.	.	−1	1	.	−1	−1	−1	1	−1	1	.	.	.	.	−1	1	−1	.
$X.106$	.	.	1	−1	.	−1	1	−1	−1	1	−1	.	.	.	.	1	−1	−1	.
$X.107$	−1	.	.	.	.	.	.	.	.	.	.	1	−1	.	.	.	.	.	.
$X.108$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.109$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.110$	.	.	.	.	1	−1	1	−1	1	1	1	.	.	.	.	−1	−1	1	1
$X.111$	.	.	.	.	1	1	1	1	−1	1	−1	.	.	.	.	1	−1	−1	−1
$X.112$	.	.	.	.	1	1	−1	1	1	−1	1	.	.	.	.	−1	1	−1	−1
$X.113$	.	.	.	.	1	−1	−1	−1	−1	−1	−1	.	.	.	.	1	1	1	1
$X.114$	.	.	1	1	−1	1	−1	1	−1	−1	−1	−1	1	.	.	1	1	1	−1

where $A = -\zeta(11)_{11}^9 - \zeta(11)_{11}^5 - \zeta(11)_{11}^4 - \zeta(11)_{11}^3 - \zeta(11)_{11} - 1$, $B = 6\zeta(3)_3 + 3$, $C = \zeta(13)_{13}^{11} + \zeta(13)_{13}^8 + \zeta(13)_{13}^7 + \zeta(13)_{13}^6 + \zeta(13)_{13}^5 + \zeta(13)_{13}^2 + 1$, $D = -\zeta(13)_{13}^{11} - \zeta(13)_{13}^8 - \zeta(13)_{13}^7 - \zeta(13)_{13}^6 - \zeta(13)_{13}^5 - \zeta(13)_{13}^2$, $E = -2\zeta(8)_8^3 - 2\zeta(8)_8$.

B.4. *Character table of* $H(2\,\mathrm{Fi}_{22}) = H_2 = \langle p_2, q_2, h_2 \rangle$

2	18	18	18	18	17	17	16	16	16	16	17	17	15	15	16	11	11	13	13	13	13	12
3	4	4	4	4	4	4	2	2	2	2	1	1	2	2	1	2	2	1	1	1	1	1
5	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.
	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r	2s	2t	2u
2P	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a
3P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r	2s	2t	2u
5P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r	2s	2t	2u
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	1	1	1	1	1
X.3	6	6	6	6	6	6	−2	−2	−2	−2	6	6	−2	−2	6	4	4	2	2	2	2	2
X.4	6	6	6	6	6	6	−2	−2	−2	−2	6	6	−2	−2	6	−4	−4	2	2	2	2	2
X.5	10	10	10	10	10	10	−6	−6	−6	−6	10	10	−6	−6	10	.	.	2	2	2	2	2
X.6	15	15	15	15	15	15	7	7	7	7	15	15	7	7	15	5	5	3	3	3	3	3
X.7	15	15	15	15	15	15	7	7	7	7	15	15	7	7	15	−5	−5	3	3	3	3	3
X.8	15	15	15	15	15	15	−1	−1	−1	−1	15	15	−1	−1	15	−5	−5	−1	−1	−1	−1	−1
X.9	15	15	15	15	15	15	−1	−1	−1	−1	15	15	−1	−1	15	5	5	−1	−1	−1	−1	−1
X.10	16	−16	16	−16	16	−16	−8	8	−8	8	.	.	8	−8	.	4	−4	4	4	−4	−4	4
X.11	16	−16	16	−16	16	−16	−8	8	−8	8	.	.	8	−8	.	−4	4	4	4	−4	−4	4
X.12	16	−16	16	−16	−16	16	8	−8	8	−8	.	.	8	−8	.	−4	4	4	4	−4	−4	−4
X.13	16	−16	16	−16	−16	16	8	−8	8	−8	.	.	8	−8	.	4	−4	4	4	−4	−4	−4
X.14	20	20	20	20	20	20	4	4	4	4	20	20	4	4	20	.	.	−4	−4	−4	−4	−4
X.15	20	20	20	20	20	20	4	4	4	4	20	20	4	4	20	10	10	4	4	4	4	4
X.16	20	20	20	20	20	20	4	4	4	4	20	20	4	4	20	−10	−10	4	4	4	4	4
X.17	24	24	24	24	24	24	8	8	8	8	24	24	8	8	24	4	4	.	.	.	.	.
X.18	24	24	24	24	24	24	8	8	8	8	24	24	8	8	24	−4	−4	.	.	.	.	.
X.19	30	30	30	30	30	30	−10	−10	−10	−10	30	30	−10	−10	30	−10	−10	2	2	2	2	2
X.20	30	30	30	30	30	30	−10	−10	−10	−10	30	30	−10	−10	30	10	10	2	2	2	2	2
X.21	32	−32	−32	32	.	.	16	16	−16	−16	.	.	.	.	.	.	.	8	−8	8	−8	.
X.22	40	40	40	40	−40	−40	16	16	16	16	8	8	−16	−16	−8	10	10	4	4	4	4	−4
X.23	40	40	40	40	−40	−40	16	16	16	16	8	8	−16	−16	−8	−10	−10	4	4	4	4	−4
X.24	40	40	40	40	−40	−40	−16	−16	−16	−16	8	8	16	16	−8	10	10	4	4	4	4	−4
X.25	40	40	40	40	−40	−40	−16	−16	−16	−16	8	8	16	16	−8	−10	−10	4	4	4	4	−4
X.26	60	60	60	60	60	60	−4	−4	−4	−4	60	60	−4	−4	60	−10	−10	4	4	4	4	4
X.27	60	60	60	60	60	60	−4	−4	−4	−4	60	60	−4	−4	60	10	10	4	4	4	4	4
X.28	60	60	60	60	60	60	12	12	12	12	60	60	12	12	60	.	.	4	4	4	4	4
X.29	64	64	64	64	64	64	.	.	.	.	64	64	.	.	64	−16	−16	.	.	.	.	.
X.30	64	64	64	64	64	64	.	.	.	.	64	64	.	.	64	16	16	.	.	.	.	.
X.31	80	80	−80	−80	.	.	−32	32	32	−32	16	−16	.	.	.	.	.	8	−8	−8	8	.
X.32	80	80	80	80	−80	−80	.	.	.	.	16	16	.	.	−16	20	20	8	8	8	8	−8
X.33	80	80	80	80	−80	−80	.	.	.	.	16	16	.	.	−16	−20	−20	8	8	8	8	−8
X.34	80	80	−80	−80	.	.	32	−32	−32	32	16	−16	.	.	.	.	.	8	−8	−8	8	.
X.35	80	80	80	80	80	80	−16	−16	−16	−16	80	80	−16	−16	80	.	.	.	.	.	.	.
X.36	81	81	81	81	81	81	9	9	9	9	81	81	9	9	81	9	9	−3	−3	−3	−3	−3
X.37	81	81	81	81	81	81	9	9	9	9	81	81	9	9	81	−9	−9	−3	−3	−3	−3	−3
X.38	90	90	90	90	90	90	−6	−6	−6	−6	90	90	−6	−6	90	.	.	−6	−6	−6	−6	−6
X.39	96	−96	96	−96	−96	96	−16	16	−16	16	.	.	−16	16	.	16	−16	8	8	−8	−8	−8
X.40	96	−96	96	−96	−96	96	−16	16	−16	16	.	.	−16	16	.	−16	16	8	8	−8	−8	−8
X.41	96	−96	96	−96	96	−96	16	−16	16	−16	.	.	−16	16	.	−16	16	8	8	−8	−8	8
X.42	96	−96	96	−96	96	−96	16	−16	16	−16	.	.	−16	16	.	16	−16	8	8	−8	−8	8
X.43	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	.	.	.
X.44	120	120	120	120	−120	−120	16	16	16	16	24	24	−16	−16	−24	−10	−10	−4	−4	−4	−4	4
X.45	120	120	120	120	−120	−120	−16	−16	−16	−16	24	24	16	16	−24	−10	−10	−4	−4	−4	−4	4
X.46	120	120	120	120	−120	−120	−16	−16	−16	−16	24	24	16	16	−24	10	10	−4	−4	−4	−4	4
X.47	120	120	120	120	−120	−120	16	16	16	16	24	24	−16	−16	−24	10	10	−4	−4	−4	−4	4
X.48	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	.	.	.
X.49	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	.	.	.
X.50	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	.	.	.
X.51	135	135	135	135	135	135	−33	−33	−33	−33	7	7	−33	−33	7	15	15	3	3	3	3	3
X.52	135	135	135	135	135	135	−33	−33	−33	−33	7	7	−33	−33	7	−15	−15	3	3	3	3	3
X.53	135	135	135	135	135	135	39	39	39	39	7	7	39	39	7	15	15	15	15	15	15	15
X.54	135	135	135	135	135	135	39	39	39	39	7	7	39	39	7	−15	−15	15	15	15	15	15
X.55	160	−160	160	−160	160	−160	48	−48	48	−48	.	.	−48	48	.	.	.	8	8	−8	−8	8
X.56	160	−160	160	−160	−160	160	−48	48	−48	48	.	.	−48	48	.	.	.	8	8	−8	−8	−8
X.57	160	160	−160	−160	.	.	.	.	.	.	32	−32	.	.	.	.	.	16	−16	−16	16	.
X.58	160	−160	−160	160	.	.	−48	−48	48	48	.	.	.	.	.	.	.	8	−8	8	−8	.
X.59	160	−160	−160	160	.	.	−48	−48	48	48	.	.	.	.	.	.	.	8	−8	8	−8	.
X.60	192	−192	−192	192	.	.	−32	−32	32	32	.	.	.	.	.	.	.	16	−16	16	−16	.
X.61	216	216	216	216	−216	−216	−48	−48	−48	−48	−8	−8	48	48	8	−6	−6	12	12	12	12	−12
X.62	216	216	216	216	−216	−216	48	48	48	48	−8	−8	−48	−48	8	−6	−6	12	12	12	12	−12
X.63	216	216	216	216	−216	−216	48	48	48	48	−8	−8	−48	−48	8	6	6	12	12	12	12	−12
X.64	216	216	216	216	−216	−216	−48	−48	−48	−48	−8	−8	48	48	8	6	6	12	12	12	12	−12
X.65	240	240	240	240	−240	−240	−32	−32	−32	−32	48	48	32	32	−48	−40	−40	8	8	8	8	−8
X.66	240	240	−240	−240	.	.	32	−32	−32	32	48	−48	.	.	.	.	.	−8	8	8	−8	.
X.67	240	240	−240	−240	.	.	−32	32	32	−32	48	−48	.	.	.	.	.	−8	8	8	−8	.
X.68	240	240	240	240	−240	−240	32	32	32	32	48	48	−32	−32	−48	40	40	8	8	8	8	−8
X.69	240	240	240	240	−240	−240	−32	−32	−32	−32	48	48	32	32	−48	40	40	8	8	8	8	−8
X.70	240	−240	240	−240	−240	240	56	−56	56	−56	.	.	56	−56	.	20	−20	12	12	−12	−12	−12
X.71	240	−240	240	−240	−240	240	56	−56	56	−56	.	.	56	−56	.	−20	20	12	12	−12	−12	−12
X.72	240	−240	240	−240	−240	240	−8	8	−8	8	.	.	−8	8	.	20	−20	−4	−4	4	4	4
X.73	240	−240	240	−240	−240	240	−8	8	−8	8	.	.	−8	8	.	−20	20	−4	−4	4	4	4
X.74	240	−240	240	−240	240	−240	8	−8	8	−8	.	.	−8	8	.	20	−20	−4	−4	4	4	−4
X.75	240	−240	240	−240	240	−240	8	−8	8	−8	.	.	−8	8	.	−20	20	−4	−4	4	4	−4
X.76	240	240	240	240	240	240	48	48	48	48	−16	−16	48	48	−16	.	.	.	.	.	.	.
X.77	240	−240	240	−240	240	−240	−56	56	−56	56	.	.	56	−56	.	−20	20	12	12	−12	−12	12
X.78	240	240	240	240	−240	−240	32	32	32	32	48	48	−32	−32	−48	−40	−40	8	8	8	8	−8
X.79	240	−240	240	−240	240	−240	−56	56	−56	56	.	.	56	−56	.	20	−20	12	12	−12	−12	12
X.80	240	240	240	240	240	240	−48	−48	−48	−48	−16	−16	−48	−48	−16	.	.	.	.	.	.	.
X.81	240	240	240	240	240	240	−48	−48	−48	−48	−16	−16	−48	−48	−16	.	.	.	.	.	.	.
X.82	270	270	270	270	270	270	6	6	6	6	14	14	6	6	14	30	30	18	18	18	18	18
X.83	270	270	270	270	270	270	6	6	6	6	14	14	6	6	14	−30	−30	18	18	18	18	18
X.84	320	320	320	320	−320	−320	64	64	64	64	64	64	−64	−64	−64	.	.	.	.	.	.	.
X.85	320	−320	320	−320	320	−320	−32	32	−32	32	.	.	32	−32	.	40	−40	16	16	−16	−16	16
X.86	320	−320	320	−320	320	−320	−32	32	−32	32	.	.	32	−32	.	−40	40	16	16	−16	−16	16
X.87	320	320	320	320	−320	−320	−64	−64	−64	−64	64	64	64	64	−64	.	.	.	.	.	.	.
X.88	320	−320	320	−320	320	−320	−32	32	−32	32	.	.	32	−32	.	.	.	−16	−16	16	16	−16
X.89	320	−320	320	−320	−320	320	32	−32	32	−32	.	.	32	−32	.	.	.	−16	−16	16	16	16
X.90	320	−320	320	−320	−320	320	32	−32	32	−32	.	.	32	−32	.	−40	40	16	16	−16	−16	−16
X.91	320	−320	320	−320	−320	320	32	−32	32	−32	.	.	32	−32	.	40	−40	16	16	−16	−16	−16
X.92	320	−320	−320	320	.	.	32	32	−32	−32	.	.	.	.	.	.	.	−16	16	−16	16	.
X.93	320	−320	−320	320	.	.	32	32	−32	−32	.	.	.	.	.	.	.	−16	16	−16	16	.
X.94	320	320	−320	−320	.	.	64	−64	−64	64	64	−64	.	.	.	.	.	.	.	.	.	.
X.95	320	320	−320	−320	.	.	64	−64	−64	64	64	−64	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	12	13	10	8	9	6	12	11	11	12	11	11	11	11	11	11	10	10	10	10	10	10	10	10
3	1	.	1	4	3	3	3	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	$2v$	$2w$	$2x$	$3a$	$3b$	$3c$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}
$2P$	$1a$	$1a$	$1a$	$3a$	$3b$	$3c$	$2a$	$2b$	$2b$	$2a$	$2j$	$2j$	$2j$	$2k$	$2k$	$2j$	$2f$	$2j$	$2k$	$2g$	$2j$	$2g$	$2b$	$2f$
$3P$	$2v$	$2w$	$2x$	$1a$	$1a$	$1a$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}
$5P$	$2v$	$2w$	$2x$	$3a$	$3b$	$3c$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	1	1	−1	1	1	1	1	−1	−1	1	−1	1	1	−1	−1	−1	1	1	−1	1	−1	1	−1	1
$X.3$	2	−2	.	−3	.	3	6	4	4	−2	.	−2	−2	.	.	.	2	−2	4	2	4	2	.	2
$X.4$	2	−2	.	−3	.	3	6	−4	−4	−2	.	−2	−2	.	.	.	2	−2	−4	2	−4	2	.	2
$X.5$	2	−6	.	1	4	−2	10	.	.	−6	.	−6	−6	.	.	.	2	−6	.	2	.	2	.	2
$X.6$	3	7	1	−3	3	.	15	5	5	7	1	7	7	1	1	1	−1	7	5	−1	5	−1	1	−1
$X.7$	3	7	−1	−3	3	.	15	−5	−5	7	−1	7	7	−1	−1	−1	−1	7	−5	−1	−5	−1	−1	−1
$X.8$	−1	−1	3	6	.	3	15	−5	−5	−1	3	−1	−1	3	3	3	3	−1	−5	3	−5	3	3	3
$X.9$	−1	−1	−3	6	.	3	15	5	5	−1	−3	−1	−1	−3	−3	−3	3	−1	5	3	5	3	−3	3
$X.10$	−4	.	.	−2	4	1	.	4	−4	.	−4	.	.	−4	4	4	4	.	.	.	.	.	.	−4
$X.11$	−4	.	.	−2	4	1	.	−4	4	.	4	.	.	4	−4	−4	4	.	.	.	.	.	.	−4
$X.12$	4	.	.	−2	4	1	.	4	−4	.	−4	.	.	4	−4	4	.	.	.	−4	.	4	.	.
$X.13$	4	.	.	−2	4	1	.	−4	4	.	4	.	.	−4	4	−4	.	.	.	−4	.	4	.	.
$X.14$	−4	4	.	−7	2	2	20	.	.	4	.	4	4	.	.	.	4	4	.	4	.	4	.	4
$X.15$	4	4	2	2	−1	5	20	10	10	4	2	4	4	2	2	2	.	4	10	.	10	.	2	.
$X.16$	4	4	−2	2	−1	5	20	−10	−10	4	−2	4	4	−2	−2	−2	.	4	−10	.	−10	.	−2	.
$X.17$	.	8	4	6	3	.	24	4	4	8	4	8	8	4	4	4	.	8	4	.	4	.	4	.
$X.18$	.	8	−4	6	3	.	24	−4	−4	8	−4	8	8	−4	−4	−4	.	8	−4	.	−4	.	−4	.
$X.19$	2	−10	2	3	3	3	30	−10	−10	−10	2	−10	−10	2	2	2	−2	−10	−10	−2	−10	−2	2	−2
$X.20$	2	−10	−2	3	3	3	30	10	10	−10	−2	−10	−10	−2	−2	−2	−2	−10	10	−2	10	−2	−2	−2
$X.21$	.	.	.	−4	8	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.22$	−4	.	6	4	7	1	.	−10	−10	.	2	4	4	−2	−2	2	.	−4	2	.	−2	.	−6	.
$X.23$	−4	.	−6	4	7	1	.	10	10	.	−2	4	4	2	2	−2	.	−4	−2	.	2	.	6	.
$X.24$	−4	.	−2	4	7	1	.	−10	−10	.	−6	−4	−4	6	6	−6	.	4	2	.	−2	.	2	.
$X.25$	−4	.	2	4	7	1	.	10	10	.	6	−4	−4	−6	−6	6	.	4	−2	.	2	.	−2	.
$X.26$	4	−4	−2	6	−3	−3	60	−10	−10	−4	−2	−4	−4	−2	−2	−2	.	−4	−10	.	−10	.	−2	.
$X.27$	4	−4	2	6	−3	−3	60	10	10	−4	2	−4	−4	2	2	2	.	−4	10	.	10	.	2	.
$X.28$	4	12	.	−3	.	−6	60	.	.	12	.	12	12	.	.	.	4	12	.	4	.	4	.	4
$X.29$	.	.	.	−8	−2	4	64	−16	−16	.	.	.	.	.	.	.	.	.	−16	.	−16	.	.	.
$X.30$	.	.	.	−8	−2	4	64	16	16	.	.	.	.	.	.	.	.	.	16	.	16	.	.	.
$X.31$	.	.	.	8	14	2	.	.	.	.	.	−8	8	.	.	.	.	.	.	.	.	.	.	.
$X.32$	−8	.	4	8	−4	2	.	−20	−20	.	−4	.	.	4	4	−4	.	.	4	.	−4	.	−4	.
$X.33$	−8	.	−4	8	−4	2	.	20	20	.	4	.	.	−4	−4	4	.	.	−4	.	4	.	4	.
$X.34$	.	.	.	8	14	2	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.
$X.35$	.	−16	.	−10	2	−4	80	.	.	−16	.	−16	−16	.	.	.	.	−16	.	.	.	.	.	.
$X.36$	−3	9	−3	.	.	.	81	9	9	9	−3	9	9	−3	−3	−3	−3	9	9	−3	9	−3	−3	−3
$X.37$	−3	9	3	.	.	.	81	−9	−9	9	3	9	9	3	3	3	−3	9	−9	−3	−9	−3	3	−3
$X.38$	−6	−6	.	9	.	.	90	.	.	−6	.	−6	−6	.	.	.	2	−6	.	2	.	2	.	2
$X.39$	8	.	.	6	.	3	.	−16	16	.	.	.	.	.	.	.	.	.	.	−8	.	8	.	.
$X.40$	8	.	.	6	.	3	.	16	−16	.	.	.	.	.	.	.	.	.	.	−8	.	8	.	.
$X.41$	−8	.	.	6	.	3	.	−16	16	.	.	.	.	.	.	.	8	.	.	.	.	.	.	−8
$X.42$	−8	.	.	6	.	3	.	16	−16	.	.	.	.	.	.	.	8	.	.	.	.	.	.	−8
$X.43$	.	−8	−8	3	6	.	8	.	.	8	8	.	.	8	8	8	−4	.	.	12	.	12	−8	−4
$X.44$	4	.	10	12	3	3	.	10	10	.	−2	4	4	2	2	−2	.	−4	−2	.	2	.	−10	.
$X.45$	4	.	2	12	3	3	.	10	10	.	−10	−4	−4	10	10	−10	.	4	−2	.	2	.	−2	.
$X.46$	4	.	−2	12	3	3	.	−10	−10	.	10	−4	−4	−10	−10	10	.	4	2	.	−2	.	2	.
$X.47$	4	.	−10	12	3	3	.	−10	−10	.	2	4	4	−2	−2	2	.	−4	2	.	−2	.	10	.
$X.48$	.	−8	8	3	6	.	8	.	.	8	−8	.	.	−8	−8	−8	12	.	.	−4	.	−4	8	12
$X.49$	.	−8	8	3	6	.	8	.	.	8	−8	.	.	−8	−8	−8	−4	.	.	12	.	12	8	−4
$X.50$	.	−8	−8	3	6	.	8	.	.	8	8	.	.	8	8	8	12	.	.	−4	.	−4	−8	12
$X.51$	3	−1	−5	.	9	.	−9	15	15	15	−5	−1	−1	−5	−5	−5	3	−1	−1	3	−1	3	−5	3
$X.52$	3	−1	5	.	9	.	−9	−15	−15	15	5	−1	−1	5	5	5	3	−1	1	3	1	3	5	3
$X.53$	15	7	7	.	9	.	−9	15	15	−9	7	−1	−1	7	7	7	3	−1	−1	3	−1	3	7	3
$X.54$	15	7	−7	.	9	.	−9	−15	−15	−9	−7	−1	−1	−7	−7	−7	3	−1	1	3	1	3	−7	3
$X.55$	−8	.	.	−2	16	−2	.	.	.	.	.	.	.	.	.	.	8	.	.	.	.	.	.	−8
$X.56$	8	.	.	−2	16	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	−8	.	8	.	.
$X.57$	.	.	.	16	−8	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.58$	.	.	.	−2	16	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.59$	.	.	.	−2	16	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.60$	.	.	.	12	.	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.61$	−12	.	−2	.	9	.	.	6	6	.	10	4	4	−10	−10	10	.	−4	2	.	−2	.	2	.
$X.62$	−12	.	−10	.	9	.	.	6	6	.	2	−4	−4	−2	−2	2	.	4	2	.	−2	.	10	.
$X.63$	−12	.	10	.	9	.	.	−6	−6	.	−2	−4	−4	2	2	−2	.	4	−2	.	2	.	−10	.
$X.64$	−12	.	2	.	9	.	.	−6	−6	.	−10	4	4	10	10	−10	.	−4	−2	.	2	.	−2	.
$X.65$	−8	.	.	−12	.	3	.	40	40	.	.	−8	−8	.	.	.	.	8	−8	.	8	.	.	.
$X.66$	.	.	.	24	6	6	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.
$X.67$	.	.	.	24	6	6	.	.	.	.	.	−8	8	.	.	.	.	.	.	.	.	.	.	.
$X.68$	−8	.	.	−12	.	3	.	−40	−40	.	.	8	8	.	.	.	.	−8	8	.	−8	.	.	.
$X.69$	−8	.	.	−12	.	3	.	−40	−40	.	.	−8	−8	.	.	.	.	8	8	.	−8	.	.	.
$X.70$	12	.	.	6	12	.	.	−20	20	.	4	.	.	−4	4	−4	.	.	.	4	.	−4	.	.
$X.71$	12	.	.	6	12	.	.	20	−20	.	−4	.	.	4	−4	4	.	.	.	4	.	−4	.	.
$X.72$	−4	.	.	−12	.	3	.	−20	20	.	−12	.	.	12	−12	12	.	.	.	−12	.	12	.	.
$X.73$	−4	.	.	−12	.	3	.	20	−20	.	12	.	.	−12	12	−12	.	.	.	−12	.	12	.	.
$X.74$	4	.	.	−12	.	3	.	20	−20	.	12	.	.	12	−12	−12	12	.	.	.	.	.	.	−12
$X.75$	4	.	.	−12	.	3	.	−20	20	.	−12	.	.	−12	12	12	12	.	.	.	.	.	.	−12
$X.76$	.	−16	.	6	12	.	16	.	.	16	.	.	.	.	.	.	−8	.	.	−8	.	−8	.	−8
$X.77$	−12	.	.	6	12	.	.	−20	20	.	4	.	.	4	−4	−4	−4	.	.	.	.	.	.	4
$X.78$	−8	.	.	−12	.	3	.	40	40	.	.	8	8	.	.	.	.	−8	−8	.	8	.	.	.
$X.79$	−12	.	.	6	12	.	.	20	−20	.	−4	.	.	−4	4	4	−4	.	.	.	.	.	.	4
$X.80$	.	16	.	6	12	.	16	.	.	−16	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.81$	.	16	.	6	12	.	16	.	.	−16	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.82$	18	6	2	.	−9	.	−18	30	30	6	2	−2	−2	2	2	2	6	−2	−2	6	−2	6	2	6
$X.83$	18	6	−2	.	−9	.	−18	−30	−30	6	−2	−2	−2	−2	−2	−2	6	−2	2	6	2	6	−2	6
$X.84$	.	.	.	−4	14	−4	.	.	.	.	.	16	16	.	.	.	.	−16	.	.	.	.	.	.
$X.85$	−16	.	.	−4	−4	5	.	40	−40	.	−8	.	.	−8	8	8	.	.	.	.	.	.	.	.
$X.86$	−16	.	.	−4	−4	5	.	−40	40	.	8	.	.	8	−8	−8	.	.	.	.	.	.	.	.
$X.87$	.	.	.	−4	14	−4	.	.	.	.	.	−16	−16	.	.	.	.	16	.	.	.	.	.	.
$X.88$	16	.	.	14	8	2	.	.	.	.	.	.	.	.	.	.	16	.	.	.	.	.	.	−16
$X.89$	−16	.	.	14	8	2	.	.	.	.	.	.	.	.	.	.	.	.	.	−16	.	16	.	.
$X.90$	16	.	.	−4	−4	5	.	40	−40	.	−8	.	.	8	−8	8	.	.	.	.	.	.	.	.
$X.91$	16	.	.	−4	−4	5	.	−40	40	.	8	.	.	−8	8	−8	.	.	.	.	.	.	.	.
$X.92$	.	.	.	14	8	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.93$	.	.	.	14	8	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.94$	.	.	.	−4	14	−4	.	.	.	.	.	16	−16	.	.	.	.	.	.	.	.	.	.	.
$X.95$	.	.	.	−4	14	−4	.	.	.	.	.	16	−16	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ *(continued)*

	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}	4_{41}	$5a$	6_1	6_2	6_3
2	11	11	9	9	9	9	9	9	10	10	9	9	9	9	9	9	9	9	9	9	9	8	8	4	8	8	8
3	.	.	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	4	4
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.
2P	$2k$	$2k$	$2r$	$2q$	$2l$	$2r$	$2l$	$2q$	$2n$	$2n$	$2q$	$2r$	$2q$	$2k$	$2j$	$2q$	$2q$	$2r$	$2q$	$2q$	$2j$	$2q$	$2q$	$5a$	$3a$	$3a$	$3a$
3P	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}	4_{41}	$5a$	$2d$	$2e$	$2e$
5P	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}	4_{41}	$1a$	6_5	6_3	6_2
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	1	1	−1	−1	1	−1	1	−1	1	1	−1	−1	−1	−1	−1	1	1	−1	1	1	1	1	1	1	1	1	1
$X.3$	2	2	−2	−2	2	−2	2	−2	2	2	2	2	2	.	.	.	.	2	.	.	2	.	.	1	−3	−3	−3
$X.4$	2	2	2	2	2	2	2	2	2	2	−2	−2	−2	.	.	.	.	−2	.	.	2	.	.	1	−3	−3	−3
$X.5$	2	2	.	.	2	.	2	.	2	2	.	.	.	.	.	−2	−2	.	−2	−2	2	−2	−2	.	1	1	1
$X.6$	3	3	3	3	−1	3	−1	3	3	3	−1	−1	−1	1	1	1	1	−1	1	1	3	1	1	.	−3	−3	−3
$X.7$	3	3	−3	−3	−1	−3	−1	−3	3	3	1	1	1	−1	−1	1	1	1	1	1	3	1	1	.	−3	−3	−3
$X.8$	−1	−1	−1	−1	3	−1	3	−1	−1	−1	−1	−1	−1	3	3	−1	−1	−1	−1	−1	−1	−1	−1	.	6	6	6
$X.9$	−1	−1	1	1	3	1	3	1	−1	−1	1	1	1	−3	−3	−1	−1	1	−1	−1	−1	−1	−1	.	6	6	6
$X.10$	.	.	−2	−2	.	2	.	2	.	.	−2	−2	2	.	.	2	−2	2	2	−2	.	2	−2	1	−2	2	2
$X.11$	.	.	2	2	.	−2	.	−2	.	.	2	2	−2	.	.	2	−2	−2	2	−2	.	2	−2	1	−2	2	2
$X.12$	.	.	2	−2	.	−2	.	2	.	.	−2	2	2	.	.	−2	2	−2	−2	2	.	2	−2	1	2	−2	−2
$X.13$	.	.	−2	2	.	2	.	−2	.	.	2	−2	−2	.	.	−2	2	2	−2	2	.	2	−2	1	2	−2	−2
$X.14$	−4	−4	.	.	4	.	4	.	−4	−4	.	.	.	.	.	.	.	.	.	.	−4	.	.	.	−7	−7	−7
$X.15$	4	4	2	2	.	2	.	2	4	4	2	2	2	2	2	.	.	2	.	.	4	.	.	.	2	2	2
$X.16$	4	4	−2	−2	.	−2	.	−2	4	4	−2	−2	−2	−2	−2	.	.	−2	.	.	4	.	.	.	2	2	2
$X.17$	.	.	.	.	.	.	.	.	.	.	.	.	.	4	4	.	.	.	.	.	.	.	.	−1	6	6	6
$X.18$	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	−4	.	.	.	.	.	.	.	.	−1	6	6	6
$X.19$	2	2	4	4	−2	4	−2	4	2	2	.	.	.	2	2	.	.	.	.	.	2	.	.	.	3	3	3
$X.20$	2	2	−4	−4	−2	−4	−2	−4	2	2	.	.	.	−2	−2	.	.	.	.	.	2	.	.	.	3	3	3
$X.21$	.	.	.	.	4	.	−4	.	.	.	.	.	.	.	.	−4	−4	.	4	4	.	.	.	2	.	.	.
$X.22$	−4	4	−4	4	.	−4	.	4	.	.	.	.	.	−2	2	2	2	.	2	2	.	−2	−2	.	−4	−4	−4
$X.23$	−4	4	4	−4	.	4	.	−4	.	.	.	.	.	2	−2	2	2	.	2	2	.	−2	−2	.	−4	−4	−4
$X.24$	−4	4	4	−4	.	4	.	−4	.	.	.	.	.	−2	2	−2	−2	.	−2	−2	.	2	2	.	−4	−4	−4
$X.25$	−4	4	−4	4	.	−4	.	4	.	.	.	.	.	2	−2	−2	−2	.	−2	−2	.	2	2	.	−4	−4	−4
$X.26$	4	4	2	2	.	2	.	2	4	4	2	2	2	−2	−2	.	.	2	.	.	4	.	.	.	6	6	6
$X.27$	4	4	−2	−2	.	−2	.	−2	4	4	−2	−2	−2	2	2	.	.	−2	.	.	4	.	.	.	6	6	6
$X.28$	4	4	.	.	4	.	4	.	4	4	.	.	.	.	.	.	.	.	.	.	4	.	.	.	−3	−3	−3
$X.29$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−8	−8	−8
$X.30$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−8	−8	−8
$X.31$	.	.	.	.	.	.	.	.	4	−4	.	.	.	.	.	−4	4	.	4	−4	.	.	.	.	.	.	.
$X.32$	−8	8	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	.	.	.	.	−8	−8	−8
$X.33$	−8	8	.	.	.	.	.	.	.	.	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	−8	−8	−8
$X.34$	.	.	.	.	.	.	.	.	4	−4	.	.	.	.	.	4	−4	.	−4	4	.	.	.	.	.	.	.
$X.35$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−10	−10	−10
$X.36$	−3	−3	3	3	−3	3	−3	3	−3	−3	−1	−1	−1	−3	−3	−1	−1	−1	−1	−1	−3	−1	−1	1	.	.	.
$X.37$	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	1	1	1	3	3	−1	−1	1	−1	−1	−3	−1	−1	1	.	.	.
$X.38$	−6	−6	.	.	2	.	2	.	−6	−6	.	.	.	.	.	2	2	.	2	2	−6	2	2	.	9	9	9
$X.39$	.	.	4	−4	.	−4	.	4	.	.	4	−4	−4	.	.	.	.	4	.	.	.	.	.	1	−6	6	6
$X.40$	.	.	−4	4	.	4	.	−4	.	.	−4	4	4	.	.	.	.	−4	.	.	.	.	.	1	−6	6	6
$X.41$	.	.	−4	−4	.	4	.	4	.	.	4	4	−4	.	.	.	.	−4	.	.	.	.	.	1	6	−6	−6
$X.42$	.	.	4	4	.	−4	.	−4	.	.	−4	−4	4	.	.	.	.	4	.	.	.	.	.	1	6	−6	−6
$X.43$	.	.	.	.	−4	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3	3
$X.44$	4	−4	4	−4	.	4	.	−4	.	.	.	.	.	−6	6	−2	−2	.	−2	−2	.	2	2	.	−12	−12	−12
$X.45$	4	−4	−4	4	.	−4	.	4	.	.	.	.	.	−6	6	2	2	.	2	2	.	−2	−2	.	−12	−12	−12
$X.46$	4	−4	4	−4	.	4	.	−4	.	.	.	.	.	6	−6	2	2	.	2	2	.	−2	−2	.	−12	−12	−12
$X.47$	4	−4	−4	4	.	−4	.	4	.	.	.	.	.	6	−6	−2	−2	.	−2	−2	.	2	2	.	−12	−12	−12
$X.48$	.	.	.	.	−4	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3	3
$X.49$	.	.	.	.	−4	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3	3
$X.50$	.	.	.	.	−4	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3	3
$X.51$	3	3	−3	−3	3	−3	3	−3	3	3	1	1	1	3	3	−3	−3	1	−3	−3	−5	−3	−3	.	.	.	.
$X.52$	3	3	3	3	3	3	3	3	3	3	−1	−1	−1	−3	−3	−3	−3	−1	−3	−3	−5	−3	−3	.	.	.	.
$X.53$	−1	−1	3	3	3	3	3	3	−1	−1	3	3	3	−1	−1	3	3	3	3	3	−1	3	3	.	.	.	.
$X.54$	−1	−1	−3	−3	3	−3	3	−3	−1	−1	−3	−3	−3	1	1	3	3	−3	3	3	−1	3	3	.	.	.	.
$X.55$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	−4	4	.	−4	4	.	−2	2	2
$X.56$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4	.	4	−4	.	−4	4	.	2	−2	−2
$X.57$	.	.	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.58$	.	.	.	.	4	.	−4	.	.	.	.	.	.	.	.	4	4	.	−4	−4	.	.	.	.	A	$\bar{A}$	A
$X.59$	.	.	.	.	4	.	−4	.	.	.	.	.	.	.	.	4	4	.	−4	−4	.	.	.	.	$\bar{A}$	A	$\bar{A}$
$X.60$	.	.	.	.	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.
$X.61$	4	−4	.	.	.	.	.	.	.	.	4	−4	4	−2	2	−2	−2	−4	−2	−2	.	2	2	1	.	.	.
$X.62$	4	−4	.	.	.	.	.	.	.	.	−4	4	−4	−2	2	2	2	4	2	2	.	−2	−2	1	.	.	.
$X.63$	4	−4	.	.	.	.	.	.	.	.	4	−4	4	2	−2	2	2	−4	2	2	.	−2	−2	1	.	.	.
$X.64$	4	−4	.	.	.	.	.	.	.	.	−4	4	−4	2	−2	−2	−2	4	−2	−2	.	2	2	1	.	.	.
$X.65$	−8	8	−8	8	.	−8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	12	12	12
$X.66$	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	−4	4	.	4	−4	.	.	.	.	.	.	.
$X.67$	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	4	−4	.	−4	4	.	.	.	.	.	.	.
$X.68$	−8	8	−8	8	.	−8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	12	12	12
$X.69$	−8	8	8	−8	.	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	12	12	12
$X.70$	.	.	−6	6	.	6	.	−6	.	.	−2	2	2	.	.	−2	2	−2	−2	2	.	2	−2	.	−6	6	6
$X.71$	.	.	6	−6	.	−6	.	6	.	.	2	−2	−2	.	.	−2	2	2	−2	2	.	2	−2	.	−6	6	6
$X.72$	.	.	−2	2	.	2	.	−2	.	.	2	−2	−2	.	.	2	−2	2	2	−2	.	−2	2	.	12	−12	−12
$X.73$	.	.	2	−2	.	−2	.	2	.	.	−2	2	2	.	.	2	−2	−2	2	−2	.	−2	2	.	12	−12	−12
$X.74$	.	.	−2	−2	.	2	.	2	.	.	−2	−2	2	.	.	−2	2	2	−2	2	.	−2	2	.	−12	12	12
$X.75$	.	.	2	2	.	−2	.	−2	.	.	2	2	−2	.	.	−2	2	−2	−2	2	.	−2	2	.	−12	12	12
$X.76$	.	.	.	.	8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	6	6	6
$X.77$	.	.	6	6	.	−6	.	−6	.	.	−2	−2	2	.	.	2	−2	2	2	−2	.	2	−2	.	6	−6	−6
$X.78$	−8	8	8	−8	.	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	12	12	12
$X.79$	.	.	−6	−6	.	6	.	6	.	.	2	2	−2	.	.	2	−2	−2	2	−2	.	2	−2	.	6	−6	−6
$X.80$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	6	6	6
$X.81$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	6	6	6
$X.82$	2	2	.	.	6	.	6	.	2	2	4	4	4	2	2	.	.	4	.	.	−6	.	.	.	.	.	.
$X.83$	2	2	.	.	6	.	6	.	2	2	−4	−4	−4	−2	−2	.	.	−4	.	.	−6	.	.	.	.	.	.
$X.84$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	4	4
$X.85$	.	.	−4	−4	.	4	.	4	.	.	−4	−4	4	.	.	.	.	4	.	.	.	.	.	.	−4	4	4
$X.86$	.	.	4	4	.	−4	.	−4	.	.	4	4	−4	.	.	.	.	−4	.	.	.	.	.	.	−4	4	4
$X.87$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	4	4
$X.88$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	14	−14	−14
$X.89$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−14	14	14
$X.90$	.	.	4	−4	.	−4	.	4	.	.	−4	4	4	.	.	.	.	−4	.	.	.	.	.	.	4	−4	−4
$X.91$	.	.	−4	4	.	4	.	−4	.	.	4	−4	−4	.	.	.	.	4	.	.	.	.	.	.	4	−4	−4
$X.92$	.	.	.	.	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	A	$\bar{A}$	A
$X.93$	.	.	.	.	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{A}$	A	$\bar{A}$
$X.94$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	F	F	$\bar{F}$
$X.95$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{F}$	$\bar{F}$	F

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

	6_{4}	6_{5}	6_{6}	6_{7}	6_{8}	6_{9}	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}	6_{22}	6_{23}	6_{24}	6_{25}	6_{26}	6_{27}	6_{28}	6_{29}
2	8	8	8	8	9	9	9	8	8	8	8	8	8	8	8	8	8	6	6	6	7	7	7	7	5	5
3	4	4	4	4	3	3	3	3	3	2	2	2	2	2	2	2	2	3	3	3	2	2	2	2	3	3
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2P	3a	3a	3a	3a	3b	3b	3b	3b	3b	3a	3a	3a	3a	3a	3a	3a	3a	3c	3c	3c	3b	3b	3b	3b	3c	3c
3P	2a	2d	2c	2b	2a	2b	2c	2e	2d	2f	2l	2g	2l	2m	2m	2h	2i	2b	2c	2a	2f	2h	2g	2i	2d	2e
5P	6_{4}	6_{1}	6_{6}	6_{7}	6_{8}	6_{9}	6_{10}	6_{11}	6_{12}	6_{13}	6_{16}	6_{15}	6_{14}	6_{18}	6_{17}	6_{19}	6_{20}	6_{21}	6_{22}	6_{23}	6_{24}	6_{25}	6_{26}	6_{27}	6_{28}	6_{29}
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.3	−3	−3	−3	−3	.	.	.	.	.	1	1	1	1	1	1	1	1	3	3	3	−2	−2	−2	−2	3	3
X.4	−3	−3	−3	−3	.	.	.	.	.	1	1	1	1	1	1	1	1	3	3	3	−2	−2	−2	−2	3	3
X.5	1	1	1	1	4	4	4	4	4	−3	−3	−3	−3	−3	−3	−3	−3	−2	−2	−2	.	.	.	.	−2	−2
X.6	−3	−3	−3	−3	3	3	3	3	3	1	1	1	1	1	1	1	1	.	.	.	1	1	1	1	.	.
X.7	−3	−3	−3	−3	3	3	3	3	3	1	1	1	1	1	1	1	1	.	.	.	1	1	1	1	.	.
X.8	6	6	6	6	.	.	.	.	.	2	2	2	2	2	2	2	2	3	3	3	2	2	2	2	3	3
X.9	6	6	6	6	.	.	.	.	.	2	2	2	2	2	2	2	2	3	3	3	2	2	2	2	3	3
X.10	2	−2	2	−2	−4	4	−4	−4	4	−2	2	2	2	−2	−2	−2	2	1	−1	−1	−2	−2	2	2	1	−1
X.11	2	−2	2	−2	−4	4	−4	−4	4	−2	2	2	2	−2	−2	−2	2	1	−1	−1	−2	−2	2	2	1	−1
X.12	2	2	2	−2	−4	4	−4	4	−4	2	2	−2	2	−2	−2	2	−2	1	−1	−1	2	2	−2	−2	−1	1
X.13	2	2	2	−2	−4	4	−4	4	−4	2	2	−2	2	−2	−2	2	−2	1	−1	−1	2	2	−2	−2	−1	1
X.14	−7	−7	−7	−7	2	2	2	2	2	1	1	1	1	1	1	1	1	2	2	2	−2	−2	−2	−2	2	2
X.15	2	2	2	2	−1	−1	−1	−1	−1	−2	−2	−2	−2	−2	−2	−2	−2	5	5	5	1	1	1	1	5	5
X.16	2	2	2	2	−1	−1	−1	−1	−1	−2	−2	−2	−2	−2	−2	−2	−2	5	5	5	1	1	1	1	5	5
X.17	6	6	6	6	3	3	3	3	3	2	2	2	2	2	2	2	2	.	.	.	−1	−1	−1	−1	.	.
X.18	6	6	6	6	3	3	3	3	3	2	2	2	2	2	2	2	2	.	.	.	−1	−1	−1	−1	.	.
X.19	3	3	3	3	3	3	3	3	3	−1	−1	−1	−1	−1	−1	−1	−1	3	3	3	−1	−1	−1	−1	3	3
X.20	3	3	3	3	3	3	3	3	3	−1	−1	−1	−1	−1	−1	−1	−1	3	3	3	−1	−1	−1	−1	3	3
X.21	4	.	−4	4	−8	−8	8	.	.	4	.	4	.	.	.	−4	−4	−2	2	−2	4	−4	4	−4	.	.
X.22	4	−4	4	4	7	7	7	−7	−7	4	−4	4	−4	−4	−4	4	4	1	1	1	1	1	1	1	−1	−1
X.23	4	−4	4	4	7	7	7	−7	−7	4	−4	4	−4	−4	−4	4	4	1	1	1	1	1	1	1	−1	−1
X.24	4	−4	4	4	7	7	7	−7	−7	−4	4	−4	4	4	4	−4	−4	1	1	1	−1	−1	−1	−1	−1	−1
X.25	4	−4	4	4	7	7	7	−7	−7	−4	4	−4	4	4	4	−4	−4	1	1	1	−1	−1	−1	−1	−1	−1
X.26	6	6	6	6	−3	−3	−3	−3	−3	2	2	2	2	2	2	2	2	−3	−3	−3	−1	−1	−1	−1	−3	−3
X.27	6	6	6	6	−3	−3	−3	−3	−3	2	2	2	2	2	2	2	2	−3	−3	−3	−1	−1	−1	−1	−3	−3
X.28	−3	−3	−3	−3	.	.	.	.	.	−3	−3	−3	−3	−3	−3	−3	−3	−6	−6	−6	.	.	.	.	−6	−6
X.29	−8	−8	−8	−8	−2	−2	−2	−2	−2	.	.	.	.	.	.	.	.	4	4	4	.	.	.	.	4	4
X.30	−8	−8	−8	−8	−2	−2	−2	−2	−2	.	.	.	.	.	.	.	.	4	4	4	.	.	.	.	4	4
X.31	8	.	−8	−8	14	−14	−14	.	.	−8	.	8	.	.	.	8	−8	−2	−2	2	−2	2	2	−2	.	.
X.32	8	−8	8	8	−4	−4	−4	4	4	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	−2	−2
X.33	8	−8	8	8	−4	−4	−4	4	4	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	−2	−2
X.34	8	.	−8	−8	14	−14	−14	.	.	8	.	−8	.	.	.	−8	8	−2	−2	2	2	−2	−2	2	.	.
X.35	−10	−10	−10	−10	2	2	2	2	2	2	2	2	2	2	2	2	2	−4	−4	−4	2	2	2	2	−4	−4
X.36	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.37	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.38	9	9	9	9	.	.	.	.	.	−3	−3	−3	−3	−3	−3	−3	−3	.	.	.	.	.	.	.	.	.
X.39	−6	−6	−6	6	.	.	.	.	.	2	2	−2	2	−2	−2	2	−2	3	−3	−3	−4	−4	4	4	−3	3
X.40	−6	−6	−6	6	.	.	.	.	.	2	2	−2	2	−2	−2	2	−2	3	−3	−3	−4	−4	4	4	−3	3
X.41	−6	6	−6	6	.	.	.	.	.	−2	2	2	2	−2	−2	−2	2	3	−3	−3	4	4	−4	−4	3	−3
X.42	−6	6	−6	6	.	.	.	.	.	−2	2	2	2	−2	−2	−2	2	3	−3	−3	4	4	−4	−4	3	−3
X.43	3	3	3	3	6	6	6	6	6	3	3	3	3	3	3	3	3	.	.	.	.	.	.	.	.	.
X.44	12	−12	12	12	3	3	3	−3	−3	4	−4	4	−4	−4	−4	4	4	3	3	3	1	1	1	1	−3	−3
X.45	12	−12	12	12	3	3	3	−3	−3	−4	4	−4	4	4	4	−4	−4	3	3	3	−1	−1	−1	−1	−3	−3
X.46	12	−12	12	12	3	3	3	−3	−3	−4	4	−4	4	4	4	−4	−4	3	3	3	−1	−1	−1	−1	−3	−3
X.47	12	−12	12	12	3	3	3	−3	−3	4	−4	4	−4	−4	−4	4	4	3	3	3	1	1	1	1	−3	−3
X.48	3	3	3	3	6	6	6	6	6	3	3	3	3	3	3	3	3	.	.	.	.	.	.	.	.	.
X.49	3	3	3	3	6	6	6	6	6	3	3	3	3	3	3	3	3	.	.	.	.	.	.	.	.	.
X.50	3	3	3	3	6	6	6	6	6	3	3	3	3	3	3	3	3	.	.	.	.	.	.	.	.	.
X.51	.	.	.	.	9	9	9	9	9	.	.	.	.	.	.	.	.	.	.	.	3	3	3	3	.	.
X.52	.	.	.	.	9	9	9	9	9	.	.	.	.	.	.	.	.	.	.	.	3	3	3	3	.	.
X.53	.	.	.	.	9	9	9	9	9	.	.	.	.	.	.	.	.	.	.	.	3	3	3	3	.	.
X.54	.	.	.	.	9	9	9	9	9	.	.	.	.	.	.	.	.	.	.	.	3	3	3	3	.	.
X.55	2	−2	2	−2	−16	16	−16	−16	16	6	−6	−6	−6	6	6	6	−6	−2	2	2	.	.	.	.	−2	2
X.56	2	2	2	−2	−16	16	−16	16	−16	−6	−6	6	−6	6	6	−6	6	−2	2	2	.	.	.	.	2	−2
X.57	16	.	−16	−16	−8	8	8	.	.	.	.	.	.	.	.	.	.	−4	−4	4	.	.	.	.	.	.
X.58	2	$\bar{A}$	−2	2	−16	−16	16	.	.	−6	B	−6	$\bar{B}$	B	$\bar{B}$	6	6	2	−2	2	.	.	.	.	.	.
X.59	2	A	−2	2	−16	−16	16	.	.	−6	$\bar{B}$	−6	B	$\bar{B}$	B	6	6	2	−2	2	.	.	.	.	.	.
X.60	−12	.	12	−12	.	.	.	.	.	4	.	4	.	.	.	−4	−4	−6	6	−6	−8	8	−8	8	.	.
X.61	.	.	.	.	9	9	9	−9	−9	.	.	.	.	.	.	.	.	.	.	.	−3	−3	−3	−3	.	.
X.62	.	.	.	.	9	9	9	−9	−9	.	.	.	.	.	.	.	.	.	.	.	3	3	3	3	.	.
X.63	.	.	.	.	9	9	9	−9	−9	.	.	.	.	.	.	.	.	.	.	.	3	3	3	3	.	.
X.64	.	.	.	.	9	9	9	−9	−9	.	.	.	.	.	.	.	.	.	.	.	−3	−3	−3	−3	.	.
X.65	−12	12	−12	−12	.	.	.	.	.	4	−4	4	−4	−4	−4	4	4	3	3	3	−2	−2	−2	−2	−3	−3
X.66	24	.	−24	−24	6	−6	−6	.	.	8	.	−8	.	.	.	−8	8	−6	−6	6	2	−2	−2	2	.	.
X.67	24	.	−24	−24	6	−6	−6	.	.	−8	.	8	.	.	.	8	−8	−6	−6	6	−2	2	2	−2	.	.
X.68	−12	12	−12	−12	.	.	.	.	.	−4	4	−4	4	4	4	−4	−4	3	3	3	2	2	2	2	−3	−3
X.69	−12	12	−12	−12	.	.	.	.	.	4	−4	4	−4	−4	−4	4	4	3	3	3	−2	−2	−2	−2	−3	−3
X.70	−6	−6	−6	6	−12	12	−12	12	−12	2	2	−2	2	−2	−2	2	−2	.	.	.	2	2	−2	−2	.	.
X.71	−6	−6	−6	6	−12	12	−12	12	−12	2	2	−2	2	−2	−2	2	−2	.	.	.	2	2	−2	−2	.	.
X.72	12	12	12	−12	.	.	.	.	.	4	4	−4	4	−4	−4	4	−4	3	−3	−3	4	4	−4	−4	−3	3
X.73	12	12	12	−12	.	.	.	.	.	4	4	−4	4	−4	−4	4	−4	3	−3	−3	4	4	−4	−4	−3	3
X.74	12	−12	12	−12	.	.	.	.	.	−4	4	4	4	−4	−4	−4	4	3	−3	−3	−4	−4	4	4	3	−3
X.75	12	−12	12	−12	.	.	.	.	.	−4	4	4	4	−4	−4	−4	4	3	−3	−3	−4	−4	4	4	3	−3
X.76	6	6	6	6	12	12	12	12	12	6	6	6	6	6	6	6	6	.	.	.	.	.	.	.	.	.
X.77	−6	6	−6	6	−12	12	−12	−12	12	−2	2	2	2	−2	−2	−2	2	.	.	.	−2	−2	2	2	.	.
X.78	−12	12	−12	−12	.	.	.	.	.	−4	4	−4	4	4	4	−4	−4	3	3	3	2	2	2	2	−3	−3
X.79	−6	6	−6	6	−12	12	−12	−12	12	−2	2	2	2	−2	−2	−2	2	.	.	.	−2	−2	2	2	.	.
X.80	6	6	6	6	12	12	12	12	12	−6	−6	−6	−6	−6	−6	−6	−6	.	.	.	.	.	.	.	.	.
X.81	6	6	6	6	12	12	12	12	12	−6	−6	−6	−6	−6	−6	−6	−6	.	.	.	.	.	.	.	.	.
X.82	.	.	.	.	−9	−9	−9	−9	−9	.	.	.	.	.	.	.	.	.	.	.	−3	−3	−3	−3	.	.
X.83	.	.	.	.	−9	−9	−9	−9	−9	.	.	.	.	.	.	.	.	.	.	.	−3	−3	−3	−3	.	.
X.84	−4	4	−4	−4	14	14	14	−14	−14	4	−4	4	−4	−4	−4	4	4	−4	−4	−4	−2	−2	−2	−2	4	4
X.85	4	−4	4	−4	4	−4	4	4	−4	4	−4	−4	−4	4	4	4	−4	5	−5	−5	−2	−2	2	2	5	−5
X.86	4	−4	4	−4	4	−4	4	4	−4	4	−4	−4	−4	4	4	4	−4	5	−5	−5	−2	−2	2	2	5	−5
X.87	−4	4	−4	−4	14	14	14	−14	−14	−4	4	−4	4	4	4	−4	−4	−4	−4	−4	2	2	2	2	4	4
X.88	−14	14	−14	14	−8	8	−8	−8	8	−2	2	2	2	−2	−2	−2	2	2	−2	−2	4	4	−4	−4	2	−2
X.89	−14	−14	−14	14	−8	8	−8	8	−8	2	2	−2	2	−2	−2	2	−2	2	−2	−2	−4	−4	4	4	−2	2
X.90	4	4	4	−4	4	−4	4	−4	4	−4	−4	4	−4	4	4	−4	4	5	−5	−5	2	2	−2	−2	−5	5
X.91	4	4	4	−4	4	−4	4	−4	4	−4	−4	4	−4	4	4	−4	4	5	−5	−5	2	2	−2	−2	−5	5
X.92	−14	$\bar{A}$	14	−14	−8	−8	8	.	.	2	A	2	$\bar{A}$	A	$\bar{A}$	−2	−2	−2	2	−2	−4	4	−4	4	.	.
X.93	−14	A	14	−14	−8	−8	8	.	.	2	$\bar{A}$	2	A	$\bar{A}$	A	−2	−2	−2	2	−2	−4	4	−4	4	.	.
X.94	−4	$\bar{F}$	4	4	14	−14	−14	.	.	4	G	−4	$\bar{G}$	$\bar{G}$	G	−4	4	4	4	−4	−2	2	2	−2	.	.
X.95	−4	F	4	4	14	−14	−14	.	.	4	$\bar{G}$	−4	G	G	$\bar{G}$	−4	4	4	4	−4	−2	2	2	−2	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	8	8	6	6	6	6	7	5	5	5	5	5	5	5	5	6	6	6	6	4	4	5	5	5	7	7	7	7
3	1	1	2	2	2	2	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2	1	1	1	1	1	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	6_{30}	6_{31}	6_{32}	6_{33}	6_{34}	6_{35}	6_{36}	6_{37}	6_{38}	6_{39}	6_{40}	6_{41}	6_{42}	6_{43}	6_{44}	6_{45}	6_{46}	6_{47}	6_{48}	6_{49}	6_{50}	6_{51}	6_{52}	6_{53}	$8a$	$8b$	$8c$	$8d$
$2P$	$3b$	$3b$	$3b$	$3b$	$3b$	$3b$	$3b$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3c$	$3b$	$3c$	4_1	4_1	4_4	4_{20}
$3P$	$2j$	$2k$	$2m$	$2o$	$2p$	$2l$	$2n$	$2f$	$2m$	$2l$	$2i$	$2m$	$2g$	$2l$	$2h$	$2q$	$2r$	$2s$	$2t$	$2o$	$2p$	$2u$	$2x$	$2v$	$8a$	$8b$	$8c$	$8d$
$5P$	6_{30}	6_{31}	6_{32}	6_{33}	6_{34}	6_{35}	6_{36}	6_{37}	6_{41}	6_{43}	6_{40}	6_{38}	6_{42}	6_{39}	6_{44}	6_{45}	6_{46}	6_{47}	6_{48}	6_{49}	6_{50}	6_{51}	6_{52}	6_{53}	$8a$	$8b$	$8c$	$8d$
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	1	1	1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	1	-1	1	-1	-1	1	1
$X.3$	.	.	-2	-2	-2	-2	.	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	-1	.	-1	.	.	2	.
$X.4$	.	.	-2	2	2	-2	.	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	.	-1	.	.	2	.
$X.5$	4	4	.	.	.	.	4	.	.	.	.	.	.	.	.	2	2	2	2	.	.	2	.	2	.	.	2	-2
$X.6$	3	3	1	-1	-1	1	3	-2	-2	-2	-2	-2	-2	-2	-2	.	.	.	.	2	2	.	1	.	1	1	-1	1
$X.7$	3	3	1	1	1	1	3	-2	-2	-2	-2	-2	-2	-2	-2	.	.	.	.	-2	-2	.	-1	.	-1	-1	-1	1
$X.8$	.	.	2	-2	-2	2	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	.	-1	3	3	3	-1
$X.9$	.	.	2	2	2	2	.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	.	-1	-3	-3	3	-1
$X.10$	.	.	-2	-2	2	2	.	1	1	-1	-1	1	-1	-1	1	1	1	-1	-1	1	-1	1	.	-1	.	.	.	.
$X.11$	.	.	-2	2	-2	2	.	1	1	-1	-1	1	-1	-1	1	1	1	-1	-1	-1	1	1	.	-1	.	.	.	.
$X.12$	.	.	-2	2	-2	2	.	-1	1	-1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	.	1	.	.	.	.
$X.13$	.	.	-2	-2	2	2	.	-1	1	-1	1	1	1	-1	-1	1	1	-1	-1	1	-1	-1	.	1	.	.	.	.
$X.14$	2	2	-2	.	.	-2	2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2	2	.	.	2	.	2	.	.	4	.
$X.15$	-1	-1	1	1	1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	1	2	2	.	.
$X.16$	-1	-1	1	-1	-1	1	-1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	1	1	1	-2	-2	.	.
$X.17$	3	3	-1	1	1	-1	3	2	2	2	2	2	2	2	2	.	.	.	.	-2	-2	.	1	.	4	4	.	.
$X.18$	3	3	-1	-1	-1	-1	3	2	2	2	2	2	2	2	2	.	.	.	.	2	2	.	-1	.	-4	-4	.	.
$X.19$	3	3	-1	-1	-1	-1	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	2	2	-2	.
$X.20$	3	3	-1	1	1	-1	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1	1	-1	-2	-2	-2	.
$X.21$	.	.	.	.	.	.	.	-2	.	.	2	.	-2	.	2	2	-2	2	-2	.	.	.	.	.	.	.	.	.
$X.22$	-1	-1	-1	1	1	-1	1	1	-1	-1	1	-1	1	-1	1	1	1	1	1	1	1	-1	3	-1	-2	2	.	-2
$X.23$	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	1	1	1	-1	-1	-1	-3	-1	2	-2	.	-2
$X.24$	-1	-1	1	1	1	1	1	-1	1	1	-1	1	-1	1	-1	1	1	1	1	1	1	-1	1	-1	2	-2	.	2
$X.25$	-1	-1	1	-1	-1	1	1	-1	1	1	-1	1	-1	1	-1	1	1	1	1	-1	-1	-1	-1	-1	-2	2	.	2
$X.26$	-3	-3	-1	-1	-1	-1	-3	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	1	1	-2	-2	.	.
$X.27$	-3	-3	-1	1	1	-1	-3	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	-1	1	2	2	.	.
$X.28$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-2	-2	-2	-2	.	.	-2	.	-2	.	.	4	.
$X.29$	-2	-2	.	2	2	.	-2	.	.	.	.	.	.	.	.	.	.	.	.	2	2	.	.	.	.	.	.	.
$X.30$	-2	-2	.	-2	-2	.	-2	.	.	.	.	.	.	.	.	.	.	.	.	-2	-2	.	.	.	.	.	.	.
$X.31$	-2	2	.	.	.	.	.	-2	.	.	-2	.	2	.	2	2	-2	-2	2	.	.	.	.	.	.	.	.	.
$X.32$	4	4	.	2	2	.	-4	.	.	.	.	.	.	.	.	2	2	2	2	2	2	-2	-2	-2	.	.	.	.
$X.33$	4	4	.	-2	-2	.	-4	.	.	.	.	.	.	.	.	2	2	2	2	-2	-2	-2	2	-2	.	.	.	.
$X.34$	-2	2	.	.	.	.	.	2	.	.	2	.	-2	.	-2	2	-2	-2	2	.	.	.	.	.	.	.	.	.
$X.35$	2	2	2	.	.	2	2	2	2	2	2	2	2	2	2	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.36$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-3	-3	-3	-1
$X.37$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3	-3	-1
$X.38$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2
$X.39$	.	.	4	4	-4	-4	.	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	1	.	-1	.	.	.	.
$X.40$	.	.	4	-4	4	-4	.	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	-1	1	1	.	-1	.	.	.	.
$X.41$	.	.	4	-4	4	-4	.	1	1	-1	-1	1	-1	-1	1	-1	-1	1	1	-1	1	-1	.	1	.	.	.	.
$X.42$	.	.	4	4	-4	-4	.	1	1	-1	-1	1	-1	-1	1	-1	-1	1	1	1	-1	-1	.	1	.	.	.	.
$X.43$	-2	-2	.	.	.	.	-2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-2	.	.	.	.	.
$X.44$	3	3	-1	-1	-1	-1	-3	1	-1	-1	1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	1	1	1	-2	2	.	2
$X.45$	3	3	1	-1	-1	1	-3	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	1	-1	1	2	-2	.	-2
$X.46$	3	3	1	1	1	1	-3	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	-1	1	1	1	1	1	-2	2	.	-2
$X.47$	3	3	-1	1	1	-1	-3	1	-1	-1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	1	-1	1	2	-2	.	2
$X.48$	-2	-2	.	.	.	.	-2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.
$X.49$	-2	-2	.	.	.	.	-2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.
$X.50$	-2	-2	.	.	.	.	-2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-2	.	.	.	.	.
$X.51$	1	1	3	3	3	3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	-1	-1	-1	1
$X.52$	1	1	3	-3	-3	3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-1	.	1	1	-1	1
$X.53$	1	1	3	3	3	3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	-1	-1	-1	-1
$X.54$	1	1	3	-3	-3	3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-1	.	1	1	-1	-1
$X.55$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	-2	-2	.	.	2	.	-2	.	.	.	.
$X.56$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	-2	-2	.	.	-2	.	2	.	.	.	.
$X.57$	8	-8	.	.	.	.	.	.	.	.	.	.	.	.	.	4	-4	-4	4	.	.	.	.	.	.	.	.	.
$X.58$	.	.	.	.	.	.	.	.	B	B	.	$\bar{B}$	.	$\bar{B}$	.	2	-2	2	-2	.	.	.	.	.	.	.	.	.
$X.59$	.	.	.	.	.	.	.	.	$\bar{B}$	$\bar{B}$	.	B	.	B	.	2	-2	2	-2	.	.	.	.	.	.	.	.	.
$X.60$	.	.	.	.	.	.	.	-2	.	.	2	.	-2	.	2	-2	2	-2	2	.	.	.	.	.	.	.	.	.
$X.61$	1	1	3	-3	-3	3	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	2	-2	.	-2
$X.62$	1	1	-3	-3	-3	-3	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-1	.	-2	2	.	2
$X.63$	1	1	-3	3	3	-3	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	2	-2	.	2
$X.64$	1	1	3	3	3	3	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-1	.	-2	2	.	-2
$X.65$	.	.	2	2	2	2	.	1	-1	-1	1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	1	.	1	.	.	.	.
$X.66$	6	-6	.	.	.	.	.	2	.	.	2	.	-2	.	-2	-2	2	2	-2	.	.	.	.	.	.	.	.	.
$X.67$	6	-6	.	.	.	.	.	-2	.	.	-2	.	2	.	2	-2	2	2	-2	.	.	.	.	.	.	.	.	.
$X.68$	.	.	-2	-2	-2	-2	.	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	-1	1	1	1	.	1	.	.	.	.
$X.69$	.	.	2	-2	-2	2	.	1	-1	-1	1	-1	1	-1	1	-1	-1	-1	-1	1	1	1	.	1	.	.	.	.
$X.70$	.	.	-2	2	-2	2	.	2	-2	2	-2	-2	-2	2	2	.	.	.	.	2	-2	.	.	.	.	.	.	.
$X.71$	.	.	-2	-2	2	2	.	2	-2	2	-2	-2	-2	2	2	.	.	.	.	-2	2	.	.	.	.	.	.	.
$X.72$	.	.	-4	-4	4	4	.	1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	-1	1	1	.	-1	.	.	.	.
$X.73$	.	.	-4	4	-4	4	.	1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	-1	1	.	-1	.	.	.	.
$X.74$	.	.	-4	-4	4	4	.	-1	-1	1	1	-1	1	1	-1	-1	-1	1	1	-1	1	-1	.	1	.	.	.	.
$X.75$	.	.	-4	4	-4	4	.	-1	-1	1	1	-1	1	1	-1	-1	-1	1	1	1	-1	-1	.	1	.	.	.	.
$X.76$	-4	-4	.	.	.	.	-4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.77$	.	.	-2	-2	2	2	.	-2	-2	2	2	-2	2	2	-2	.	.	.	.	-2	2	.	.	.	.	.	.	.
$X.78$	.	.	-2	2	2	-2	.	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	1	.	1	.	.	.	.
$X.79$	.	.	-2	2	-2	2	.	-2	-2	2	2	-2	2	2	-2	.	.	.	.	2	-2	.	.	.	.	.	.	.
$X.80$	-4	-4	.	.	.	.	-4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.81$	-4	-4	.	.	.	.	-4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.82$	-1	-1	-3	-3	-3	-3	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	-1	.	-2	-2	-2	.
$X.83$	-1	-1	-3	3	3	-3	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	2	2	-2	.
$X.84$	-2	-2	2	.	.	2	2	-2	2	2	-2	2	-2	2	-2	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.85$	.	.	-2	-2	2	2	.	1	1	-1	-1	1	-1	-1	1	1	1	-1	-1	1	-1	1	.	-1	.	.	.	.
$X.86$	.	.	-2	2	-2	2	.	1	1	-1	-1	1	-1	-1	1	1	1	-1	-1	-1	1	1	.	-1	.	.	.	.
$X.87$	-2	-2	-2	.	.	-2	2	2	-2	-2	2	-2	2	-2	2	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.88$	.	.	4	.	.	-4	.	-2	-2	2	2	-2	2	2	-2	2	2	-2	-2	.	.	2	.	-2	.	.	.	.
$X.89$	.	.	4	.	.	-4	.	2	-2	2	-2	-2	-2	2	2	2	2	-2	-2	.	.	-2	.	2	.	.	.	.
$X.90$	.	.	-2	2	-2	2	.	-1	1	-1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	.	1	.	.	.	.
$X.91$	.	.	-2	-2	2	2	.	-1	1	-1	1	1	1	-1	-1	1	1	-1	-1	1	-1	-1	.	1	.	.	.	.
$X.92$	.	.	.	.	.	.	.	2	.	.	-2	.	2	.	-2	2	-2	2	-2	.	.	.	.	.	.	.	.	.
$X.93$	.	.	.	.	.	.	.	2	.	.	-2	.	2	.	-2	2	-2	2	-2	.	.	.	.	.	.	.	.	.
$X.94$	-2	2	.	.	.	.	.	-2	$\bar{B}$	B	-2	B	2	$\bar{B}$	2	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.95$	-2	2	.	.	.	.	.	-2	B	$\bar{B}$	-2	$\bar{B}$	2	B	2	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,Fi_{22})$ (continued)

2	7	7	7	6	6	6	6	6	6	6	3	4	4	4
3	.	.	.	.	.	.	.	.	.	.	2	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	1	1	1
	$8e$	$8f$	$8g$	$8h$	$8i$	$8j$	$8k$	$8l$	$8m$	$8n$	$9a$	$10a$	$10b$	$10c$
$2P$	4_{20}	4_{20}	4_{20}	4_{11}	4_{11}	4_{16}	4_{19}	4_{27}	4_{27}	4_{16}	$9a$	$5a$	$5a$	$5a$
$3P$	$8e$	$8f$	$8g$	$8h$	$8i$	$8j$	$8k$	$8l$	$8m$	$8n$	$3a$	$10a$	$10b$	$10c$
$5P$	$8e$	$8f$	$8g$	$8h$	$8i$	$8j$	$8k$	$8l$	$8m$	$8n$	$9a$	$2b$	$2a$	$2c$
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	−1	1	−1	−1	−1	−1	−1	1	1	−1	1	1	1	1
$X.3$	−2	.	2	.	.	.	2	.	.	.	.	1	1	1
$X.4$	2	.	−2	.	.	.	−2	.	.	.	.	1	1	1
$X.5$	.	−2	.	.	.	.	.	−2	−2	.	1	.	.	.
$X.6$	3	1	−1	−1	−1	−1	−1	1	1	−1	.	.	.	.
$X.7$	−3	1	1	1	1	1	1	1	1	1	.	.	.	.
$X.8$	−1	−1	−1	1	1	1	−1	−1	−1	1	.	.	.	.
$X.9$	1	−1	1	−1	−1	−1	1	−1	−1	−1	.	.	.	.
$X.10$	.	.	.	.	.	2	.	.	.	−2	1	1	−1	−1
$X.11$	.	.	.	.	.	−2	.	.	.	2	1	1	−1	−1
$X.12$	.	.	.	−2	2	.	.	.	.	.	1	1	−1	−1
$X.13$	.	.	.	2	−2	.	.	.	.	.	1	1	−1	−1
$X.14$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.15$	2	.	2	.	.	.	2	.	.	.	−1	.	.	.
$X.16$	−2	.	−2	.	.	.	−2	.	.	.	−1	.	.	.
$X.17$	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1
$X.18$	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1
$X.19$	4	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.20$	−4	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.21$	.	.	.	.	.	.	.	.	.	.	2	−2	−2	2
$X.22$	.	2	.	.	.	.	.	.	.	.	1	.	.	.
$X.23$	.	2	.	.	.	.	.	.	.	.	1	.	.	.
$X.24$	.	−2	.	.	.	.	.	.	.	.	1	.	.	.
$X.25$	.	−2	.	.	.	.	.	.	.	.	1	.	.	.
$X.26$	2	.	2	.	.	.	2	.	.	.	.	.	.	.
$X.27$	−2	.	−2	.	.	.	−2	.	.	.	.	.	.	.
$X.28$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.29$	.	.	.	.	.	.	.	.	.	.	1	−1	−1	−1
$X.30$	.	.	.	.	.	.	.	.	.	.	1	−1	−1	−1
$X.31$	.	.	.	.	.	.	.	−2	2	.	2	.	.	.
$X.32$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.33$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.34$	.	.	.	.	.	.	.	2	−2	.	2	.	.	.
$X.35$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.36$	3	−1	−1	1	1	1	−1	−1	−1	1	.	1	1	1
$X.37$	−3	−1	1	−1	−1	−1	1	−1	−1	−1	.	1	1	1
$X.38$	.	2	.	.	.	.	.	2	2	.	.	.	.	.
$X.39$	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1
$X.40$	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1
$X.41$	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1
$X.42$	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1
$X.43$	.	.	.	2	2	−2	.	.	.	−2	.	.	.	.
$X.44$	.	−2	.	.	.	.	.	.	.	.	.	.	.	.
$X.45$	.	2	.	.	.	.	.	.	.	.	.	.	.	.
$X.46$	.	2	.	.	.	.	.	.	.	.	.	.	.	.
$X.47$	.	−2	.	.	.	.	.	.	.	.	.	.	.	.
$X.48$	.	.	.	2	2	−2	.	.	.	−2	.	.	.	.
$X.49$	.	.	.	−2	−2	2	.	.	.	2	.	.	.	.
$X.50$	.	.	.	−2	−2	2	.	.	.	2	.	.	.	.
$X.51$	1	1	−3	−1	−1	−1	1	1	1	−1	.	.	.	.
$X.52$	−1	1	3	1	1	1	−1	1	1	1	.	.	.	.
$X.53$	−1	−1	−1	1	1	1	−1	−1	−1	1	.	.	.	.
$X.54$	1	−1	1	−1	−1	−1	1	−1	−1	−1	.	.	.	.
$X.55$	.	.	.	.	.	.	.	.	.	.	1	.	.	.
$X.56$	.	.	.	.	.	.	.	.	.	.	1	.	.	.
$X.57$	.	.	.	.	.	.	.	.	.	.	−2	.	.	.
$X.58$	.	.	.	.	.	.	.	.	.	.	1	.	.	.
$X.59$	.	.	.	.	.	.	.	.	.	.	1	.	.	.
$X.60$	.	.	.	.	.	.	.	.	.	.	.	−2	−2	2
$X.61$	.	2	.	.	.	.	.	.	.	.	.	1	1	1
$X.62$	.	−2	.	.	.	.	.	.	.	.	.	1	1	1
$X.63$	.	−2	.	.	.	.	.	.	.	.	.	1	1	1
$X.64$	.	2	.	.	.	.	.	.	.	.	.	1	1	1
$X.65$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.66$	.	.	.	.	.	.	.	−2	2	.	.	.	.	.
$X.67$	.	.	.	.	.	.	.	2	−2	.	.	.	.	.
$X.68$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.69$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.70$	.	.	.	−2	2	.	.	.	.	.	.	.	.	.
$X.71$	.	.	.	2	−2	.	.	.	.	.	.	.	.	.
$X.72$	.	.	.	−2	2	.	.	.	.	.	.	.	.	.
$X.73$	.	.	.	2	−2	.	.	.	.	.	.	.	.	.
$X.74$	.	.	.	.	.	−2	.	.	.	2	.	.	.	.
$X.75$	.	.	.	.	.	2	.	.	.	−2	.	.	.	.
$X.76$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.77$	.	.	.	.	.	2	.	.	.	−2	.	.	.	.
$X.78$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.79$	.	.	.	.	.	−2	.	.	.	2	.	.	.	.
$X.80$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.81$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.82$	.	.	−4	.	.	.	.	.	.	.	.	.	.	.
$X.83$	.	.	4	.	.	.	.	.	.	.	.	.	.	.
$X.84$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.85$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.86$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.87$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.88$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.89$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.90$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.91$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.92$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.93$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.94$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.
$X.95$	.	.	.	.	.	.	.	.	.	.	−1	.	.	.

2	3	3	3	3	7	7	5	6	6	6	6	6	6
3	.	.	.	.	2	2	3	2	2	2	1	1	1
5	1	1	1	1	.	.	.	.	.	.	.	.	.
	$10d$	$10e$	$10f$	$10g$	12_1	12_2	12_3	12_4	12_5	12_6	12_7	12_8	12_9
$2P$	$5a$	$5a$	$5a$	$5a$	6_8	6_8	6_4	6_9	6_9	6_8	6_{30}	6_{30}	6_{30}
$3P$	$10d$	$10e$	$10f$	$10g$	4_1	4_1	4_1	4_3	4_2	4_1	4_5	4_6	4_7
$5P$	$2o$	$2d$	$2p$	$2e$	12_1	12_2	12_3	12_4	12_5	12_6	12_7	12_8	12_9
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	−1	1	−1	1	1	1	1	−1	−1	1	−1	1	1
$X.3$	−1	1	−1	1	.	.	−3	−2	−2	.	.	−2	−2
$X.4$	1	1	1	1	.	.	−3	2	2	.	.	−2	−2
$X.5$	.	.	.	.	4	4	1	.	.	4	.	.	.
$X.6$	.	.	.	.	3	3	−3	−1	−1	3	1	1	1
$X.7$	.	.	.	.	3	3	−3	1	1	3	−1	1	1
$X.8$	.	.	.	.	.	.	6	−2	−2	.	.	2	2
$X.9$	.	.	.	.	.	.	6	2	2	.	.	2	2
$X.10$	−1	1	1	−1	.	.	.	2	−2	.	2	.	.
$X.11$	1	1	−1	−1	.	.	.	−2	2	.	−2	.	.
$X.12$	1	−1	−1	1	.	.	.	2	−2	.	2	.	.
$X.13$	−1	−1	1	1	.	.	.	−2	2	.	−2	.	.
$X.14$	.	.	.	.	2	2	−7	.	.	2	.	−2	−2
$X.15$	.	.	.	.	−1	−1	2	1	1	−1	−1	1	1
$X.16$	.	.	.	.	−1	−1	2	−1	−1	−1	1	1	1
$X.17$	−1	−1	−1	−1	3	3	6	1	1	3	1	−1	−1
$X.18$	1	−1	1	−1	3	3	6	−1	−1	3	−1	−1	−1
$X.19$	.	.	.	.	3	3	3	−1	−1	3	−1	−1	−1
$X.20$	.	.	.	.	3	3	3	1	1	3	1	−1	−1
$X.21$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.22$	.	.	.	.	3	3	.	−1	−1	−3	−1	1	1
$X.23$	.	.	.	.	3	3	.	1	1	−3	1	1	1
$X.24$	.	.	.	.	3	3	.	−1	−1	−3	−3	−1	−1
$X.25$	.	.	.	.	3	3	.	1	1	−3	3	−1	−1
$X.26$	.	.	.	.	−3	−3	6	−1	−1	−3	1	−1	−1
$X.27$	.	.	.	.	−3	−3	6	1	1	−3	−1	−1	−1
$X.28$	.	.	.	.	.	.	−3	.	.	.	.	.	.
$X.29$	−1	−1	−1	−1	−2	−2	−8	2	2	−2	.	.	.
$X.30$	1	−1	1	−1	−2	−2	−8	−2	−2	−2	.	.	.
$X.31$	.	.	.	.	6	−6	.	.	.	.	.	−2	2
$X.32$	.	.	.	.	.	.	.	−2	−2	.	2	.	.
$X.33$	.	.	.	.	.	.	.	2	2	.	−2	.	.
$X.34$	.	.	.	.	6	−6	.	.	.	.	.	2	−2
$X.35$	.	.	.	.	2	2	−10	.	.	2	.	2	2
$X.36$	−1	1	−1	1	.	.	.	.	.	.	.	.	.
$X.37$	1	1	1	1	.	.	.	.	.	.	.	.	.
$X.38$	.	.	.	.	.	.	9	.	.	.	.	.	.
$X.39$	1	−1	−1	1	.	.	.	4	−4	.	.	.	.
$X.40$	−1	−1	1	1	.	.	.	−4	4	.	.	.	.
$X.41$	−1	1	1	−1	.	.	.	4	−4	.	.	.	.
$X.42$	1	1	−1	−1	.	.	.	−4	4	.	.	.	.
$X.43$	.	.	.	.	2	2	−1	.	.	2	2	.	.
$X.44$	.	.	.	.	3	3	.	1	1	−3	1	1	1
$X.45$	.	.	.	.	3	3	.	1	1	−3	−1	−1	−1
$X.46$	.	.	.	.	3	3	.	−1	−1	−3	1	−1	−1
$X.47$	.	.	.	.	3	3	.	−1	−1	−3	−1	1	1
$X.48$	.	.	.	.	2	2	−1	.	.	2	−2	.	.
$X.49$	.	.	.	.	2	2	−1	.	.	2	−2	.	.
$X.50$	.	.	.	.	2	2	−1	.	.	2	2	.	.
$X.51$	.	.	.	.	−3	−3	.	3	3	−3	1	−1	−1
$X.52$	.	.	.	.	−3	−3	.	−3	−3	−3	−1	−1	−1
$X.53$	.	.	.	.	−3	−3	.	3	3	−3	1	−1	−1
$X.54$	.	.	.	.	−3	−3	.	−3	−3	−3	−1	−1	−1
$X.55$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.56$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.57$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.58$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.59$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.60$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.61$	−1	−1	−1	−1	−3	−3	.	3	3	3	1	1	1
$X.62$	−1	−1	−1	−1	−3	−3	.	3	3	3	−1	−1	−1
$X.63$	1	−1	1	−1	−3	−3	.	−3	−3	3	1	−1	−1
$X.64$	1	−1	1	−1	−3	−3	.	−3	−3	3	−1	1	1
$X.65$	.	.	.	.	.	.	.	−2	−2	.	.	−2	−2
$X.66$	.	.	.	.	6	−6	.	.	.	.	.	2	−2
$X.67$	.	.	.	.	6	−6	.	.	.	.	.	−2	2
$X.68$	.	.	.	.	.	.	.	2	2	.	.	2	2
$X.69$	.	.	.	.	.	.	.	2	2	.	.	−2	−2
$X.70$	.	.	.	.	.	.	.	2	−2	.	−2	.	.
$X.71$	.	.	.	.	.	.	.	−2	2	.	2	.	.
$X.72$	.	.	.	.	.	.	.	−4	4	.	.	.	.
$X.73$	.	.	.	.	.	.	.	4	−4	.	.	.	.
$X.74$	.	.	.	.	.	.	.	4	−4	.	.	.	.
$X.75$	.	.	.	.	.	.	.	−4	4	.	.	.	.
$X.76$	.	.	.	.	4	4	−2	.	.	4	.	.	.
$X.77$	.	.	.	.	.	.	.	2	−2	.	−2	.	.
$X.78$	.	.	.	.	.	.	.	−2	−2	.	.	2	2
$X.79$	.	.	.	.	.	.	.	−2	2	.	2	.	.
$X.80$	.	.	.	.	4	4	−2	.	.	4	.	.	.
$X.81$	.	.	.	.	4	4	−2	.	.	4	.	.	.
$X.82$	.	.	.	.	3	3	.	−3	−3	3	−1	1	1
$X.83$	.	.	.	.	3	3	.	3	3	3	1	1	1
$X.84$	.	.	.	.	6	6	.	.	.	−6	.	−2	−2
$X.85$	.	.	.	.	.	.	.	2	−2	.	−2	.	.
$X.86$	.	.	.	.	.	.	.	−2	2	.	2	.	.
$X.87$	.	.	.	.	6	6	.	.	.	−6	.	2	2
$X.88$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.89$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.90$	.	.	.	.	.	.	.	2	−2	.	−2	.	.
$X.91$	.	.	.	.	.	.	.	−2	2	.	2	.	.
$X.92$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.93$	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.94$	.	.	.	.	6	−6	.	.	.	.	.	−2	2
$X.95$	.	.	.	.	6	−6	.	.	.	.	.	−2	2

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	6	6	6	4	4	5	5	5	5	5	5	5	5	5	5	5	5	5	4	4	4
3	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	12_{10}	12_{11}	12_{12}	12_{13}	12_{14}	12_{15}	12_{16}	12_{17}	12_{18}	12_{19}	12_{20}	12_{21}	12_{22}	12_{23}	12_{24}	12_{25}	12_{26}	12_{27}	12_{28}	12_{29}	12_{30}
$2P$	6_{31}	6_{31}	6_{30}	6_{21}	6_{21}	6_{13}	6_{30}	6_{14}	6_{31}	6_{14}	6_{15}	6_{30}	6_{4}	6_{16}	6_{16}	6_{9}	6_{15}	6_{13}	6_{46}	6_{45}	6_{46}
$3P$	4_{8}	4_{9}	4_{10}	4_{2}	4_{3}	4_{11}	4_{12}	4_{23}	4_{13}	4_{25}	4_{14}	4_{15}	4_{4}	4_{25}	4_{23}	4_{17}	4_{16}	4_{18}	4_{21}	4_{22}	4_{24}
$5P$	12_{10}	12_{11}	12_{12}	12_{13}	12_{14}	12_{15}	12_{16}	12_{24}	12_{18}	12_{23}	12_{20}	12_{21}	12_{22}	12_{19}	12_{17}	12_{25}	12_{26}	12_{27}	12_{28}	12_{29}	12_{30}
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	−1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	1	1	1	−1	1	1	−1	−1	−1
$X.3$	.	.	.	1	1	−1	−2	−1	−2	−1	−1	−2	1	−1	−1	.	−1	−1	1	1	1
$X.4$	.	.	.	−1	−1	−1	−2	−1	2	−1	−1	2	1	−1	−1	.	−1	−1	−1	−1	−1
$X.5$	.	.	.	.	.	−1	.	−1	.	−1	−1	.	−3	−1	−1	.	−1	−1	.	.	.
$X.6$	1	1	1	2	2	−1	1	−1	−1	−1	−1	−1	1	−1	−1	1	−1	−1	.	.	.
$X.7$	−1	−1	−1	−2	−2	−1	1	−1	1	−1	−1	1	1	−1	−1	−1	−1	−1	.	.	.
$X.8$	.	.	.	1	1	.	2	.	−2	.	.	−2	2	.	.	.	.	.	−1	−1	−1
$X.9$	.	.	.	−1	−1	.	2	.	2	.	.	2	2	.	.	.	.	.	1	1	1
$X.10$	2	−2	−2	1	−1	−2	.	.	.	.	.	.	.	.	.	.	.	2	1	1	−1
$X.11$	−2	2	2	−1	1	−2	.	.	.	.	.	.	.	.	.	.	.	2	−1	−1	1
$X.12$	−2	2	−2	1	−1	.	.	.	.	.	2	.	.	.	.	.	−2	.	−1	1	1
$X.13$	2	−2	2	−1	1	.	.	.	.	.	2	.	.	.	.	.	−2	.	1	−1	−1
$X.14$	.	.	.	.	.	1	−2	1	.	1	1	.	1	1	1	.	1	1	.	.	.
$X.15$	−1	−1	−1	1	1	.	1	.	1	.	.	1	−2	.	.	−1	.	.	−1	−1	−1
$X.16$	1	1	1	−1	−1	.	1	.	−1	.	.	−1	−2	.	.	1	.	.	1	1	1
$X.17$	1	1	1	−2	−2	.	−1	.	1	.	.	1	2	.	.	1	.	.	.	.	.
$X.18$	−1	−1	−1	2	2	.	−1	.	−1	.	.	−1	2	.	.	−1	.	.	.	.	.
$X.19$	−1	−1	−1	−1	−1	1	−1	1	−1	1	1	−1	−1	1	1	−1	1	1	1	1	1
$X.20$	1	1	1	1	1	1	−1	1	1	1	1	1	−1	1	1	1	1	1	−1	−1	−1
$X.21$	.	.	.	.	.	.	.	−2	.	2	.	.	.	2	−2	.	.	.	.	.	.
$X.22$	1	1	−1	−1	−1	.	−1	.	−1	.	.	1	.	.	.	−3	.	.	−1	1	−1
$X.23$	−1	−1	1	1	1	.	−1	.	1	.	.	−1	.	.	.	3	.	.	1	−1	1
$X.24$	3	3	−3	−1	−1	.	1	.	−1	.	.	1	.	.	.	−1	.	.	1	−1	1
$X.25$	−3	−3	3	1	1	.	1	.	1	.	.	−1	.	.	.	1	.	.	−1	1	−1
$X.26$	1	1	1	−1	−1	.	−1	.	−1	.	.	−1	2	.	.	1	.	.	−1	−1	−1
$X.27$	−1	−1	−1	1	1	.	−1	.	1	.	.	1	2	.	.	−1	.	.	1	1	1
$X.28$	.	.	.	.	.	1	.	1	.	1	1	.	−3	1	1	.	1	1	.	.	.
$X.29$	.	.	.	2	2	.	.	.	2	.	.	2	.	.	.	.	.	.	.	.	.
$X.30$	.	.	.	−2	−2	.	.	.	−2	.	.	−2	.	.	.	.	.	.	.	.	.
$X.31$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.32$	−2	−2	2	−2	−2	.	.	.	−2	.	.	2	.	.	.	2	.	.	.	.	.
$X.33$	2	2	−2	2	2	.	.	.	2	.	.	−2	.	.	.	−2	.	.	.	.	.
$X.34$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.35$	.	.	.	.	.	.	2	.	.	.	.	.	2	.	.	.	.	.	.	.	.
$X.36$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.37$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.38$	.	.	.	.	.	−1	.	−1	.	−1	−1	.	−3	−1	−1	.	−1	−1	.	.	.
$X.39$	.	.	.	−1	1	.	.	.	.	.	−2	.	.	.	.	.	2	.	1	−1	−1
$X.40$	.	.	.	1	−1	.	.	.	.	.	−2	.	.	.	.	.	2	.	−1	1	1
$X.41$	.	.	.	−1	1	2	.	.	.	.	.	.	.	.	.	.	.	−2	−1	−1	1
$X.42$	.	.	.	1	−1	2	.	.	.	.	.	.	.	.	.	.	.	−2	1	1	−1
$X.43$	2	2	2	.	.	−1	.	−1	.	−1	3	.	−1	−1	−1	−2	3	−1	.	.	.
$X.44$	−1	−1	1	1	1	.	−1	.	1	.	.	−1	.	.	.	−1	.	.	1	−1	1
$X.45$	1	1	−1	1	1	.	1	.	1	.	.	−1	.	.	.	1	.	.	−1	1	−1
$X.46$	−1	−1	1	−1	−1	.	1	.	−1	.	.	1	.	.	.	−1	.	.	1	−1	1
$X.47$	1	1	−1	−1	−1	.	−1	.	−1	.	.	1	.	.	.	1	.	.	−1	1	−1
$X.48$	−2	−2	−2	.	.	3	.	−1	.	−1	−1	.	−1	−1	−1	2	−1	3	.	.	.
$X.49$	−2	−2	−2	.	.	−1	.	−1	.	−1	3	.	−1	−1	−1	2	3	−1	.	.	.
$X.50$	2	2	2	.	.	3	.	−1	.	−1	−1	.	−1	−1	−1	−2	−1	3	.	.	.
$X.51$	1	1	1	.	.	.	−1	.	−1	.	.	−1	.	.	.	1	.	.	.	.	.
$X.52$	−1	−1	−1	.	.	.	−1	.	1	.	.	1	.	.	.	−1	.	.	.	.	.
$X.53$	1	1	1	.	.	.	−1	.	−1	.	.	−1	.	.	.	1	.	.	.	.	.
$X.54$	−1	−1	−1	.	.	.	−1	.	1	.	.	1	.	.	.	−1	.	.	.	.	.
$X.55$	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.
$X.56$	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	2	.	.	.	.
$X.57$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.58$	.	.	.	.	.	.	.	C	.	$-C$	.	.	.	$-\bar{C}$	$\bar{C}$	.	.	.	.	.	.
$X.59$	.	.	.	.	.	.	.	$\bar{C}$	.	$-\bar{C}$	.	.	.	$-C$	C	.	.	.	.	.	.
$X.60$	.	.	.	.	.	.	.	2	.	−2	.	.	.	−2	2	.	.	.	.	.	.
$X.61$	−1	−1	1	.	.	.	−1	.	−1	.	.	1	.	.	.	−1	.	.	.	.	.
$X.62$	1	1	−1	.	.	.	1	.	−1	.	.	1	.	.	.	1	.	.	.	.	.
$X.63$	−1	−1	1	.	.	.	1	.	1	.	.	−1	.	.	.	−1	.	.	.	.	.
$X.64$	1	1	−1	.	.	.	−1	.	1	.	.	−1	.	.	.	1	.	.	.	.	.
$X.65$	.	.	.	1	1	.	2	.	−2	.	.	2	.	.	.	.	.	.	1	−1	1
$X.66$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.67$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.68$	.	.	.	−1	−1	.	−2	.	2	.	.	−2	.	.	.	.	.	.	1	−1	1
$X.69$	.	.	.	−1	−1	.	2	.	2	.	.	−2	.	.	.	.	.	.	−1	1	−1
$X.70$	2	−2	2	−2	2	.	.	.	.	.	−2	.	.	.	.	.	2	.	.	.	.
$X.71$	−2	2	−2	2	−2	.	.	.	.	.	−2	.	.	.	.	.	2	.	.	.	.
$X.72$	.	.	.	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1
$X.73$	.	.	.	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1
$X.74$	.	.	.	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	−1
$X.75$	.	.	.	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1
$X.76$	.	.	.	.	.	−2	.	2	.	2	−2	.	−2	2	2	.	−2	−2	.	.	.
$X.77$	−2	2	2	−2	2	2	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.
$X.78$	.	.	.	1	1	.	−2	.	−2	.	.	2	.	.	.	.	.	.	−1	1	−1
$X.79$	2	−2	−2	2	−2	2	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.
$X.80$	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.
$X.81$	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.
$X.82$	−1	−1	−1	.	.	.	1	.	1	.	.	1	.	.	.	−1	.	.	.	.	.
$X.83$	1	1	1	.	.	.	1	.	−1	.	.	−1	.	.	.	1	.	.	.	.	.
$X.84$	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.85$	−2	2	2	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1
$X.86$	2	−2	−2	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	−1
$X.87$	.	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.88$	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.
$X.89$	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	−2	.	.	.	.
$X.90$	2	−2	2	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1
$X.91$	−2	2	−2	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1
$X.92$	.	.	.	.	.	.	.	$-C$	.	C	.	.	.	$\bar{C}$	$-\bar{C}$	.	.	.	.	.	.
$X.93$	.	.	.	.	.	.	.	$-\bar{C}$	.	$\bar{C}$	.	.	.	C	$-C$	.	.	.	.	.	.
$X.94$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.95$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	4	5	5	3	3	3	3	3	3	3	3	3	4	4
3	1	.	.	2	2	2	2	2	2	2	.	.	1	1
5	.	.	.	.	.	.	.	.	.	.	1	1	.	.
	12_{31}	$16a$	$16b$	$18a$	$18b$	$18c$	$18d$	$18e$	$18f$	$18g$	$20a$	$20b$	$24a$	$24b$
$2P$	6_{45}	$8c$	$8c$	$9a$	$9a$	$9a$	$9a$	$9a$	$9a$	$9a$	$10a$	$10a$	12_1	12_2
$3P$	4_{26}	$16a$	$16b$	6_2	6_1	6_3	6_6	6_5	6_7	6_4	$20a$	$20b$	$8a$	$8b$
$5P$	12_{31}	$16b$	$16a$	$18c$	$18e$	$18a$	$18d$	$18b$	$18f$	$18g$	4_2	4_3	$24a$	$24b$
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	−1	−1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1
$X.3$	1	.	.	.	.	.	.	.	.	.	−1	−1	.	.
$X.4$	−1	.	.	.	.	.	.	.	.	.	1	1	.	.
$X.5$	.	.	.	1	1	1	1	1	1	1	.	.	.	.
$X.6$	.	−1	−1	.	.	.	.	.	.	.	.	.	1	1
$X.7$	.	1	1	.	.	.	.	.	.	.	.	.	−1	−1
$X.8$	−1	1	1	.	.	.	.	.	.	.	.	.	.	.
$X.9$	1	−1	−1	.	.	.	.	.	.	.	.	.	.	.
$X.10$	−1	.	.	−1	1	−1	−1	1	1	−1	−1	1	.	.
$X.11$	1	.	.	−1	1	−1	−1	1	1	−1	1	−1	.	.
$X.12$	−1	.	.	1	−1	1	−1	−1	1	−1	−1	1	.	.
$X.13$	1	.	.	1	−1	1	−1	−1	1	−1	1	−1	.	.
$X.14$	.	.	.	−1	−1	−1	−1	−1	−1	−1	.	.	.	.
$X.15$	−1	.	.	−1	−1	−1	−1	−1	−1	−1	.	.	−1	−1
$X.16$	1	.	.	−1	−1	−1	−1	−1	−1	−1	.	.	1	1
$X.17$	.	.	.	.	.	.	.	.	.	.	−1	−1	1	1
$X.18$	.	.	.	.	.	.	.	.	.	.	1	1	−1	−1
$X.19$	1	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.20$	−1	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.21$	.	.	.	.	.	.	2	.	−2	−2	.	.	.	.
$X.22$	1	.	.	−1	−1	−1	1	−1	1	1	.	.	1	−1
$X.23$	−1	.	.	−1	−1	−1	1	−1	1	1	.	.	−1	1
$X.24$	−1	.	.	−1	−1	−1	1	−1	1	1	.	.	−1	1
$X.25$	1	.	.	−1	−1	−1	1	−1	1	1	.	.	1	−1
$X.26$	−1	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.27$	1	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.28$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.29$	.	.	.	1	1	1	1	1	1	1	−1	−1	.	.
$X.30$	.	.	.	1	1	1	1	1	1	1	1	1	.	.
$X.31$	.	.	.	.	.	.	−2	.	−2	2	.	.	.	.
$X.32$	.	.	.	1	1	1	−1	1	−1	−1	.	.	.	.
$X.33$	.	.	.	1	1	1	−1	1	−1	−1	.	.	.	.
$X.34$	.	.	.	.	.	.	−2	.	−2	2	.	.	.	.
$X.35$	.	.	.	−1	−1	−1	−1	−1	−1	−1	.	.	.	.
$X.36$	.	1	1	.	.	.	.	.	.	.	−1	−1	.	.
$X.37$	.	−1	−1	.	.	.	.	.	.	.	1	1	.	.
$X.38$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.39$	1	.	.	.	.	.	.	.	.	.	−1	1	.	.
$X.40$	−1	.	.	.	.	.	.	.	.	.	1	−1	.	.
$X.41$	1	.	.	.	.	.	.	.	.	.	−1	1	.	.
$X.42$	−1	.	.	.	.	.	.	.	.	.	1	−1	.	.
$X.43$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.44$	−1	.	.	.	.	.	.	.	.	.	.	.	1	−1
$X.45$	1	.	.	.	.	.	.	.	.	.	.	.	−1	1
$X.46$	−1	.	.	.	.	.	.	.	.	.	.	.	1	−1
$X.47$	1	.	.	.	.	.	.	.	.	.	.	.	−1	1
$X.48$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.49$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.50$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.51$	.	1	1	.	.	.	.	.	.	.	.	.	−1	−1
$X.52$	.	−1	−1	.	.	.	.	.	.	.	.	.	1	1
$X.53$	.	−1	−1	.	.	.	.	.	.	.	.	.	−1	−1
$X.54$	.	1	1	.	.	.	.	.	.	.	.	.	1	1
$X.55$	.	.	.	−1	1	−1	−1	1	1	−1	.	.	.	.
$X.56$	.	.	.	1	−1	1	−1	−1	1	−1	.	.	.	.
$X.57$	.	.	.	.	.	.	2	.	2	−2	.	.	.	.
$X.58$	.	.	.	D	$\bar{D}$	$\bar{D}$	1	D	−1	−1	.	.	.	.
$X.59$	.	.	.	$\bar{D}$	D	D	1	$\bar{D}$	−1	−1	.	.	.	.
$X.60$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.61$	.	.	.	.	.	.	.	.	.	.	1	1	−1	1
$X.62$	.	.	.	.	.	.	.	.	.	.	1	1	1	−1
$X.63$	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	1
$X.64$	.	.	.	.	.	.	.	.	.	.	−1	−1	1	−1
$X.65$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.66$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.67$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.68$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.69$	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.70$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.71$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.72$	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.73$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.74$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.75$	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.76$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.77$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.78$	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.79$	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.80$	.	E	$\bar{E}$	.	.	.	.	.	.	.	.	.	.	.
$X.81$	.	$\bar{E}$	E	.	.	.	.	.	.	.	.	.	.	.
$X.82$	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.83$	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.84$	.	.	.	1	1	1	−1	1	−1	−1	.	.	.	.
$X.85$	1	.	.	1	−1	1	1	−1	−1	1	.	.	.	.
$X.86$	−1	.	.	1	−1	1	1	−1	−1	1	.	.	.	.
$X.87$	.	.	.	1	1	1	−1	1	−1	−1	.	.	.	.
$X.88$	.	.	.	1	−1	1	1	−1	−1	1	.	.	.	.
$X.89$	.	.	.	−1	1	−1	1	1	−1	1	.	.	.	.
$X.90$	1	.	.	−1	1	−1	1	1	−1	1	.	.	.	.
$X.91$	−1	.	.	−1	1	−1	1	1	−1	1	.	.	.	.
$X.92$	.	.	.	D	$\bar{D}$	$\bar{D}$	−1	D	1	1	.	.	.	.
$X.93$	.	.	.	$\bar{D}$	D	D	−1	$\bar{D}$	1	1	.	.	.	.
$X.94$	.	.	.	D	D	$\bar{D}$	1	$\bar{D}$	1	−1	.	.	.	.
$X.95$	.	.	.	$\bar{D}$	$\bar{D}$	D	1	D	1	−1	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	18	18	18	18	17	17	16	16	16	16	17	17	15	15	16	11	11	13	13
3	4	4	4	4	4	4	2	2	2	2	1	1	2	2	1	2	2	1	1
5	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	1	1	.	.
	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r
2P	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a	1a
3P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r
5P	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n	2o	2p	2q	2r
X.96	320	320	−320	−320	.	.	−64	64	64	−64	64	−64	.	.	.	.	.	.	.
X.97	320	320	−320	−320	.	.	−64	64	64	−64	64	−64	.	.	.	.	.	.	.
X.98	384	−384	384	−384	384	−384	−64	64	−64	64	.	.	64	−64	.	16	−16	.	.
X.99	384	−384	384	−384	−384	384	64	−64	64	−64	.	.	64	−64	.	−16	16	.	.
X.100	384	−384	384	−384	384	−384	−64	64	−64	64	.	.	64	−64	.	−16	16	.	.
X.101	384	−384	384	−384	−384	384	64	−64	64	−64	.	.	64	−64	.	16	−16	.	.
X.102	405	405	405	405	405	405	45	45	45	45	21	21	45	45	21	45	45	9	9
X.103	405	405	405	405	405	405	−27	−27	−27	−27	21	21	−27	−27	21	45	45	−3	−3
X.104	405	405	405	405	405	405	−27	−27	−27	−27	21	21	−27	−27	21	−45	−45	−3	−3
X.105	405	405	405	405	405	405	45	45	45	45	21	21	45	45	21	−45	−45	9	9
X.106	432	432	−432	−432	.	.	96	−96	−96	96	−16	16	.	.	.	.	.	24	−24
X.107	432	432	−432	−432	.	.	−96	96	96	−96	−16	16	.	.	.	.	.	24	−24
X.108	480	480	480	480	−480	−480	−32	−32	−32	−32	96	96	32	32	−96	−40	−40	.	.
X.109	480	480	−480	−480	.	.	64	−64	−64	64	96	−96	.	.	.	.	.	16	−16
X.110	480	480	480	480	−480	−480	.	.	.	.	96	96	.	.	−96	.	.	−16	−16
X.111	480	−480	480	−480	480	−480	80	−80	80	−80	.	.	−80	80	.	−40	40	8	8
X.112	480	−480	480	−480	−480	480	−80	80	−80	80	.	.	−80	80	.	−40	40	8	8
X.113	480	480	480	480	−480	−480	32	32	32	32	96	96	−32	−32	−96	−40	−40	.	.
X.114	480	−480	480	−480	−480	480	−80	80	−80	80	.	.	−80	80	.	40	−40	8	8
X.115	480	480	−480	−480	.	.	−64	64	64	−64	96	−96	.	.	.	.	.	16	−16
X.116	480	−480	480	−480	480	−480	80	−80	80	−80	.	.	−80	80	.	40	−40	8	8
X.117	480	480	480	480	−480	−480	−32	−32	−32	−32	96	96	32	32	−96	40	40	.	.
X.118	480	−480	−480	480	.	.	112	112	−112	−112	.	.	.	.	.	.	.	24	−24
X.119	480	480	480	480	−480	−480	32	32	32	32	96	96	−32	−32	−96	40	40	.	.
X.120	480	−480	−480	480	.	.	−16	−16	16	16	.	.	.	.	.	.	.	−8	8
X.121	480	480	−480	−480	.	.	.	.	.	.	96	−96	.	.	.	.	.	−16	16
X.122	480	480	−480	−480	.	.	.	.	.	.	96	−96	.	.	.	.	.	−16	16
X.123	540	540	540	540	540	540	60	60	60	60	28	28	60	60	28	−30	−30	−12	−12
X.124	540	540	540	540	540	540	−84	−84	−84	−84	28	28	−84	−84	28	30	30	12	12
X.125	540	540	540	540	540	540	−84	−84	−84	−84	28	28	−84	−84	28	−30	−30	12	12
X.126	540	540	540	540	540	540	60	60	60	60	28	28	60	60	28	30	30	−12	−12
X.127	640	640	640	640	−640	−640	.	.	.	.	128	128	.	.	−128	.	.	.	.
X.128	640	−640	−640	640	.	.	64	64	−64	−64	.	.	.	.	.	.	.	32	−32
X.129	640	640	−640	−640	.	.	.	.	.	.	128	−128	.	.	.	.	.	.	.
X.130	640	640	−640	−640	.	.	.	.	.	.	128	−128	.	.	.	.	.	.	.
X.131	720	720	720	720	720	720	−48	−48	−48	−48	−48	−48	−48	−48	−48	.	.	.	.
X.132	720	720	720	720	720	720	−48	−48	−48	−48	−48	−48	−48	−48	−48	.	.	.	.
X.133	720	720	720	720	720	720	−48	−48	−48	−48	−48	−48	−48	−48	−48	.	.	.	.
X.134	720	720	720	720	720	720	−48	−48	−48	−48	−48	−48	−48	−48	−48	.	.	.	.
X.135	768	−768	−768	768	.	.	128	128	−128	−128	.	.	.	.	.	.	.	.	.
X.136	810	810	810	810	810	810	90	90	90	90	42	42	90	90	42	.	.	18	18
X.137	810	810	810	810	810	810	18	18	18	18	42	42	18	18	42	.	.	−18	−18
X.138	810	810	810	810	810	810	−54	−54	−54	−54	42	42	−54	−54	42	.	.	−6	−6
X.139	810	810	810	810	810	810	18	18	18	18	42	42	18	18	42	.	.	−18	−18
X.140	864	864	864	864	−864	−864	96	96	96	96	−32	−32	−96	−96	32	24	24	.	.
X.141	864	864	864	864	−864	−864	−96	−96	−96	−96	−32	−32	96	96	32	−24	−24	.	.
X.142	864	864	864	864	−864	−864	96	96	96	96	−32	−32	−96	−96	32	−24	−24	.	.
X.143	864	864	864	864	−864	−864	−96	−96	−96	−96	−32	−32	96	96	32	24	24	.	.
X.144	960	960	−960	−960	.	.	−64	64	64	−64	192	−192	.	.	.	.	.	.	.
X.145	960	960	960	960	960	960	.	.	.	.	−64	−64	.	.	−64	.	.	.	.
X.146	960	960	−960	−960	.	.	64	−64	−64	64	192	−192	.	.	.	.	.	.	.
X.147	960	−960	960	−960	960	−960	−96	96	−96	96	.	.	96	−96	.	.	.	16	16
X.148	960	−960	−960	960	.	.	−160	−160	160	160	.	.	.	.	.	.	.	16	−16
X.149	960	960	960	960	960	960	.	.	.	.	−64	−64	.	.	−64	.	.	.	.
X.150	960	−960	960	−960	−960	960	96	−96	96	−96	.	.	96	−96	.	.	.	16	16
X.151	960	−960	960	−960	−960	960	−32	32	−32	32	.	.	−32	32	.	−40	40	16	16
X.152	960	−960	960	−960	960	−960	32	−32	32	−32	.	.	−32	32	.	−40	40	16	16
X.153	960	−960	960	−960	−960	960	−32	32	−32	32	.	.	−32	32	.	40	−40	16	16
X.154	960	−960	960	−960	960	−960	32	−32	32	−32	.	.	−32	32	.	40	−40	16	16
X.155	960	−960	−960	960	.	.	96	96	−96	−96	.	.	.	.	.	.	.	16	−16
X.156	960	−960	−960	960	.	.	96	96	−96	−96	.	.	.	.	.	.	.	16	−16
X.157	1024	−1024	1024	−1024	1024	−1024	.	.	.	.	.	.	.	.	.	−64	64	.	.
X.158	1024	−1024	1024	−1024	−1024	1024	.	.	.	.	.	.	.	.	.	−64	64	.	.
X.159	1024	−1024	1024	−1024	−1024	1024	.	.	.	.	.	.	.	.	.	64	−64	.	.
X.160	1024	−1024	1024	−1024	1024	−1024	.	.	.	.	.	.	.	.	.	64	−64	.	.
X.161	1080	1080	1080	1080	−1080	−1080	48	48	48	48	−40	−40	−48	−48	40	30	30	12	12
X.162	1080	1080	1080	1080	−1080	−1080	−48	−48	−48	−48	−40	−40	48	48	40	30	30	12	12
X.163	1080	1080	1080	1080	1080	1080	−24	−24	−24	−24	56	56	−24	−24	56	−60	−60	.	.
X.164	1080	1080	1080	1080	−1080	−1080	−48	−48	−48	−48	−40	−40	48	48	40	−30	−30	12	12
X.165	1080	1080	1080	1080	1080	1080	−24	−24	−24	−24	56	56	−24	−24	56	60	60	.	.
X.166	1080	1080	1080	1080	−1080	−1080	48	48	48	48	−40	−40	−48	−48	40	−30	−30	12	12
X.167	1280	−1280	1280	−1280	1280	−1280	128	−128	128	−128	.	.	−128	128	.	.	.	.	.
X.168	1280	−1280	1280	−1280	−1280	1280	−128	128	−128	128	.	.	−128	128	.	.	.	.	.
X.169	1280	−1280	−1280	1280	.	.	−128	−128	128	128	.	.	.	.	.	.	.	.	.
X.170	1280	−1280	−1280	1280	.	.	−128	−128	128	128	.	.	.	.	.	.	.	.	.
X.171	1296	1296	1296	1296	−1296	−1296	.	.	.	.	−48	−48	.	.	48	36	36	−24	−24
X.172	1296	−1296	1296	−1296	1296	−1296	−72	72	−72	72	.	.	72	−72	.	−36	36	−12	−12
X.173	1296	−1296	1296	−1296	−1296	1296	72	−72	72	−72	.	.	72	−72	.	−36	36	−12	−12
X.174	1296	1296	1296	1296	−1296	−1296	.	.	.	.	−48	−48	.	.	48	−36	−36	−24	−24
X.175	1296	−1296	1296	−1296	−1296	1296	72	−72	72	−72	.	.	72	−72	.	36	−36	−12	−12
X.176	1296	−1296	1296	−1296	1296	−1296	−72	72	−72	72	.	.	72	−72	.	36	−36	−12	−12
X.177	1440	−1440	1440	−1440	1440	−1440	48	−48	48	−48	.	.	−48	48	.	.	.	−24	−24
X.178	1440	−1440	1440	−1440	−1440	1440	−48	48	−48	48	.	.	−48	48	.	.	.	−24	−24
X.179	1440	1440	1440	1440	1440	1440	96	96	96	96	−96	−96	96	96	−96	.	.	.	.
X.180	1440	−1440	−1440	1440	.	.	−48	−48	48	48	.	.	.	.	.	.	.	−24	24
X.181	1440	−1440	−1440	1440	.	.	−48	−48	48	48	.	.	.	.	.	.	.	−24	24
X.182	1728	1728	−1728	−1728	.	.	−192	192	192	−192	−64	64	.	.	.	.	.	.	.
X.183	1728	1728	−1728	−1728	.	.	192	−192	−192	192	−64	64	.	.	.	.	.	.	.
X.184	1920	−1920	−1920	1920	.	.	−64	−64	64	64	.	.	.	.	.	.	.	32	−32
X.185	2048	−2048	−2048	2048	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.186	2160	2160	−2160	−2160	.	.	−96	96	96	−96	−80	80	.	.	.	.	.	24	−24
X.187	2160	2160	−2160	−2160	.	.	96	−96	−96	96	−80	80	.	.	.	.	.	24	−24
X.188	2592	−2592	−2592	2592	.	.	144	144	−144	−144	.	.	.	.	.	.	.	−24	24
X.189	2592	2592	−2592	−2592	.	.	.	.	.	.	−96	96	.	.	.	.	.	−48	48

Character table of $H(2\,Fi_{22})$ *(continued)*

2	13	13	12	12	13	10	8	9	6	12	11	11	12	11	11	11	11	11	11	10	10	10	10
3	1	1	1	1	.	1	4	3	3	3	2	2	1	1	1	1	1	1	1	1	1	1	1
5	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.
	$2s$	$2t$	$2u$	$2v$	$2w$	$2x$	$3a$	$3b$	$3c$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}
$2P$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$3a$	$3b$	$3c$	$2a$	$2b$	$2b$	$2a$	$2j$	$2j$	$2j$	$2k$	$2k$	$2j$	$2f$	$2j$	$2k$	$2g$
$3P$	$2s$	$2t$	$2u$	$2v$	$2w$	$2x$	$1a$	$1a$	$1a$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}
$5P$	$2s$	$2t$	$2u$	$2v$	$2w$	$2x$	$3a$	$3b$	$3c$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}
$X.96$	.	.	.	.	.	.	−4	14	−4	.	.	.	.	.	−16	16	.	.	.	.	.	.	.
$X.97$	.	.	.	.	.	.	−4	14	−4	.	.	.	.	.	−16	16	.	.	.	.	.	.	.
$X.98$	.	.	.	.	.	.	−12	12	.	.	16	−16	.	−16	.	.	−16	16	16	.	.	.	.
$X.99$	.	.	.	.	.	.	−12	12	.	.	16	−16	.	−16	.	.	16	−16	16	.	.	.	.
$X.100$	.	.	.	.	.	.	−12	12	.	.	−16	16	.	16	.	.	16	−16	−16	.	.	.	.
$X.101$	.	.	.	.	.	.	−12	12	.	.	−16	16	.	16	.	.	−16	16	−16	.	.	.	.
$X.102$	9	9	9	9	13	9	.	.	.	−27	45	45	−3	9	−3	−3	9	9	9	−3	−3	−3	−3
$X.103$	−3	−3	−3	−3	5	−3	.	.	.	−27	45	45	21	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
$X.104$	−3	−3	−3	−3	5	3	.	.	.	−27	−45	−45	21	3	−3	−3	3	3	3	−3	−3	3	−3
$X.105$	9	9	9	9	13	−9	.	.	.	−27	−45	−45	−3	−9	−3	−3	−9	−9	−9	−3	−3	3	−3
$X.106$	−24	24	.	.	.	.	.	18	.	.	.	.	.	.	−8	8	.	.	.	.	.	.	.
$X.107$	−24	24	.	.	.	.	.	18	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.
$X.108$	.	.	.	.	.	8	12	−3	.	.	40	40	.	8	−8	−8	−8	−8	8	.	8	−8	.
$X.109$	−16	16	.	.	.	.	−24	.	6	.	.	.	.	.	16	−16	.	.	.	.	.	.	.
$X.110$	−16	−16	16	16	.	.	−24	.	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.111$	−8	−8	8	−8	.	.	−6	12	3	.	−40	40	.	−8	.	.	−8	8	8	−8	.	.	.
$X.112$	−8	−8	−8	8	.	.	−6	12	3	.	40	−40	.	8	.	.	−8	8	−8	.	.	.	8
$X.113$	.	.	.	.	.	−8	12	−3	.	.	40	40	.	−8	8	8	8	8	−8	.	−8	−8	.
$X.114$	−8	−8	−8	8	.	.	−6	12	3	.	−40	40	.	−8	.	.	8	−8	8	.	.	.	8
$X.115$	−16	16	.	.	.	.	−24	.	6	.	.	.	.	.	−16	16	.	.	.	.	.	.	.
$X.116$	−8	−8	8	−8	.	.	−6	12	3	.	40	−40	.	8	.	.	8	−8	−8	−8	.	.	.
$X.117$	.	.	.	.	.	−8	12	−3	.	.	−40	−40	.	−8	−8	−8	8	8	−8	.	8	8	.
$X.118$	24	−24	.	.	.	.	12	24	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.119$	.	.	.	.	.	8	12	−3	.	.	−40	−40	.	8	8	8	−8	−8	8	.	−8	8	.
$X.120$	−8	8	.	.	.	.	−24	.	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.121$	16	−16	.	.	.	.	−24	.	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.122$	16	−16	.	.	.	.	−24	.	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.123$	−12	−12	−12	−12	−4	2	.	9	.	−36	−30	−30	−36	2	4	4	2	2	2	.	4	2	.
$X.124$	12	12	12	12	−20	−2	.	9	.	−36	30	30	12	−2	4	4	−2	−2	−2	.	4	−2	.
$X.125$	12	12	12	12	−20	2	.	9	.	−36	−30	−30	12	2	4	4	2	2	2	.	4	2	.
$X.126$	−12	−12	−12	−12	−4	−2	.	9	.	−36	30	30	−36	−2	4	4	−2	−2	−2	.	4	−2	.
$X.127$	.	.	.	.	.	.	−8	−8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.128$	32	−32	.	.	.	.	−8	−8	10	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.129$	.	.	.	.	.	.	−8	−8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.130$	.	.	.	.	.	.	−8	−8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.131$	.	.	.	.	16	.	−9	.	.	48	.	.	−16	.	.	.	.	.	.	24	.	.	−8
$X.132$	.	.	.	.	16	.	−9	.	.	48	.	.	−16	.	.	.	.	.	.	−8	.	.	24
$X.133$	.	.	.	.	16	.	−9	.	.	48	.	.	−16	.	.	.	.	.	.	−8	.	.	−8
$X.134$	.	.	.	.	16	.	−9	.	.	48	.	.	−16	.	.	.	.	.	.	−8	.	.	−8
$X.135$	.	.	.	.	.	.	−24	24	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.136$	18	18	18	18	26	.	.	.	.	−54	.	.	−6	.	−6	−6	.	.	.	−6	−6	.	−6
$X.137$	−18	−18	−18	−18	18	−12	.	.	.	−54	.	.	18	−12	−6	−6	−12	−12	−12	6	−6	.	6
$X.138$	−6	−6	−6	−6	10	.	.	.	.	−54	.	.	42	.	−6	−6	.	.	.	−6	−6	.	−6
$X.139$	−18	−18	−18	−18	18	12	.	.	.	−54	.	.	18	12	−6	−6	12	12	12	6	−6	.	6
$X.140$	.	.	.	.	.	8	.	9	.	.	−24	−24	.	8	−8	−8	−8	−8	8	.	8	−8	.
$X.141$	.	.	.	.	.	8	.	9	.	.	24	24	.	8	8	8	−8	−8	8	.	−8	8	.
$X.142$	.	.	.	.	.	−8	.	9	.	.	24	24	.	−8	−8	−8	8	8	−8	.	8	8	.
$X.143$	.	.	.	.	.	−8	.	9	.	.	−24	−24	.	−8	8	8	8	8	−8	.	−8	−8	.
$X.144$	.	.	.	.	.	.	24	−6	.	.	.	.	.	.	−16	16	.	.	.	.	.	.	.
$X.145$	.	.	.	.	.	−16	24	−6	.	64	.	.	.	16	.	.	16	16	16	.	.	.	.
$X.146$	.	.	.	.	.	.	24	−6	.	.	.	.	.	.	16	−16	.	.	.	.	.	.	.
$X.147$	−16	−16	16	−16	.	.	6	.	−6	.	.	.	.	.	.	.	.	.	.	16	.	.	.
$X.148$	16	−16	.	.	.	.	−12	24	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.149$	.	.	.	.	.	16	24	−6	.	64	.	.	.	−16	.	.	−16	−16	−16	.	.	.	.
$X.150$	−16	−16	−16	16	.	.	6	.	−6	.	.	.	.	.	.	.	.	.	.	.	.	.	−16
$X.151$	−16	−16	−16	16	.	.	−12	−12	−3	.	40	−40	.	−8	.	.	8	−8	8	.	.	.	.
$X.152$	−16	−16	16	−16	.	.	−12	−12	−3	.	−40	40	.	8	.	.	8	−8	−8	.	.	.	.
$X.153$	−16	−16	−16	16	.	.	−12	−12	−3	.	−40	40	.	8	.	.	−8	8	−8	.	.	.	.
$X.154$	−16	−16	16	−16	.	.	−12	−12	−3	.	40	−40	.	−8	.	.	−8	8	8	.	.	.	.
$X.155$	16	−16	.	.	.	.	6	.	−6	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.156$	16	−16	.	.	.	.	6	.	−6	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.157$	.	.	.	.	.	.	16	−8	4	.	−64	64	.	.	.	.	.	.	.	.	.	.	.
$X.158$	.	.	.	.	.	.	16	−8	4	.	64	−64	.	.	.	.	.	.	.	.	.	.	.
$X.159$	.	.	.	.	.	.	16	−8	4	.	−64	64	.	.	.	.	.	.	.	.	.	.	.
$X.160$	.	.	.	.	.	.	16	−8	4	.	64	−64	.	.	.	.	.	.	.	.	.	.	.
$X.161$	12	12	−12	−12	.	10	.	−9	.	.	−30	−30	.	−2	−4	−4	2	2	−2	.	4	−10	.
$X.162$	12	12	−12	−12	.	2	.	−9	.	.	−30	−30	.	−10	4	4	10	10	−10	.	−4	−10	.
$X.163$	.	.	.	.	−24	4	.	−9	.	−72	−60	−60	−24	4	8	8	4	4	4	.	8	4	.
$X.164$	12	12	−12	−12	.	−2	.	−9	.	.	30	30	.	10	4	4	−10	−10	10	.	−4	10	.
$X.165$	.	.	.	.	−24	−4	.	−9	.	−72	60	60	−24	−4	8	8	−4	−4	−4	.	8	−4	.
$X.166$	12	12	−12	−12	.	−10	.	−9	.	.	30	30	.	2	−4	−4	−2	−2	2	.	4	10	.
$X.167$	.	.	.	.	.	.	20	8	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.168$	.	.	.	.	.	.	20	8	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.169$	.	.	.	.	.	.	20	8	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.170$	.	.	.	.	.	.	20	8	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.171$	−24	−24	24	24	.	−12	.	.	.	.	−36	−36	.	12	.	.	−12	−12	12	.	.	−12	.
$X.172$	12	12	−12	12	.	.	.	.	.	.	−36	36	.	−12	.	.	−12	12	12	−12	.	.	.
$X.173$	12	12	12	−12	.	.	.	.	.	.	36	−36	.	12	.	.	−12	12	−12	.	.	.	12
$X.174$	−24	−24	24	24	.	12	.	.	.	.	36	36	.	−12	.	.	12	12	−12	.	.	12	.
$X.175$	12	12	12	−12	.	.	.	.	.	.	−36	36	.	−12	.	.	12	−12	12	.	.	.	12
$X.176$	12	12	−12	12	.	.	.	.	.	.	36	−36	.	12	.	.	12	−12	−12	−12	.	.	.
$X.177$	24	24	−24	24	.	.	−18	.	.	.	.	.	.	.	.	.	.	.	.	8	.	.	.
$X.178$	24	24	24	−24	.	.	−18	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−8
$X.179$	.	.	.	.	−32	.	−18	.	.	96	.	.	32	.	.	.	.	.	.	.	.	.	.
$X.180$	−24	24	.	.	.	.	−18	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.181$	−24	24	.	.	.	.	−18	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.182$	.	.	.	.	.	.	.	18	.	.	.	.	.	.	16	−16	.	.	.	.	.	.	.
$X.183$	.	.	.	.	.	.	.	18	.	.	.	.	.	.	−16	16	.	.	.	.	.	.	.
$X.184$	32	−32	.	.	.	.	−24	−24	−6	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.185$	.	.	.	.	.	.	32	−16	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.186$	−24	24	.	.	.	.	.	−18	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.
$X.187$	−24	24	.	.	.	.	.	−18	.	.	.	.	.	.	−8	8	.	.	.	.	.	.	.
$X.188$	−24	24	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.189$	48	−48	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	10	10	10	10	11	11	9	9	9	9	9	9	10	10	9	9	9	9	9	9	9	9	9	9	9	8
3	1	1	1	1	.	.	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}
$2P$	$2j$	$2g$	$2b$	$2f$	$2k$	$2k$	$2r$	$2q$	$2l$	$2r$	$2l$	$2q$	$2n$	$2n$	$2q$	$2r$	$2q$	$2k$	$2j$	$2q$	$2q$	$2r$	$2q$	$2q$	$2j$	$2q$
$3P$	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}
$5P$	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}
$X.96$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.97$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.98$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.99$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.100$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.101$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.102$	−3	−3	9	−3	9	9	3	3	−3	3	−3	3	−7	−7	−1	−1	−1	1	1	−1	−1	−1	−1	−1	1	−1
$X.103$	−3	−3	−3	−3	13	13	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	5	5	1	1	−3	1	1	−3	1
$X.104$	3	−3	3	−3	13	13	3	3	−3	3	−3	3	−3	−3	3	3	3	−5	−5	1	1	3	1	1	−3	1
$X.105$	3	−3	−9	−3	9	9	−3	−3	−3	−3	−3	−3	−7	−7	1	1	1	−1	−1	−1	−1	1	−1	−1	1	−1
$X.106$	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	4	−4	.	−4	4	.	.
$X.107$	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	−4	4	.	4	−4	.	.
$X.108$	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.109$	.	.	.	.	.	.	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.
$X.110$	.	.	.	.	16	−16	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.111$	.	.	.	8	.	.	−8	−8	.	8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.112$	.	−8	.	.	.	.	−8	8	.	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.113$	8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.114$	.	−8	.	.	.	.	8	−8	.	−8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.115$	.	.	.	.	.	.	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.
$X.116$	.	.	.	8	.	.	8	8	.	−8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.117$	−8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.118$	.	.	.	.	.	.	.	.	−4	.	4	.	.	.	.	.	.	.	.	−4	−4	.	4	4	.	.
$X.119$	−8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.120$	.	.	.	.	.	.	.	.	12	.	−12	.	.	.	.	.	.	.	.	4	4	.	−4	−4	.	.
$X.121$	.	.	.	.	.	.	.	.	.	.	.	.	−8	8	.	.	.	.	.	.	.	.	.	.	.	.
$X.122$	.	.	.	.	.	.	.	.	.	.	.	.	−8	8	.	.	.	.	.	.	.	.	.	.	.	.
$X.123$	2	.	2	.	4	4	−6	−6	.	−6	.	−6	4	4	2	2	2	2	2	.	.	2	.	.	−4	.
$X.124$	−2	.	−2	.	−4	−4	−6	−6	.	−6	.	−6	−4	−4	2	2	2	−2	−2	.	.	2	.	.	4	.
$X.125$	2	.	2	.	−4	−4	6	6	.	6	.	6	−4	−4	−2	−2	−2	2	2	.	.	−2	.	.	4	.
$X.126$	−2	.	−2	.	4	4	6	6	.	6	.	6	4	4	−2	−2	−2	−2	−2	.	.	−2	.	.	−4	.
$X.127$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.128$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.129$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.130$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.131$	.	−8	.	24	.	.	.	.	−8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.132$	.	24	.	−8	.	.	.	.	−8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.133$	.	−8	.	−8	.	.	.	.	8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.134$	.	−8	.	−8	.	.	.	.	8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.135$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.136$	.	−6	.	−6	−14	−14	.	.	−6	.	−6	.	2	2	.	.	.	.	.	−2	−2	.	−2	−2	2	−2
$X.137$	.	6	−12	6	−2	−2	6	6	6	6	6	6	−2	−2	2	2	2	4	4	.	.	2	.	.	6	.
$X.138$	.	−6	.	−6	−6	−6	.	.	−6	.	−6	.	10	10	.	.	.	.	.	2	2	.	2	2	−6	2
$X.139$	.	6	12	6	−2	−2	−6	−6	6	−6	6	−6	−2	−2	−2	−2	−2	−4	−4	.	.	−2	.	.	6	.
$X.140$	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.141$	−8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.142$	−8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.143$	8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.144$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.145$	.	.	−16	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.146$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.147$	.	.	.	−16	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.148$	.	.	.	.	.	.	.	.	−8	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.149$	.	.	16	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.150$	.	16	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.151$	.	.	.	.	.	.	−4	4	.	4	.	−4	.	.	4	−4	−4	.	.	.	.	4	.	.	.	.
$X.152$	.	.	.	.	.	.	−4	−4	.	4	.	4	.	.	−4	−4	4	.	.	.	.	4	.	.	.	.
$X.153$	.	.	.	.	.	.	4	−4	.	−4	.	4	.	.	−4	4	4	.	.	.	.	−4	.	.	.	.
$X.154$	.	.	.	.	.	.	4	4	.	−4	.	−4	.	.	4	4	−4	.	.	.	.	−4	.	.	.	.
$X.155$	.	.	.	.	.	.	.	.	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.156$	.	.	.	.	.	.	.	.	8	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.157$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.158$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.159$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.160$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.161$	10	.	−10	.	4	−4	.	.	.	.	.	.	.	.	−4	4	−4	2	−2	−2	−2	4	−2	−2	.	2
$X.162$	10	.	−2	.	4	−4	.	.	.	.	.	.	.	.	4	−4	4	2	−2	2	2	−4	2	2	.	−2
$X.163$	4	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	4	.	.	.	.	.	.	.
$X.164$	−10	.	2	.	4	−4	.	.	.	.	.	.	.	.	−4	4	−4	−2	2	2	2	4	2	2	.	−2
$X.165$	−4	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	−4	.	.	.	.	.	.	.
$X.166$	−10	.	10	.	4	−4	.	.	.	.	.	.	.	.	4	−4	4	−2	2	−2	−2	−4	−2	−2	.	2
$X.167$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.168$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.169$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.170$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.171$	12	.	12	.	−8	8	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	.	.
$X.172$	.	.	.	12	.	.	6	6	.	−6	.	−6	.	.	−2	−2	2	.	.	−2	2	2	−2	2	.	−2
$X.173$	.	−12	.	.	.	.	6	−6	.	−6	.	6	.	.	2	−2	−2	.	.	2	−2	2	2	−2	.	−2
$X.174$	−12	.	−12	.	−8	8	.	.	.	.	.	.	.	.	.	.	.	4	−4	.	.	.	.	.	.	.
$X.175$	.	−12	.	.	.	.	−6	6	.	6	.	−6	.	.	−2	2	2	.	.	2	−2	−2	2	−2	.	−2
$X.176$	.	.	.	12	.	.	−6	−6	.	6	.	6	.	.	2	2	−2	.	.	−2	2	−2	−2	2	.	−2
$X.177$	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4	.	4	−4	.	4
$X.178$	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	−4	4	.	4
$X.179$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.180$	.	.	.	.	.	.	.	.	4	.	−4	.	.	.	.	.	.	.	.	−4	−4	.	4	4	.	.
$X.181$	.	.	.	.	.	.	.	.	4	.	−4	.	.	.	.	.	.	.	.	−4	−4	.	4	4	.	.
$X.182$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.183$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.184$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.185$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.186$	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	4	−4	.	−4	4	.	.
$X.187$	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	−4	4	.	4	−4	.	.
$X.188$	.	.	.	.	.	.	.	.	−12	.	12	.	.	.	.	.	.	.	.	4	4	.	−4	−4	.	.
$X.189$	.	.	.	.	.	.	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,Fi_{22})$ (continued)

2	8	4	8	8	8	8	8	8	8	9	9	9	8	8	8	8	8	8	8	8	8	8	6	6	6
3	.	.	4	4	4	4	4	4	4	3	3	3	3	3	2	2	2	2	2	2	2	2	3	3	3
5	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	4_{41}	$5a$	6_1	6_2	6_3	6_4	6_5	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{14}	6_{15}	6_{16}	6_{17}	6_{18}	6_{19}	6_{20}	6_{21}	6_{22}	6_{23}
$2P$	$2q$	$5a$	$3a$	$3a$	$3a$	$3a$	$3a$	$3a$	$3a$	$3b$	$3b$	$3b$	$3b$	$3b$	$3a$	$3a$	$3a$	$3a$	$3a$	$3a$	$3a$	$3a$	$3c$	$3c$	$3c$
$3P$	4_{41}	$5a$	$2d$	$2e$	$2e$	$2a$	$2d$	$2c$	$2b$	$2a$	$2b$	$2c$	$2e$	$2d$	$2f$	$2l$	$2g$	$2l$	$2m$	$2m$	$2h$	$2i$	$2b$	$2c$	$2a$
$5P$	4_{41}	$1a$	6_5	6_3	6_2	6_4	6_1	6_6	6_7	6_8	6_9	6_{10}	6_{11}	6_{12}	6_{13}	6_{16}	6_{15}	6_{14}	6_{18}	6_{17}	6_{19}	6_{20}	6_{21}	6_{22}	6_{23}
$X.96$	.	.	F	F	F	−4	F	4	4	14	−14	−14	.	.	−4	G	4	G	G	G	4	−4	4	4	−4
$X.97$	.	.	$\bar F$	$\bar F$	F	−4	F	4	4	14	−14	−14	.	.	−4	G	4	$\bar G$	$\bar G$	G	4	−4	4	4	−4
$X.98$	.	−1	−12	12	12	12	−12	12	−12	−12	12	−12	−12	12	−4	4	4	4	−4	−4	−4	4	.	.	.
$X.99$	.	−1	12	−12	−12	12	12	12	−12	−12	12	−12	12	−12	4	4	−4	4	−4	−4	4	−4	.	.	.
$X.100$	.	−1	−12	12	12	12	−12	12	−12	−12	12	−12	−12	12	−4	4	4	4	−4	−4	−4	4	.	.	.
$X.101$	.	−1	12	−12	−12	12	12	12	−12	−12	12	−12	12	−12	4	4	−4	4	−4	−4	4	−4	.	.	.
$X.102$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.103$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.104$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.105$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.106$	.	2	.	.	.	.	.	.	.	18	−18	−18	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.107$	.	2	.	.	.	.	.	.	.	18	−18	−18	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.108$	.	.	−12	−12	−12	12	−12	12	12	−3	−3	−3	3	3	4	−4	4	−4	−4	−4	4	4	.	.	.
$X.109$	.	.	.	.	.	−24	.	24	24	.	.	.	.	.	−8	.	8	.	.	.	8	−8	−6	−6	6
$X.110$	.	.	24	24	24	−24	24	−24	−24	.	.	.	.	.	.	.	.	.	.	.	.	.	6	6	6
$X.111$	.	.	−6	6	6	6	−6	6	−6	−12	12	−12	−12	12	2	−2	−2	−2	2	2	2	−2	3	−3	−3
$X.112$	.	.	6	−6	−6	6	6	6	−6	−12	12	−12	12	−12	−2	−2	2	−2	2	2	−2	2	3	−3	−3
$X.113$	.	.	−12	−12	−12	12	−12	12	12	−3	−3	−3	3	3	−4	4	−4	4	4	4	−4	−4	.	.	.
$X.114$	.	.	6	−6	−6	6	6	6	−6	−12	12	−12	12	−12	−2	−2	2	−2	2	2	−2	2	3	−3	−3
$X.115$	.	.	.	.	.	−24	.	24	24	.	.	.	.	.	8	.	−8	.	.	.	−8	8	−6	−6	6
$X.116$	.	.	−6	6	6	6	−6	6	−6	−12	12	−12	−12	12	2	−2	−2	−2	2	2	2	−2	3	−3	−3
$X.117$	.	.	−12	−12	−12	12	−12	12	12	−3	−3	−3	3	3	4	−4	4	−4	−4	−4	4	4	.	.	.
$X.118$	.	.	.	.	.	−12	.	12	−12	−24	−24	24	.	.	4	.	4	.	.	.	−4	−4	.	.	.
$X.119$	.	.	−12	−12	−12	12	−12	12	12	−3	−3	−3	3	3	−4	4	−4	4	4	4	−4	−4	.	.	.
$X.120$	.	.	.	.	.	24	.	−24	24	.	.	.	.	.	8	.	8	.	.	.	−8	−8	−6	6	−6
$X.121$	.	.	.	.	.	−24	.	24	24	.	.	.	.	.	.	H	.	$\bar H$	$\bar H$	H	.	.	−6	−6	6
$X.122$	.	.	.	.	.	−24	.	24	24	.	.	.	.	.	.	$\bar H$	.	H	H	$\bar H$	.	.	−6	−6	6
$X.123$	.	.	.	.	.	.	.	.	.	9	9	9	9	9	.	.	.	.	.	.	.	.	.	.	.
$X.124$	.	.	.	.	.	.	.	.	.	9	9	9	9	9	.	.	.	.	.	.	.	.	.	.	.
$X.125$	.	.	.	.	.	.	.	.	.	9	9	9	9	9	.	.	.	.	.	.	.	.	.	.	.
$X.126$	.	.	.	.	.	.	.	.	.	9	9	9	9	9	.	.	.	.	.	.	.	.	.	.	.
$X.127$	.	.	8	8	8	−8	8	−8	−8	−8	−8	−8	8	8	.	.	.	.	.	.	.	.	−8	−8	−8
$X.128$	.	.	.	.	.	8	.	−8	8	8	8	−8	.	.	−8	.	−8	.	.	.	8	8	−10	10	−10
$X.129$	.	.	I	I	$\bar I$	−8	$\bar I$	8	8	−8	8	8	.	.	.	.	.	.	.	.	.	.	8	8	−8
$X.130$	.	.	$\bar I$	$\bar I$	I	−8	I	8	8	−8	8	8	.	.	.	.	.	.	.	.	.	.	8	8	−8
$X.131$	.	.	−9	−9	−9	−9	−9	−9	−9	.	.	.	.	.	3	3	3	3	3	3	3	3	.	.	.
$X.132$	.	.	−9	−9	−9	−9	−9	−9	−9	.	.	.	.	.	3	3	3	3	3	3	3	3	.	.	.
$X.133$	.	.	−9	−9	−9	−9	−9	−9	−9	.	.	.	.	.	3	3	3	3	3	3	3	3	.	.	.
$X.134$	.	.	−9	−9	−9	−9	−9	−9	−9	.	.	.	.	.	3	3	3	3	3	3	3	3	.	.	.
$X.135$	.	−2	.	.	.	24	.	−24	24	−24	−24	24	.	.	8	.	8	.	.	.	−8	−8	.	.	.
$X.136$	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.137$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.138$	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.139$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.140$	.	−1	.	.	.	.	.	.	.	9	9	9	−9	−9	.	.	.	.	.	.	.	.	.	.	.
$X.141$	.	−1	.	.	.	.	.	.	.	9	9	9	−9	−9	.	.	.	.	.	.	.	.	.	.	.
$X.142$	.	−1	.	.	.	.	.	.	.	9	9	9	−9	−9	.	.	.	.	.	.	.	.	.	.	.
$X.143$	.	−1	.	.	.	.	.	.	.	9	9	9	−9	−9	.	.	.	.	.	.	.	.	.	.	.
$X.144$	.	.	.	.	.	24	.	−24	−24	−6	6	6	.	.	8	.	−8	.	.	.	−8	8	.	.	.
$X.145$	.	.	24	24	24	24	24	24	24	−6	−6	−6	−6	−6	.	.	.	.	.	.	.	.	.	.	.
$X.146$	.	.	.	.	.	24	.	−24	−24	−6	6	6	.	.	−8	.	8	.	.	.	8	−8	.	.	.
$X.147$	.	.	6	−6	−6	−6	6	−6	6	.	.	.	.	.	6	−6	−6	−6	6	6	6	−6	−6	6	6
$X.148$	.	.	.	.	.	12	.	−12	12	−24	−24	24	.	.	−4	.	−4	.	.	.	4	4	−6	6	−6
$X.149$	.	.	24	24	24	24	24	24	24	−6	−6	−6	−6	−6	.	.	.	.	.	.	.	.	.	.	.
$X.150$	.	.	−6	6	6	−6	−6	−6	6	.	.	.	.	.	−6	−6	6	−6	6	6	−6	6	−6	6	6
$X.151$	.	.	12	−12	−12	12	12	12	−12	12	−12	12	−12	12	4	4	−4	4	−4	−4	4	−4	−3	3	3
$X.152$	.	.	−12	12	12	12	−12	12	−12	12	−12	12	12	−12	−4	4	4	4	−4	−4	−4	4	−3	3	3
$X.153$	.	.	12	−12	−12	12	12	12	−12	12	−12	12	−12	12	4	4	−4	4	−4	−4	4	−4	−3	3	3
$X.154$	.	.	−12	12	12	12	−12	12	−12	12	−12	12	12	−12	−4	4	4	4	−4	−4	−4	4	−3	3	3
$X.155$	.	.	K	$\bar K$	K	−6	$\bar K$	6	−6	.	.	.	.	.	−6	$\bar B$	−6	B	$\bar B$	B	6	6	6	−6	6
$X.156$	.	.	$\bar K$	K	$\bar K$	−6	K	6	−6	.	.	.	.	.	−6	B	−6	$\bar B$	B	$\bar B$	6	6	6	−6	6
$X.157$	.	−1	16	−16	−16	−16	16	−16	16	8	−8	8	8	−8	.	.	.	.	.	.	.	.	4	−4	−4
$X.158$	.	−1	−16	16	16	−16	−16	−16	16	8	−8	8	−8	8	.	.	.	.	.	.	.	.	4	−4	−4
$X.159$	.	−1	−16	16	16	−16	−16	−16	16	8	−8	8	−8	8	.	.	.	.	.	.	.	.	4	−4	−4
$X.160$	.	−1	16	−16	−16	−16	16	−16	16	8	−8	8	8	−8	.	.	.	.	.	.	.	.	4	−4	−4
$X.161$	2	.	.	.	.	.	.	.	.	−9	−9	−9	9	9	.	.	.	.	.	.	.	.	.	.	.
$X.162$	−2	.	.	.	.	.	.	.	.	−9	−9	−9	9	9	.	.	.	.	.	.	.	.	.	.	.
$X.163$	.	.	.	.	.	.	.	.	.	−9	−9	−9	−9	−9	.	.	.	.	.	.	.	.	.	.	.
$X.164$	−2	.	.	.	.	.	.	.	.	−9	−9	−9	9	9	.	.	.	.	.	.	.	.	.	.	.
$X.165$	.	.	.	.	.	.	.	.	.	−9	−9	−9	−9	−9	.	.	.	.	.	.	.	.	.	.	.
$X.166$	2	.	.	.	.	.	.	.	.	−9	−9	−9	9	9	.	.	.	.	.	.	.	.	.	.	.
$X.167$	.	.	20	−20	−20	−20	20	−20	20	−8	8	−8	−8	8	−4	4	4	4	−4	−4	−4	4	−4	4	4
$X.168$	.	.	−20	20	20	−20	−20	−20	20	−8	8	−8	8	−8	4	4	−4	4	−4	−4	4	−4	−4	4	4
$X.169$	.	.	$\bar F$	F	$\bar F$	−20	F	20	−20	−8	−8	8	.	.	4	$\bar G$	4	G	$\bar G$	G	−4	−4	4	−4	4
$X.170$	.	.	F	$\bar F$	F	−20	$\bar F$	20	−20	−8	−8	8	.	.	4	G	4	$\bar G$	G	$\bar G$	−4	−4	4	−4	4
$X.171$	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.172$	2	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.173$	2	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.174$	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.175$	2	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.176$	2	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.177$	−4	.	−18	18	18	18	−18	18	−18	.	.	.	.	.	6	−6	−6	−6	6	6	6	−6	.	.	.
$X.178$	−4	.	18	−18	−18	18	18	18	−18	.	.	.	.	.	−6	−6	6	−6	6	6	−6	6	.	.	.
$X.179$	.	.	−18	−18	−18	−18	−18	−18	−18	.	.	.	.	.	−6	−6	−6	−6	−6	−6	−6	−6	.	.	.
$X.180$	.	.	K	$\bar K$	K	18	$\bar K$	−18	18	.	.	.	.	.	−6	$\bar A$	−6	A	$\bar A$	A	6	6	.	.	.
$X.181$	.	.	$\bar K$	K	$\bar K$	18	K	−18	18	.	.	.	.	.	−6	A	−6	$\bar A$	A	$\bar A$	6	6	.	.	.
$X.182$	.	−2	.	.	.	.	.	.	.	18	−18	−18	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.183$	.	−2	.	.	.	.	.	.	.	18	−18	−18	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.184$	.	.	.	.	.	24	.	−24	24	24	24	−24	.	.	8	.	8	.	.	.	−8	−8	6	−6	6
$X.185$	.	−2	.	.	.	−32	.	32	−32	16	16	−16	.	.	.	.	.	.	.	.	.	.	−8	8	−8
$X.186$	.	.	.	.	.	.	.	.	.	−18	18	18	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.187$	.	.	.	.	.	.	.	.	.	−18	18	18	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.188$	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.189$	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	7	7	7	7	5	5	8	8	6	6	6	6	7	5	5	5	5	5	5	5	5	6	6	6	6	4	4
3	2	2	2	2	3	3	1	1	2	2	2	2	1	2	2	2	2	2	2	2	2	1	1	1	1	2	2
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	6_{24}	6_{25}	6_{26}	6_{27}	6_{28}	6_{29}	6_{30}	6_{31}	6_{32}	6_{33}	6_{34}	6_{35}	6_{36}	6_{37}	6_{38}	6_{39}	6_{40}	6_{41}	6_{42}	6_{43}	6_{44}	6_{45}	6_{46}	6_{47}	6_{48}	6_{49}	6_{50}
2P	3b	3b	3b	3b	3c	3c	3b	3b	3b	3b	3b	3b	3b	3c	3c	3c	3c	3c	3c	3c	3c	3c	3c	3c	3c	3c	3c
3P	2f	2h	2g	2i	2d	2e	2j	2k	2m	2o	2p	2l	2n	2f	2m	2l	2i	2m	2g	2l	2h	2q	2r	2s	2t	2o	2p
5P	6_{24}	6_{25}	6_{26}	6_{27}	6_{28}	6_{29}	6_{30}	6_{31}	6_{32}	6_{33}	6_{34}	6_{35}	6_{36}	6_{37}	6_{41}	6_{43}	6_{40}	6_{38}	6_{42}	6_{39}	6_{44}	6_{45}	6_{46}	6_{47}	6_{48}	6_{49}	6_{50}
$X.96$	2	−2	−2	2	.	.	−2	2	.	.	.	.	.	2	B	B	2	B	−2	B	−2	.	.	.	.	.	.
$X.97$	2	−2	−2	2	.	.	−2	2	.	.	.	.	.	2	$\bar B$	B	2	B	−2	$\bar B$	−2	.	.	.	.	.	.
$X.98$	2	2	−2	−2	.	.	.	.	2	−2	2	−2	.	2	2	−2	−2	2	−2	−2	2	.	.	.	.	−2	2
$X.99$	−2	−2	2	2	.	.	.	.	2	2	−2	−2	.	−2	2	−2	2	2	2	−2	−2	.	.	.	.	2	−2
$X.100$	2	2	−2	−2	.	.	.	.	2	2	−2	−2	.	2	2	−2	−2	2	−2	−2	2	.	.	.	.	2	−2
$X.101$	−2	−2	2	2	.	.	.	.	2	−2	2	−2	.	−2	2	−2	2	2	2	−2	−2	.	.	.	.	−2	2
$X.102$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.103$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.104$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.105$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.106$	6	−6	−6	6	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.107$	−6	6	6	−6	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.108$	1	1	1	1	.	.	−3	−3	−1	−1	−1	−1	3	−2	2	2	−2	2	−2	2	−2	.	.	.	.	2	2
$X.109$	4	−4	−4	4	.	.	.	.	.	.	.	.	.	−2	.	.	−2	.	2	.	2	−2	2	2	−2	.	.
$X.110$	.	.	.	.	−6	−6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	2	2	.	.
$X.111$	2	2	−2	−2	3	−3	.	.	2	2	−2	−2	.	−1	−1	1	1	−1	1	1	−1	−1	−1	1	1	−1	1
$X.112$	−2	−2	2	2	−3	3	.	.	2	2	−2	−2	.	1	−1	1	−1	−1	−1	1	1	−1	−1	1	1	−1	1
$X.113$	−1	−1	−1	−1	.	.	−3	−3	1	−1	−1	1	3	2	−2	−2	2	−2	2	−2	2	.	.	.	.	2	2
$X.114$	−2	−2	2	2	−3	3	.	.	2	−2	2	−2	.	1	−1	1	−1	−1	−1	1	1	−1	−1	1	1	1	−1
$X.115$	−4	4	4	−4	.	.	.	.	.	.	.	.	.	2	.	.	2	.	−2	.	−2	−2	2	2	−2	.	.
$X.116$	2	2	−2	−2	3	−3	.	.	2	−2	2	−2	.	−1	−1	1	1	−1	1	1	−1	−1	−1	1	1	1	−1
$X.117$	1	1	1	1	.	.	−3	−3	−1	1	1	−1	3	−2	2	2	−2	2	−2	2	−2	.	.	.	.	−2	−2
$X.118$	4	−4	4	−4	.	.	.	.	.	.	.	.	.	4	.	.	−4	.	4	.	−4	.	.	.	.	.	.
$X.119$	−1	−1	−1	−1	.	.	−3	−3	1	1	1	1	3	2	−2	−2	2	−2	2	−2	2	.	.	.	.	−2	−2
$X.120$	8	−8	8	−8	.	.	.	.	.	.	.	.	.	2	.	.	−2	.	2	.	−2	−2	2	−2	2	.	.
$X.121$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	B	$\bar B$	.	$\bar B$	.	B	.	2	−2	−2	2	.	.
$X.122$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar B$	B	.	B	.	$\bar B$	.	2	−2	−2	2	.	.
$X.123$	−3	−3	−3	−3	.	.	1	1	−3	3	3	−3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.124$	−3	−3	−3	−3	.	.	1	1	−3	−3	−3	−3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.125$	−3	−3	−3	−3	.	.	1	1	−3	3	3	−3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.126$	−3	−3	−3	−3	.	.	1	1	−3	−3	−3	−3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.127$	.	.	.	.	8	8	8	8	.	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.128$	4	−4	4	−4	.	.	.	.	.	.	.	.	.	−2	.	.	2	.	−2	.	2	2	−2	2	−2	.	.
$X.129$	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.130$	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.131$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.132$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.133$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.134$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.135$	−4	4	−4	4	.	.	.	.	.	.	.	.	.	−4	.	.	4	.	−4	.	4	.	.	.	.	.	.
$X.136$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.137$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.138$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.139$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.140$	−3	−3	−3	−3	.	.	1	1	3	3	3	3	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.141$	3	3	3	3	.	.	1	1	−3	−3	−3	−3	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.142$	−3	−3	−3	−3	.	.	1	1	3	−3	−3	3	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.143$	3	3	3	3	.	.	1	1	−3	3	3	−3	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.144$	2	−2	−2	2	.	.	−6	6	.	.	.	.	.	−4	.	.	−4	.	4	.	4	.	.	.	.	.	.
$X.145$	.	.	.	.	.	.	2	2	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.146$	−2	2	2	−2	.	.	−6	6	.	.	.	.	.	4	.	.	4	.	−4	.	−4	.	.	.	.	.	.
$X.147$	.	.	.	.	−6	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2	2	2	.	.
$X.148$	−4	4	−4	4	.	.	.	.	.	.	.	.	.	2	.	.	−2	.	2	.	−2	−2	2	−2	2	.	.
$X.149$	.	.	.	.	.	.	2	2	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.150$	.	.	.	.	6	−6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2	2	2	.	.
$X.151$	−2	−2	2	2	3	−3	.	.	2	2	−2	−2	.	1	−1	1	−1	−1	−1	1	1	1	1	−1	−1	−1	1
$X.152$	2	2	−2	−2	−3	3	.	.	2	2	−2	−2	.	−1	−1	1	1	−1	1	1	−1	1	1	−1	−1	−1	1
$X.153$	−2	−2	2	2	3	−3	.	.	2	−2	2	−2	.	1	−1	1	−1	−1	−1	1	1	1	1	−1	−1	1	−1
$X.154$	2	2	−2	−2	−3	3	.	.	2	−2	2	−2	.	−1	−1	1	1	−1	1	1	−1	1	1	−1	−1	1	−1
$X.155$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar B$	$\bar B$	.	B	.	B	.	−2	2	−2	2	.	.
$X.156$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	B	B	.	$\bar B$	.	$\bar B$	.	−2	2	−2	2	.	.
$X.157$	.	.	.	.	4	−4	.	.	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2
$X.158$	.	.	.	.	−4	4	.	.	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2
$X.159$	.	.	.	.	−4	4	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.160$	.	.	.	.	4	−4	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.161$	3	3	3	3	.	.	−1	−1	−3	−3	−3	−3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.162$	−3	−3	−3	−3	.	.	−1	−1	3	−3	−3	3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.163$	3	3	3	3	.	.	−1	−1	3	−3	−3	3	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.164$	−3	−3	−3	−3	.	.	−1	−1	3	3	3	3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.165$	3	3	3	3	.	.	−1	−1	3	3	3	3	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.166$	3	3	3	3	.	.	−1	−1	−3	3	3	−3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.167$	−4	−4	4	4	−4	4	.	.	−4	.	.	4	.	2	2	−2	−2	2	−2	−2	2	.	.	.	.	.	.
$X.168$	4	4	−4	−4	4	−4	.	.	−4	.	.	4	.	−2	2	−2	2	2	2	−2	−2	.	.	.	.	.	.
$X.169$	4	−4	4	−4	.	.	.	.	.	.	.	.	.	−2	$\bar B$	$\bar B$	2	B	−2	B	2	.	.	.	.	.	.
$X.170$	4	−4	4	−4	.	.	.	.	.	.	.	.	.	−2	B	B	2	$\bar B$	−2	$\bar B$	2	.	.	.	.	.	.
$X.171$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.172$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.173$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.174$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.175$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.176$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.177$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.178$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.179$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.180$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.181$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.182$	6	−6	−6	6	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.183$	−6	6	6	−6	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.184$	−4	4	−4	4	.	.	.	.	.	.	.	.	.	2	.	.	−2	.	2	.	−2	2	−2	2	−2	.	.
$X.185$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.186$	−6	6	6	−6	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.187$	6	−6	−6	6	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.188$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.189$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	5	5	5	7	7	7	7	7	7	7	6	6	6	6	6	6	6	3	4	4	4	3	3	3	3	7	7
3	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	2	2
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	1	1	.	.
	6_{51}	6_{52}	6_{53}	$8a$	$8b$	$8c$	$8d$	$8e$	$8f$	$8g$	$8h$	$8i$	$8j$	$8k$	$8l$	$8m$	$8n$	$9a$	$10a$	$10b$	$10c$	$10d$	$10e$	$10f$	$10g$	12_1	12_2
$2P$	$3c$	$3b$	$3c$	4_1	4_1	4_4	4_{20}	4_{20}	4_{20}	4_{20}	4_{11}	4_{11}	4_{16}	4_{19}	4_{27}	4_{27}	4_{16}	$9a$	$5a$	$5a$	$5a$	$5a$	$5a$	$5a$	$5a$	6_8	6_8
$3P$	$2u$	$2x$	$2v$	$8a$	$8b$	$8c$	$8d$	$8e$	$8f$	$8g$	$8h$	$8i$	$8j$	$8k$	$8l$	$8m$	$8n$	$3a$	$10a$	$10b$	$10c$	$10d$	$10e$	$10f$	$10g$	4_1	4_1
$5P$	6_{51}	6_{52}	6_{53}	$8a$	$8b$	$8c$	$8d$	$8e$	$8f$	$8g$	$8h$	$8i$	$8j$	$8k$	$8l$	$8m$	$8n$	$9a$	$2b$	$2a$	$2c$	$2o$	$2d$	$2p$	$2e$	12_1	12_2
$X.96$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.	.	.	.	.	.	6	−6
$X.97$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.	.	.	.	.	.	6	−6
$X.98$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	1	−1	−1	1	.	.
$X.99$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	−1	1	1	−1	.	.
$X.100$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	−1	−1	1	1	.	.
$X.101$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	1	1	−1	−1	.	.
$X.102$	.	.	.	−3	−3	1	3	−1	3	3	−1	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.	.	.
$X.103$	.	.	.	−3	−3	1	−3	1	−3	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.
$X.104$	.	.	.	3	3	1	−3	−1	−3	−1	−1	−1	−1	−1	1	1	−1	.	.	.	.	.	.	.	.	.	.
$X.105$	.	.	.	3	3	1	3	1	3	−3	1	1	1	1	−1	−1	1	.	.	.	.	.	.	.	.	.	.
$X.106$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2	.	.	−2	2	−2	.	.	.	.	−6	6
$X.107$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	.	.	−2	2	−2	.	.	.	.	−6	6
$X.108$	.	−1	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−3	−3
$X.109$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.110$	−2	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.111$	−1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.112$	1	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.113$	.	1	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−3	−3
$X.114$	1	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.115$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.116$	−1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.117$	.	1	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−3	−3
$X.118$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.119$	.	−1	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−3	−3
$X.120$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.121$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.122$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.123$	.	−1	.	−2	−2	.	.	2	.	2	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	−3	−3
$X.124$	.	1	.	2	2	.	.	2	.	2	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	−3	−3
$X.125$	.	−1	.	−2	−2	.	.	−2	.	−2	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	−3	−3
$X.126$	.	1	.	2	2	.	.	−2	.	−2	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	−3	−3
$X.127$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.
$X.128$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.129$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.
$X.130$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.
$X.131$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.132$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.133$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.134$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.135$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	−2	.	.	.	.	.	.
$X.136$	.	.	.	.	.	2	−2	.	−2	.	.	.	.	.	2	2	.	.	.	.	.	.	.	.	.	.	.
$X.137$	.	.	.	.	.	−2	.	−2	.	2	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.138$	.	.	.	.	.	2	2	.	2	.	.	.	.	.	−2	−2	.	.	.	.	.	.	.	.	.	.	.
$X.139$	.	.	.	.	.	−2	.	2	.	−2	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.140$	.	−1	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	−1	1	−1	1	−3	−3
$X.141$	.	−1	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	1	1	1	1	−3	−3
$X.142$	.	1	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	1	1	1	1	−3	−3
$X.143$	.	1	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	−1	1	−1	1	−3	−3
$X.144$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−6	6
$X.145$	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2
$X.146$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−6	6
$X.147$	−2	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.148$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.149$	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2
$X.150$	2	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.151$	−1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.152$	1	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.153$	−1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.154$	1	.	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.155$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.156$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.157$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	1	1	−1	−1	1	.	.
$X.158$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	1	1	1	−1	−1	.	.
$X.159$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	1	−1	1	1	−1	.	.
$X.160$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	1	−1	−1	1	1	.	.
$X.161$	.	1	.	2	−2	.	−2	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3
$X.162$	.	−1	.	−2	2	.	2	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3
$X.163$	.	1	.	−4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3
$X.164$	.	1	.	2	−2	.	2	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3
$X.165$	.	−1	.	4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3
$X.166$	.	−1	.	−2	2	.	−2	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	3
$X.167$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.	.	.	.	.	.	.	.
$X.168$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.	.	.	.	.	.	.	.
$X.169$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.	.	.	.	.	.	.	.
$X.170$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.	.	.	.	.	.	.	.
$X.171$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	−1	1	−1	.	.
$X.172$	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	2	.	1	−1	−1	−1	1	1	−1	.	.
$X.173$	.	.	.	.	.	.	.	.	.	.	−2	2	.	.	.	.	.	.	1	−1	−1	−1	−1	1	1	.	.
$X.174$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	−1	−1	−1	−1	.	.
$X.175$	.	.	.	.	.	.	.	.	.	.	2	−2	.	.	.	.	.	.	1	−1	−1	1	−1	−1	1	.	.
$X.176$	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	−2	.	1	−1	−1	1	1	−1	−1	.	.
$X.177$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.178$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.179$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.180$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.181$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.182$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	2	.	.	.	.	−6	6
$X.183$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	2	.	.	.	.	−6	6
$X.184$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.185$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	2	−2	.	.	.	.	.	.
$X.186$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	6	−6
$X.187$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	6	−6
$X.188$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2	2	.	.	.	.	.	.
$X.189$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2	−2	.	.	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

2	5	6	6	6	6	6	6	6	6	6	4	4	5	5	5	5	5	5	5	5	5	5
3	3	2	2	2	1	1	1	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	12_3	12_4	12_5	12_6	12_7	12_8	12_9	12_{10}	12_{11}	12_{12}	12_{13}	12_{14}	12_{15}	12_{16}	12_{17}	12_{18}	12_{19}	12_{20}	12_{21}	12_{22}	12_{23}	12_{24}
$2P$	6_4	6_9	6_9	6_8	6_{30}	6_{30}	6_{30}	6_{31}	6_{31}	6_{30}	6_{21}	6_{21}	6_{13}	6_{30}	6_{14}	6_{31}	6_{14}	6_{15}	6_{30}	6_4	6_{16}	6_{16}
$3P$	4_1	4_3	4_2	4_1	4_5	4_6	4_7	4_8	4_9	4_{10}	4_2	4_3	4_{11}	4_{12}	4_{23}	4_{13}	4_{25}	4_{14}	4_{15}	4_4	4_{25}	4_{23}
$5P$	12_3	12_4	12_5	12_6	12_7	12_8	12_9	12_{10}	12_{11}	12_{12}	12_{13}	12_{14}	12_{15}	12_{16}	12_{24}	12_{18}	12_{23}	12_{20}	12_{21}	12_{22}	12_{19}	12_{17}
$X.96$	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.97$	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.98$	.	2	−2	.	2	.	.	2	−2	−2	−2	2	.	.	.	.	.	.	.	.	.	.
$X.99$	.	2	−2	.	2	.	.	−2	2	−2	−2	2	.	.	.	.	.	.	.	.	.	.
$X.100$	.	−2	2	.	−2	.	.	−2	2	2	2	−2	.	.	.	.	.	.	.	.	.	.
$X.101$	.	−2	2	.	−2	.	.	2	−2	2	2	−2	.	.	.	.	.	.	.	.	.	.
$X.102$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.103$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.104$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.105$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.106$	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.107$	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.108$	.	1	1	3	−1	1	1	1	1	−1	−2	−2	.	−1	.	1	.	.	−1	.	.	.
$X.109$	.	.	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.110$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.111$	.	−2	2	.	−2	.	.	−2	2	2	−1	1	−2	.	.	.	.	.	.	.	.	.
$X.112$	.	2	−2	.	2	.	.	−2	2	−2	1	−1	.	.	.	.	.	2	.	.	.	.
$X.113$	.	1	1	3	1	−1	−1	−1	−1	1	−2	−2	.	1	.	1	.	.	−1	.	.	.
$X.114$	.	−2	2	.	−2	.	.	2	−2	2	−1	1	.	.	.	.	.	2	.	.	.	.
$X.115$	.	.	.	.	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.116$	.	2	−2	.	2	.	.	2	−2	−2	1	−1	−2	.	.	.	.	.	.	.	.	.
$X.117$	.	−1	−1	3	1	1	1	−1	−1	1	2	2	.	−1	.	−1	.	.	1	.	.	.
$X.118$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	−2	.	.	.	−2	2
$X.119$	.	−1	−1	3	−1	−1	−1	1	1	−1	2	2	.	1	.	−1	.	.	1	.	.	.
$X.120$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.121$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.122$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.123$	.	3	3	−3	−1	1	1	−1	−1	−1	.	.	.	1	.	−1	.	.	−1	.	.	.
$X.124$	.	−3	−3	−3	1	1	1	1	1	1	.	.	.	1	.	1	.	.	1	.	.	.
$X.125$	.	3	3	−3	−1	1	1	−1	−1	−1	.	.	.	1	.	−1	.	.	−1	.	.	.
$X.126$	.	−3	−3	−3	1	1	1	1	1	1	.	.	.	1	.	1	.	.	1	.	.	.
$X.127$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.128$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.129$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.130$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.131$	3	.	.	.	.	.	.	.	.	.	.	.	−3	.	1	.	1	1	.	−1	1	1
$X.132$	3	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	1	−3	.	−1	1	1
$X.133$	3	.	.	.	.	.	.	.	.	.	.	.	1	.	J	.	J	1	.	−1	$\bar{J}$	$\bar{J}$
$X.134$	3	.	.	.	.	.	.	.	.	.	.	.	1	.	$\bar{J}$	.	$\bar{J}$	1	.	−1	J	J
$X.135$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.136$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.137$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.138$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.139$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.140$	.	−3	−3	3	−1	1	1	1	1	−1	.	.	.	−1	.	1	.	.	−1	.	.	.
$X.141$	.	3	3	3	−1	−1	−1	1	1	−1	.	.	.	1	.	−1	.	.	1	.	.	.
$X.142$	.	3	3	3	1	1	1	−1	−1	1	.	.	.	−1	.	−1	.	.	1	.	.	.
$X.143$	.	−3	−3	3	1	−1	−1	−1	−1	1	.	.	.	1	.	1	.	.	−1	.	.	.
$X.144$	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.145$	−8	.	.	−2	−2	.	.	−2	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.
$X.146$	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.147$	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.148$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	2	.	.	.	2	−2
$X.149$	−8	.	.	−2	2	.	.	2	2	2	.	.	.	.	.	.	.	.	.	.	.	.
$X.150$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.
$X.151$	.	2	−2	.	−2	.	.	2	−2	2	1	−1	.	.	.	.	.	.	.	.	.	.
$X.152$	.	−2	2	.	2	.	.	2	−2	−2	−1	1	.	.	.	.	.	.	.	.	.	.
$X.153$	.	−2	2	.	2	.	.	−2	2	−2	−1	1	.	.	.	.	.	.	.	.	.	.
$X.154$	.	2	−2	.	−2	.	.	−2	2	2	1	−1	.	.	.	.	.	.	.	.	.	.
$X.155$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$-\bar{C}$	.	$\bar{C}$	.	.	.	C	$-C$
$X.156$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$-C$	.	C	.	.	.	$\bar{C}$	$-\bar{C}$
$X.157$	.	4	−4	.	.	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.
$X.158$	.	−4	4	.	.	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.
$X.159$	.	4	−4	.	.	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.
$X.160$	.	−4	4	.	.	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.
$X.161$	.	3	3	−3	1	−1	−1	−1	−1	1	.	.	.	1	.	−1	.	.	1	.	.	.
$X.162$	.	3	3	−3	−1	1	1	1	1	−1	.	.	.	−1	.	−1	.	.	1	.	.	.
$X.163$	.	−3	−3	3	1	−1	−1	1	1	1	.	.	.	−1	.	1	.	.	1	.	.	.
$X.164$	.	−3	−3	−3	1	1	1	−1	−1	1	.	.	.	−1	.	1	.	.	−1	.	.	.
$X.165$	.	3	3	3	−1	−1	−1	−1	−1	−1	.	.	.	−1	.	−1	.	.	−1	.	.	.
$X.166$	.	−3	−3	−3	−1	−1	−1	1	1	−1	.	.	.	1	.	1	.	.	−1	.	.	.
$X.167$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.168$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.169$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.170$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.171$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.172$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.173$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.174$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.175$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.176$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.177$	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.
$X.178$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.
$X.179$	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.
$X.180$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{C}$	.	$-\bar{C}$	.	.	.	$-C$	C
$X.181$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	C	.	$-C$	.	.	.	$-\bar{C}$	$\bar{C}$
$X.182$	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.183$	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.184$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.185$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.186$	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.187$	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.188$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.189$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $H(2\,\mathrm{Fi}_{22})$ (continued)

	12_{25}	12_{26}	12_{27}	12_{28}	12_{29}	12_{30}	12_{31}	$16a$	$16b$	$18a$	$18b$	$18c$	$18d$	$18e$	$18f$	$18g$	$20a$	$20b$	$24a$	$24b$
2	5	5	5	4	4	4	4	5	5	3	3	3	3	3	3	3	3	3	4	4
3	1	1	1	1	1	1	1	.	.	2	2	2	2	2	2	2	.	.	1	1
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.
$2P$	6_9	6_{15}	6_{13}	6_{46}	6_{45}	6_{46}	6_{45}	$8c$	$8c$	$9a$	$9a$	$9a$	$9a$	$9a$	$9a$	$9a$	$10a$	$10a$	12_1	12_2
$3P$	4_{17}	4_{16}	4_{18}	4_{21}	4_{22}	4_{24}	4_{26}	$16a$	$16b$	6_2	6_1	6_3	6_6	6_5	6_7	6_4	$20a$	$20b$	$8a$	$8b$
$5P$	12_{25}	12_{26}	12_{27}	12_{28}	12_{29}	12_{30}	12_{31}	$16b$	$16a$	$18c$	$18e$	$18a$	$18d$	$18b$	$18f$	$18g$	4_2	4_3	$24a$	$24b$
$X.96$	.	.	.	.	.	.	.	.	.	D	D	$\bar{D}$	1	$\bar{D}$	1	−1	.	.	.	.
$X.97$	.	.	.	.	.	.	.	.	.	$\bar{D}$	$\bar{D}$	D	1	D	1	−1	.	.	.	.
$X.98$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	.	.
$X.99$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	.	.
$X.100$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	.	.
$X.101$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	.	.
$X.102$	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.
$X.103$	.	.	.	.	.	.	.	−1	−1	.	.	.	.	.	.	.	.	.	.	.
$X.104$	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.
$X.105$	.	.	.	.	.	.	.	−1	−1	.	.	.	.	.	.	.	.	.	.	.
$X.106$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.107$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.108$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1
$X.109$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.110$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.111$	.	.	2	1	1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.112$	.	−2	.	1	−1	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.113$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1
$X.114$	.	−2	.	−1	1	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.115$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.116$	.	.	2	−1	−1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.117$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1
$X.118$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.119$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1
$X.120$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.121$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.122$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.123$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.124$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.125$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.126$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.127$	.	.	.	.	.	.	.	.	.	−1	−1	−1	1	−1	1	1	.	.	.	.
$X.128$	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	2	2	.	.	.	.
$X.129$	.	.	.	.	.	.	.	.	.	$\bar{D}$	$\bar{D}$	D	−1	D	−1	1	.	.	.	.
$X.130$	.	.	.	.	.	.	.	.	.	D	D	$\bar{D}$	−1	$\bar{D}$	−1	1	.	.	.	.
$X.131$	.	1	−3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.132$	.	−3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.133$	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.134$	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.135$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.136$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.137$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.138$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.139$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.140$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	−1
$X.141$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	−1
$X.142$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	1
$X.143$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	−1	1
$X.144$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.145$	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.146$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.147$	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.148$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.149$	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.150$	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.151$	.	.	.	−1	1	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.152$	.	.	.	−1	−1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.153$	.	.	.	1	−1	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.154$	.	.	.	1	1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.155$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.156$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.157$	.	.	.	.	.	.	.	.	.	−1	1	−1	−1	1	1	−1	1	−1	.	.
$X.158$	.	.	.	.	.	.	.	.	.	1	−1	1	−1	−1	1	−1	−1	1	.	.
$X.159$	.	.	.	.	.	.	.	.	.	1	−1	1	−1	−1	1	−1	1	−1	.	.
$X.160$	.	.	.	.	.	.	.	.	.	−1	1	−1	−1	1	1	−1	−1	1	.	.
$X.161$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1
$X.162$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1
$X.163$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.164$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1
$X.165$	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.166$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1
$X.167$	.	.	.	.	.	.	.	.	.	1	−1	1	1	−1	−1	1	.	.	.	.
$X.168$	.	.	.	.	.	.	.	.	.	−1	1	−1	1	1	−1	1	.	.	.	.
$X.169$	.	.	.	.	.	.	.	.	.	D	$\bar{D}$	$\bar{D}$	−1	D	1	1	.	.	.	.
$X.170$	.	.	.	.	.	.	.	.	.	$\bar{D}$	D	D	−1	$\bar{D}$	1	1	.	.	.	.
$X.171$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	.	.
$X.172$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	.	.
$X.173$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	.	.
$X.174$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	.
$X.175$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	.	.
$X.176$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	.	.
$X.177$	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.178$	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.179$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.180$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.181$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.182$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.183$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.184$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.185$	.	.	.	.	.	.	.	.	.	.	.	.	2	.	−2	−2	.	.	.	.
$X.186$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.187$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.188$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.189$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

where $A=-12\zeta(3)_3-6$, $B=4\zeta(3)_3+2$, $C=2\zeta(3)_3+2$, $D=2\zeta(3)_3+1$, $E=-2\zeta(8)_8^3-2\zeta(8)_8$, $F=-24\zeta(3)_3-12$, $G=-8\zeta(3)_3-4$, $H=-16\zeta(3)_3-8$, $I=-48\zeta(3)_3-24$, $J=4\zeta(3)_3+1$, $K=-36\zeta(3)_3-18$.

B.5. *Character table of* $D(2\,\mathrm{Fi}_{22}) = D_2 = \langle p_2, q_2\rangle \cong \langle p_1, q_1\rangle$

2	18	18	18	18	17	17	16	16	16	16	17	17	15	15	16	13	13	13	13	14	14	12	12
3	1	1	1	1	1	1	1	1	1	1	.	.	1	1	.	1	1	1	1	.	.	1	1
5	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	$1a$	2_1	2_2	2_3	2_4	2_5	2_6	2_7	2_8	2_9	2_{10}	2_{11}	2_{12}	2_{13}	2_{14}	2_{15}	2_{16}	2_{17}	2_{18}	2_{19}	2_{20}	2_{21}	2_{22}
$2P$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$
$3P$	$1a$	2_1	2_2	2_3	2_4	2_5	2_6	2_7	2_8	2_9	2_{10}	2_{11}	2_{12}	2_{13}	2_{14}	2_{15}	2_{16}	2_{17}	2_{18}	2_{19}	2_{20}	2_{21}	2_{22}
$5P$	$1a$	2_1	2_2	2_3	2_4	2_5	2_6	2_7	2_8	2_9	2_{10}	2_{11}	2_{12}	2_{13}	2_{14}	2_{15}	2_{16}	2_{17}	2_{18}	2_{19}	2_{20}	2_{21}	2_{22}
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.3$	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
$X.4$	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
$X.5$	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
$X.6$	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
$X.7$	5	5	5	5	5	5	−3	−3	−3	−3	5	5	−3	−3	5	1	1	1	1	5	5	1	1
$X.8$	5	5	5	5	5	5	−3	−3	−3	−3	5	5	−3	−3	5	1	1	1	1	5	5	1	1
$X.9$	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	−2	6	6	6	6	−2	6
$X.10$	10	10	10	10	10	10	2	2	2	2	10	10	2	2	10	−2	−2	−2	−2	10	10	−2	−2
$X.11$	10	10	10	10	10	10	2	2	2	2	10	10	2	2	10	−2	−2	−2	−2	10	10	−2	−2
$X.12$	10	10	10	10	10	10	2	2	2	2	10	10	2	2	10	−2	−2	−2	−2	10	10	−2	−2
$X.13$	10	10	10	10	10	10	2	2	2	2	10	10	2	2	10	−2	−2	−2	−2	10	10	−2	−2
$X.14$	10	10	10	10	10	10	−6	−6	−6	−6	10	10	−6	−6	10	2	2	2	2	10	10	2	2
$X.15$	15	15	15	15	15	15	−9	−9	−9	−9	15	15	−9	−9	15	3	3	3	3	15	15	3	3
$X.16$	15	15	15	15	15	15	−9	−9	−9	−9	15	15	−9	−9	15	3	3	3	3	15	15	3	3
$X.17$	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	−1	−1	15	15
$X.18$	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	−1	−1	15	15
$X.19$	15	15	15	15	15	15	−9	−9	−9	−9	15	15	−9	−9	15	3	3	3	3	−1	−1	3	3
$X.20$	15	15	15	15	15	15	−9	−9	−9	−9	15	15	−9	−9	15	3	3	3	3	−1	−1	3	3
$X.21$	15	15	15	15	15	15	−9	−9	−9	−9	15	15	−9	−9	15	3	3	3	3	−1	−1	3	3
$X.22$	15	15	15	15	15	15	−9	−9	−9	−9	15	15	−9	−9	15	3	3	3	3	−1	−1	3	3
$X.23$	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	−1	−1	15	15
$X.24$	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	−1	−1	15	15
$X.25$	16	−16	16	−16	−16	16	8	8	−8	−8	.	.	−8	8	.	4	4	−4	−4	.	.	−4	4
$X.26$	16	−16	16	−16	16	−16	−8	−8	8	8	.	.	−8	8	.	4	4	−4	−4	.	.	4	−4
$X.27$	16	−16	16	−16	−16	16	8	8	−8	−8	.	.	−8	8	.	4	4	−4	−4	.	.	−4	4
$X.28$	16	−16	16	−16	16	−16	−8	−8	8	8	.	.	−8	8	.	4	4	−4	−4	.	.	4	−4
$X.29$	20	20	20	20	20	20	4	4	4	4	20	20	4	4	20	−4	−4	−4	−4	20	20	−4	−4
$X.30$	20	20	20	20	20	20	4	4	4	4	20	20	4	4	20	−4	−4	−4	−4	20	20	−4	−4
$X.31$	30	30	30	30	30	30	6	6	6	6	30	30	6	6	30	−6	−6	−6	−6	−2	−2	−6	−6
$X.32$	30	30	30	30	30	30	6	6	6	6	30	30	6	6	30	−6	−6	−6	−6	−2	−2	−6	−6
$X.33$	30	30	30	30	30	30	−18	−18	−18	−18	30	30	−18	−18	30	6	6	6	6	−2	−2	6	6
$X.34$	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	−2	−2	30	30
$X.35$	30	30	30	30	30	30	6	6	6	6	30	30	6	6	30	−6	−6	−6	−6	−2	−2	−6	−6
$X.36$	30	30	30	30	30	30	6	6	6	6	30	30	6	6	30	−6	−6	−6	−6	−2	−2	−6	−6
$X.37$	32	−32	−32	32	.	.	16	−16	16	−16	.	.	.	.	.	−8	8	8	−8	.	.	.	.
$X.38$	40	40	40	40	−40	−40	−16	−16	−16	−16	8	8	16	16	−8	4	4	4	4	8	8	−4	−4
$X.39$	40	40	40	40	−40	−40	−16	−16	−16	−16	8	8	16	16	−8	4	4	4	4	8	8	−4	−4
$X.40$	40	40	40	40	−40	−40	−16	−16	−16	−16	8	8	16	16	−8	4	4	4	4	8	8	−4	−4
$X.41$	40	40	40	40	−40	−40	−16	−16	−16	−16	8	8	16	16	−8	4	4	4	4	8	8	−4	−4
$X.42$	40	40	40	40	−40	−40	16	16	16	16	8	8	−16	−16	−8	4	4	4	4	8	8	−4	−4
$X.43$	40	40	40	40	−40	−40	16	16	16	16	8	8	−16	−16	−8	4	4	4	4	8	8	−4	−4
$X.44$	40	40	40	40	−40	−40	16	16	16	16	8	8	−16	−16	−8	4	4	4	4	8	8	−4	−4
$X.45$	40	40	40	40	−40	−40	16	16	16	16	8	8	−16	−16	−8	4	4	4	4	8	8	−4	−4
$X.46$	60	60	60	60	60	60	12	12	12	12	60	60	12	12	60	−12	−12	−12	−12	−4	−4	−12	−12
$X.47$	60	60	60	60	60	60	12	12	12	12	60	60	12	12	60	−12	−12	−12	−12	−4	−4	−12	−12
$X.48$	60	60	60	60	60	60	−36	−36	−36	−36	60	60	−36	−36	60	12	12	12	12	−4	−4	12	12
$X.49$	60	60	60	60	60	60	−36	−36	−36	−36	60	60	−36	−36	60	12	12	12	12	−4	−4	12	12
$X.50$	60	60	60	60	60	60	12	12	12	12	60	60	12	12	60	−12	−12	−12	−12	−4	−4	−12	−12
$X.51$	60	60	60	60	60	60	12	12	12	12	60	60	12	12	60	−12	−12	−12	−12	−4	−4	−12	−12
$X.52$	64	−64	64	−64	−64	64	32	32	−32	−32	.	.	−32	32	.	16	16	−16	−16	.	.	−16	16
$X.53$	64	−64	64	−64	64	−64	−32	−32	32	32	.	.	−32	32	.	16	16	−16	−16	.	.	16	−16
$X.54$	64	−64	64	−64	64	−64	−32	−32	32	32	.	.	−32	32	.	16	16	−16	−16	.	.	16	−16
$X.55$	64	−64	64	−64	−64	64	32	32	−32	−32	.	.	−32	32	.	16	16	−16	−16	.	.	−16	16
$X.56$	80	80	80	80	−80	−80	−32	−32	−32	−32	16	16	32	32	−16	8	8	8	8	16	16	−8	−8
$X.57$	80	−80	80	−80	−80	80	40	40	−40	−40	.	.	−40	40	.	20	20	−20	−20	.	.	−20	20
$X.58$	80	80	80	80	−80	−80	.	.	.	.	16	16	.	.	−16	−8	−8	−8	−8	16	16	8	8
$X.59$	80	−80	80	−80	80	−80	24	24	−24	−24	.	.	24	−24	.	4	4	−4	−4	.	.	4	−4
$X.60$	80	−80	80	−80	80	−80	−40	−40	40	40	.	.	−40	40	.	20	20	−20	−20	.	.	20	−20
$X.61$	80	80	80	80	−80	−80	32	32	32	32	16	16	−32	−32	−16	8	8	8	8	16	16	−8	−8
$X.62$	80	−80	80	−80	80	−80	24	24	−24	−24	.	.	24	−24	.	4	4	−4	−4	.	.	4	−4
$X.63$	80	−80	80	−80	−80	80	−24	−24	24	24	.	.	24	−24	.	4	4	−4	−4	.	.	−4	4
$X.64$	80	80	−80	−80	.	.	32	−32	−32	32	−16	16	.	.	.	−8	8	−8	8	−16	16	.	.
$X.65$	80	80	80	80	−80	−80	32	32	32	32	16	16	−32	−32	−16	8	8	8	8	16	16	−8	−8
$X.66$	80	−80	80	−80	−80	80	40	40	−40	−40	.	.	−40	40	.	20	20	−20	−20	.	.	−20	20
$X.67$	80	80	−80	−80	.	.	−32	32	32	−32	−16	16	.	.	.	−8	8	−8	8	−16	16	.	.
$X.68$	80	−80	80	−80	−80	80	−24	−24	24	24	.	.	24	−24	.	4	4	−4	−4	.	.	−4	4
$X.69$	80	80	80	80	−80	−80	.	.	.	.	16	16	.	.	−16	−8	−8	−8	−8	16	16	8	8
$X.70$	80	80	−80	−80	.	.	−32	32	32	−32	−16	16	.	.	.	−8	8	−8	8	−16	16	.	.
$X.71$	80	80	80	80	−80	−80	−32	−32	−32	−32	16	16	32	32	−16	8	8	8	8	16	16	−8	−8
$X.72$	80	−80	80	−80	80	−80	−40	−40	40	40	.	.	−40	40	.	20	20	−20	−20	.	.	20	−20
$X.73$	80	80	−80	−80	.	.	32	−32	−32	32	−16	16	.	.	.	−8	8	−8	8	−16	16	.	.
$X.74$	96	−96	96	−96	96	−96	−48	−48	48	48	.	.	−48	48	.	24	24	−24	−24	.	.	24	−24
$X.75$	96	−96	96	−96	−96	96	48	48	−48	−48	.	.	−48	48	.	24	24	−24	−24	.	.	−24	24
$X.76$	96	96	96	96	−96	−96	.	.	.	.	−32	−32	.	.	32	.	.	.	.	.	.	.	.
$X.77$	96	96	96	96	−96	−96	.	.	.	.	−32	−32	.	.	32	.	.	.	.	.	.	.	.
$X.78$	96	96	96	96	−96	−96	.	.	.	.	−32	−32	.	.	32	.	.	.	.	.	.	.	.
$X.79$	96	96	96	96	−96	−96	.	.	.	.	−32	−32	.	.	32	.	.	.	.	.	.	.	.
$X.80$	96	−96	−96	96	.	.	48	−48	48	−48	.	.	.	.	.	−24	24	24	−24	.	.	.	.
$X.81$	96	−96	−96	96	.	.	48	−48	48	−48	.	.	.	.	.	−24	24	24	−24	.	.	.	.
$X.82$	120	120	120	120	−120	−120	48	48	48	48	24	24	−48	−48	−24	12	12	12	12	−8	−8	−12	−12
$X.83$	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	8	8	.	.
$X.84$	120	120	120	120	−120	−120	−48	−48	−48	−48	24	24	48	48	−24	12	12	12	12	−8	−8	−12	−12
$X.85$	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	8	8	.	.
$X.86$	120	120	120	120	−120	−120	48	48	48	48	24	24	−48	−48	−24	12	12	12	12	−8	−8	−12	−12
$X.87$	120	120	120	120	120	120	−24	−24	−24	−24	−8	−8	−24	−24	−8	.	.	.	.	8	8	.	.
$X.88$	120	120	120	120	120	120	−24	−24	−24	−24	−8	−8	−24	−24	−8	.	.	.	.	8	8	.	.
$X.89$	120	120	120	120	−120	−120	−48	−48	−48	−48	24	24	48	48	−24	12	12	12	12	−8	−8	−12	−12
$X.90$	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	−8	−8	.	.
$X.91$	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	−8	−8	.	.
$X.92$	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	8	8	.	.
$X.93$	120	120	120	120	−120	−120	−48	−48	−48	−48	24	24	48	48	−24	12	12	12	12	−8	−8	−12	−12

Character table of $D(2\,\mathrm{Fi}_{22})$ (continued)

2	13	13	11	11	12	12	12	12	12	11	11	10	6	11	11	12	12	11	11	11	11	11	11	11	11
3	.	.	1	1	.	.	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	2_{23}	2_{24}	2_{25}	2_{26}	2_{27}	2_{28}	2_{29}	2_{30}	2_{31}	2_{32}	2_{33}	2_{34}	$3a$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}
$2P$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$3a$	2_2	2_2	2_1	2_1	2_{10}	2_{10}	2_{10}	2_{11}	2_{10}	2_{11}	2_{11}	2_{11}
$3P$	2_{23}	2_{24}	2_{25}	2_{26}	2_{27}	2_{28}	2_{29}	2_{30}	2_{31}	2_{32}	2_{33}	2_{34}	$1a$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}
$5P$	2_{23}	2_{24}	2_{25}	2_{26}	2_{27}	2_{28}	2_{29}	2_{30}	2_{31}	2_{32}	2_{33}	2_{34}	$3a$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9	4_{10}	4_{11}	4_{12}
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	1	1	−1	−1	1	1	1	1	1	1	1	−1	1	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1
$X.3$	4	4	2	2	.	.	.	.	4	.	.	2	1	2	2	4	4	4	4	2	2	2	2	4	4
$X.4$	4	4	−2	−2	.	.	.	.	4	.	.	−2	1	−2	−2	4	4	4	4	−2	−2	−2	−2	4	4
$X.5$	5	5	−1	−1	1	1	1	1	5	1	1	−1	−1	−1	−1	5	5	5	5	−1	−1	−1	−1	5	5
$X.6$	5	5	1	1	1	1	1	1	5	1	1	1	−1	1	1	5	5	5	5	1	1	1	1	5	5
$X.7$	5	−3	−3	−3	1	1	1	1	−3	1	1	1	2	−3	−3	−3	5	1	1	1	1	1	1	−3	−3
$X.8$	5	−3	3	3	1	1	1	1	−3	1	1	−1	2	3	3	−3	5	1	1	−1	−1	−1	−1	−3	−3
$X.9$	6	6	.	.	−2	−2	−2	−2	6	−2	−2	.	.	.	.	6	6	6	6	.	.	.	.	6	6
$X.10$	10	2	−4	−4	2	2	2	2	2	2	2	.	1	−4	−4	2	10	−2	−2	.	.	.	.	2	2
$X.11$	10	2	−2	−2	−2	−2	−2	−2	2	−2	−2	2	1	−2	−2	2	10	−2	−2	2	2	2	2	2	2
$X.12$	10	2	4	4	2	2	2	2	2	2	2	.	1	4	4	2	10	−2	−2	.	.	.	.	2	2
$X.13$	10	2	2	2	−2	−2	−2	−2	2	−2	−2	−2	1	2	2	2	10	−2	−2	−2	−2	−2	−2	2	2
$X.14$	10	−6	.	.	2	2	2	2	−6	2	2	.	−2	.	.	−6	10	2	2	.	.	.	.	−6	−6
$X.15$	15	−9	−3	−3	−1	−1	−1	−1	−9	−1	−1	1	.	−3	−3	−9	15	3	3	1	1	1	1	−9	−9
$X.16$	15	−9	3	3	−1	−1	−1	−1	−9	−1	−1	−1	.	3	3	−9	15	3	3	−1	−1	−1	−1	−9	−9
$X.17$	−1	15	−3	−3	−1	−1	−1	−1	−1	−1	−1	−3	.	−3	−3	−1	−1	−1	−1	−3	−3	−3	−3	−1	−1
$X.18$	−1	15	3	3	−1	−1	−1	−1	−1	−1	−1	3	.	3	3	−1	−1	−1	−1	3	3	3	3	−1	−1
$X.19$	−1	−9	−3	−3	−1	−1	−1	−1	7	−1	−1	1	.	−3	−3	7	−1	3	3	1	1	1	1	−1	−1
$X.20$	−1	−9	3	3	−1	−1	−1	−1	7	−1	−1	−1	.	3	3	7	−1	3	3	−1	−1	−1	−1	−1	−1
$X.21$	−1	−9	3	3	3	3	3	3	7	3	3	−1	.	3	3	7	−1	3	3	−1	−1	−1	−1	−1	−1
$X.22$	−1	−9	−3	−3	3	3	3	3	7	3	3	1	.	−3	−3	7	−1	3	3	1	1	1	1	−1	−1
$X.23$	−1	15	3	3	3	3	3	3	−1	3	3	3	.	3	3	−1	−1	−1	−1	3	3	3	3	−1	−1
$X.24$	−1	15	−3	−3	3	3	3	3	−1	3	3	−3	.	−3	−3	−1	−1	−1	−1	−3	−3	−3	−3	−1	−1
$X.25$	.	.	4	−4	4	4	−4	−4	.	−4	4	.	1	−4	4	.	.	.	.	−4	4	4	−4	.	.
$X.26$	.	.	−4	4	4	4	−4	−4	.	4	−4	.	1	−4	4	.	.	.	.	4	4	−4	−4	.	.
$X.27$	.	.	−4	4	4	4	−4	−4	.	−4	4	.	1	4	−4	.	.	.	.	4	−4	−4	4	.	.
$X.28$	.	.	4	−4	4	4	−4	−4	.	4	−4	.	1	4	−4	.	.	.	.	−4	−4	4	4	.	.
$X.29$	20	4	−2	−2	.	.	.	.	4	.	.	−2	−1	−2	−2	4	20	−4	−4	−2	−2	−2	−2	4	4
$X.30$	20	4	2	2	.	.	.	.	4	.	.	2	−1	2	2	4	20	−4	−4	2	2	2	2	4	4
$X.31$	−2	6	.	.	−2	−2	−2	−2	6	−2	−2	−4	.	.	.	6	−2	10	10	−4	−4	−4	−4	−2	−2
$X.32$	−2	6	.	.	−2	−2	−2	−2	6	−2	−2	4	.	.	.	6	−2	10	10	4	4	4	4	−2	−2
$X.33$	−2	−18	.	.	−2	−2	−2	−2	14	−2	−2	.	.	.	.	14	−2	6	6	.	.	.	.	−2	−2
$X.34$	−2	30	.	.	−2	−2	−2	−2	−2	−2	−2	.	.	.	.	−2	−2	−2	−2	.	.	.	.	−2	−2
$X.35$	−2	6	−6	−6	2	2	2	2	6	2	2	−2	.	−6	−6	6	−2	10	10	−2	−2	−2	−2	−2	−2
$X.36$	−2	6	6	6	2	2	2	2	6	2	2	2	.	6	6	6	−2	10	10	2	2	2	2	−2	−2
$X.37$	.	.	.	.	8	−8	8	−8	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.
$X.38$	−8	.	10	10	4	4	4	4	.	−4	−4	−2	1	−10	−10	.	.	4	−4	6	−6	6	−6	−4	−4
$X.39$	−8	.	−10	−10	4	4	4	4	.	−4	−4	2	1	10	10	.	.	4	−4	−6	6	−6	6	−4	−4
$X.40$	−8	.	−2	−2	−4	−4	−4	−4	.	4	4	2	1	2	2	.	.	4	−4	−6	6	−6	6	−4	−4
$X.41$	−8	.	2	2	−4	−4	−4	−4	.	4	4	−2	1	−2	−2	.	.	4	−4	6	−6	6	−6	−4	−4
$X.42$	−8	.	10	10	4	4	4	4	.	−4	−4	6	1	−10	−10	.	.	4	−4	−2	2	−2	2	4	4
$X.43$	−8	.	−10	−10	4	4	4	4	.	−4	−4	−6	1	10	10	.	.	4	−4	2	−2	2	−2	4	4
$X.44$	−8	.	−2	−2	−4	−4	−4	−4	.	4	4	−6	1	2	2	.	.	4	−4	2	−2	2	−2	4	4
$X.45$	−8	.	2	2	−4	−4	−4	−4	.	4	4	6	1	−2	−2	.	.	4	−4	−2	2	−2	2	4	4
$X.46$	−4	12	.	.	4	4	4	4	12	4	4	.	.	.	.	12	−4	−12	−12	.	.	.	.	−4	−4
$X.47$	−4	12	6	6	.	.	.	.	−20	.	.	−2	.	6	6	−20	−4	4	4	−2	−2	−2	−2	4	4
$X.48$	−4	−36	−6	−6	.	.	.	.	−4	.	.	2	.	−6	−6	−4	−4	−4	−4	2	2	2	2	4	4
$X.49$	−4	−36	6	6	.	.	.	.	−4	.	.	−2	.	6	6	−4	−4	−4	−4	−2	−2	−2	−2	4	4
$X.50$	−4	12	−6	−6	.	.	.	.	−20	.	.	2	.	−6	−6	−20	−4	4	4	2	2	2	2	4	4
$X.51$	−4	12	.	.	−4	−4	−4	−4	12	−4	−4	.	.	.	.	12	−4	−12	−12	.	.	.	.	−4	−4
$X.52$	.	.	−8	8	.	.	.	.	.	.	.	.	1	8	−8	.	.	.	.	8	−8	−8	8	.	.
$X.53$	.	.	8	−8	.	.	.	.	.	.	.	.	1	8	−8	.	.	.	.	−8	−8	8	8	.	.
$X.54$	.	.	−8	8	.	.	.	.	.	.	.	.	1	−8	8	.	.	.	.	8	8	−8	−8	.	.
$X.55$	.	.	8	−8	.	.	.	.	.	.	.	.	1	−8	8	.	.	.	.	−8	8	8	−8	.	.
$X.56$	−16	.	8	8	.	.	.	.	.	.	.	.	−1	−8	−8	.	.	8	−8	.	.	.	.	−8	−8
$X.57$	.	.	4	−4	4	4	−4	−4	.	−4	4	.	−1	−4	4	.	.	.	.	−4	4	4	−4	.	.
$X.58$	−16	.	12	12	.	.	.	.	.	.	.	−4	2	−12	−12	.	.	−8	8	−4	4	−4	4	.	.
$X.59$	.	.	−12	12	4	4	−4	−4	.	4	−4	.	2	−12	12	.	.	.	.	−4	−4	4	4	.	.
$X.60$	.	.	4	−4	4	4	−4	−4	.	4	−4	.	−1	4	−4	.	.	.	.	−4	−4	4	4	.	.
$X.61$	−16	.	−8	−8	.	.	.	.	.	.	.	.	−1	8	8	.	.	8	−8	.	.	.	.	8	8
$X.62$	.	.	12	−12	4	4	−4	−4	.	4	−4	.	2	12	−12	.	.	.	.	4	4	−4	−4	.	.
$X.63$	.	.	−12	12	4	4	−4	−4	.	−4	4	.	2	12	−12	.	.	.	.	−4	4	4	−4	.	.
$X.64$	.	.	.	.	−8	8	8	−8	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	−8	8
$X.65$	−16	.	8	8	.	.	.	.	.	.	.	.	−1	−8	−8	.	.	8	−8	.	.	.	.	8	8
$X.66$	.	.	−4	4	4	4	−4	−4	.	−4	4	.	−1	4	−4	.	.	.	.	4	−4	−4	4	.	.
$X.67$	.	.	.	.	−8	8	8	−8	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	8	−8
$X.68$	.	.	12	−12	4	4	−4	−4	.	−4	4	.	2	−12	12	.	.	.	.	4	−4	−4	4	.	.
$X.69$	−16	.	−12	−12	.	.	.	.	.	.	.	4	2	12	12	.	.	−8	8	4	−4	4	−4	.	.
$X.70$	.	.	.	.	8	−8	−8	8	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	8	−8
$X.71$	−16	.	−8	−8	.	.	.	.	.	.	.	.	−1	8	8	.	.	8	−8	.	.	.	.	−8	−8
$X.72$	.	.	−4	4	4	4	−4	−4	.	4	−4	.	−1	−4	4	.	.	.	.	4	4	−4	−4	.	.
$X.73$	.	.	.	.	8	−8	−8	8	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	−8	8
$X.74$	.	.	.	.	−8	−8	8	8	.	−8	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.75$	.	.	.	.	−8	−8	8	8	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.76$	.	.	.	.	8	8	8	8	.	−8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.77$	.	.	.	.	8	8	8	8	.	−8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.78$	.	.	.	.	−8	−8	−8	−8	.	8	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.79$	.	.	.	.	−8	−8	−8	−8	.	8	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.80$	.	.	.	.	−8	8	−8	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.81$	.	.	.	.	−8	8	−8	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.82$	8	.	−6	−6	4	4	4	4	.	−4	−4	−10	.	6	6	.	.	−4	4	−2	2	−2	2	−4	−4
$X.83$	8	−8	12	12	−4	−4	−4	−4	8	−4	−4	4	.	12	12	−8	−8	.	.	4	4	4	4	.	.
$X.84$	8	.	6	6	−4	−4	−4	−4	.	4	4	−6	.	−6	−6	.	.	−4	4	2	−2	2	−2	4	4
$X.85$	8	−8	−12	−12	−4	−4	−4	−4	8	−4	−4	−4	.	−12	−12	−8	−8	.	.	−4	−4	−4	−4	.	.
$X.86$	8	.	−6	−6	−4	−4	−4	−4	.	4	4	−2	.	6	6	.	.	−4	4	6	−6	6	−6	−4	−4
$X.87$	8	8	12	12	.	.	.	.	−8	.	.	−4	.	12	12	8	−8	.	.	−4	−4	−4	−4	.	.
$X.88$	8	8	12	12	.	.	.	.	−8	.	.	−4	.	12	12	8	−8	.	.	−4	−4	−4	−4	.	.
$X.89$	8	.	6	6	4	4	4	4	.	−4	−4	2	.	−6	−6	.	.	−4	4	10	−10	10	−10	4	4
$X.90$	−8	−8	.	.	.	.	.	.	−8	.	.	8	.	.	.	8	8	.	.	−8	−8	−8	−8	.	.
$X.91$	−8	−8	.	.	.	.	.	.	−8	.	.	−8	.	.	.	8	8	.	.	8	8	8	8	.	.
$X.92$	8	−8	−12	−12	12	12	12	12	8	12	12	−4	.	−12	−12	−8	−8	.	.	−4	−4	−4	−4	.	.
$X.93$	8	.	−6	−6	4	4	4	4	.	−4	−4	−2	.	6	6	.	.	−4	4	−10	10	−10	10	4	4

Character table of $D(2\,\mathrm{Fi}_{22})$ *(continued)*

2	9	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10	9	9	9	9	9	9	9	9	9	9	9
3	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}
2P	2_{15}	2_{16}	2_{15}	2_{16}	2_{6}	2_{10}	2_{14}	2_{8}	2_{8}	2_{14}	2_{10}	2_{11}	2_{11}	2_{10}	2_{2}	2_{6}	2_{19}	2_{19}	2_{16}	2_{16}	2_{15}	2_{11}	2_{16}	2_{14}	2_{16}	2_{13}	2_{16}
3P	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}
5P	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}	4_{36}	4_{37}	4_{38}	4_{39}
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	−1	−1	−1	−1	1	1	1	1	1	1	1	−1	1	−1	−1	1	1	1	1	1	−1	1	−1	1	−1	1	1
X.3	2	2	2	2	.	.	4	.	.	4	.	2	4	2	2	.	.	.	.	.	2	4	2	.	2	.	.
X.4	−2	−2	−2	−2	.	.	4	.	.	4	.	−2	4	−2	−2	.	.	.	.	.	−2	4	−2	.	−2	.	.
X.5	−1	−1	−1	−1	1	1	5	1	1	5	1	−1	5	−1	−1	1	1	1	1	1	−1	5	−1	1	−1	1	1
X.6	1	1	1	1	1	1	5	1	1	5	1	1	5	1	1	1	1	1	1	1	1	5	1	1	1	1	1
X.7	3	3	3	3	1	1	1	1	1	1	1	−3	−3	−3	1	1	1	1	−1	−1	−1	1	−1	1	−1	1	−1
X.8	−3	−3	−3	−3	1	1	1	1	1	1	1	3	−3	3	−1	1	1	1	−1	−1	1	1	1	1	1	1	−1
X.9	.	.	.	.	−2	−2	6	−2	−2	6	−2	.	6	.	.	−2	−2	−2	−2	−2	.	6	.	−2	.	−2	−2
X.10	−2	−2	−2	−2	−2	2	−2	−2	−2	−2	2	−4	2	−4	.	−2	2	2	.	.	2	−2	2	2	2	−2	.
X.11	−4	−4	−4	−4	2	−2	−2	2	2	−2	−2	−2	2	−2	2	2	−2	−2	.	.	.	−2	.	−2	.	2	.
X.12	2	2	2	2	−2	2	−2	−2	−2	−2	2	4	2	4	.	−2	2	2	.	.	−2	−2	−2	2	−2	−2	.
X.13	4	4	4	4	2	−2	−2	2	2	−2	−2	2	2	2	−2	2	−2	−2	.	.	.	−2	.	−2	.	2	.
X.14	.	.	.	.	2	2	2	2	2	2	2	.	−6	.	.	2	2	2	−2	−2	.	2	.	2	.	2	−2
X.15	3	3	3	3	−1	−1	3	−1	−1	3	−1	−3	−9	−3	1	−1	−1	−1	1	1	−1	3	−1	−1	−1	−1	1
X.16	−3	−3	−3	−3	−1	−1	3	−1	−1	3	−1	3	−9	3	−1	−1	−1	−1	1	1	1	3	1	−1	1	−1	1
X.17	−3	−3	−3	−3	−1	−1	−1	−1	−1	−1	−1	−3	−1	−3	−3	−1	3	3	−1	−1	−3	−1	−3	−1	−3	−1	−1
X.18	3	3	3	3	−1	−1	−1	−1	−1	−1	−1	3	−1	3	3	−1	3	3	−1	−1	3	−1	3	−1	3	−1	−1
X.19	3	3	3	3	−1	−1	3	−1	−1	3	−1	−3	−1	−3	1	−1	3	3	1	1	−1	−5	−1	−1	−1	−1	1
X.20	−3	−3	−3	−3	−1	−1	3	−1	−1	3	−1	3	−1	3	−1	−1	3	3	1	1	1	−5	1	−1	1	−1	1
X.21	−3	−3	−3	−3	3	3	3	3	3	3	3	3	−1	3	−1	3	−1	−1	−3	−3	1	−5	1	3	1	3	−3
X.22	3	3	3	3	3	3	3	3	3	3	3	−3	−1	−3	1	3	−1	−1	−3	−3	−1	−5	−1	3	−1	3	−3
X.23	3	3	3	3	3	3	−1	3	3	−1	3	3	−1	3	3	3	−1	−1	3	3	3	−1	3	3	3	3	3
X.24	−3	−3	−3	−3	3	3	−1	3	3	−1	3	−3	−1	−3	−3	3	−1	−1	3	3	−3	−1	−3	3	−3	3	3
X.25	−2	2	2	−2	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	−2	−2	2	.	−2	.	2	.	2
X.26	2	2	−2	−2	4	.	.	.	.	.	.	.	.	.	.	−4	.	.	2	2	−2	.	−2	.	2	.	−2
X.27	2	−2	−2	2	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	−2	−2	−2	.	2	.	−2	.	2
X.28	−2	−2	2	2	4	.	.	.	.	.	.	.	.	.	.	−4	.	.	2	2	2	.	2	.	−2	.	−2
X.29	2	2	2	2	.	.	−4	.	.	−4	.	−2	4	−2	−2	.	.	.	.	.	2	−4	2	.	2	.	.
X.30	−2	−2	−2	−2	.	.	−4	.	.	−4	.	2	4	2	2	.	.	.	.	.	−2	−4	−2	.	−2	.	.
X.31	6	6	6	6	2	−2	−6	2	2	−6	−2	.	−2	.	−4	2	−2	−2	.	.	2	2	2	−2	2	2	.
X.32	−6	−6	−6	−6	2	−2	−6	2	2	−6	−2	.	−2	.	4	2	−2	−2	.	.	−2	2	−2	−2	−2	2	.
X.33	.	.	.	.	−2	−2	6	−2	−2	6	−2	.	−2	.	.	−2	−2	−2	2	2	.	−10	.	−2	.	−2	2
X.34	.	.	.	.	−2	−2	−2	−2	−2	−2	−2	.	−2	.	.	−2	−2	−2	−2	−2	.	−2	.	−2	.	−2	−2
X.35	.	.	.	.	−2	2	−6	−2	−2	−6	2	−6	−2	−6	−2	−2	2	2	.	.	4	2	4	2	4	−2	.
X.36	.	.	.	.	−2	2	−6	−2	−2	−6	2	6	−2	6	2	−2	2	2	.	.	−4	2	−4	2	−4	−2	.
X.37	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4	.	.	.	.	.	−4	4
X.38	4	−4	4	−4	.	−4	.	.	.	.	4	−2	4	2	2	.	−4	4	−2	−2	.	.	.	.	.	.	−2
X.39	−4	4	−4	4	.	−4	.	.	.	.	4	2	4	−2	−2	.	−4	4	−2	−2	.	.	.	.	.	.	−2
X.40	4	−4	4	−4	.	4	.	.	.	.	−4	−6	4	6	−2	.	4	−4	2	2	.	.	.	.	.	.	2
X.41	−4	4	−4	4	.	4	.	.	.	.	−4	6	4	−6	2	.	4	−4	2	2	.	.	.	.	.	.	2
X.42	−4	4	−4	4	.	−4	.	.	.	.	4	−2	−4	2	−6	.	−4	4	2	2	.	.	.	.	.	.	2
X.43	4	−4	4	−4	.	−4	.	.	.	.	4	2	−4	−2	6	.	−4	4	2	2	.	.	.	.	.	.	2
X.44	−4	4	−4	4	.	4	.	.	.	.	−4	−6	−4	6	6	.	4	−4	−2	−2	.	.	.	.	.	.	−2
X.45	4	−4	4	−4	.	4	.	.	.	.	−4	6	−4	−6	−6	.	4	−4	−2	−2	.	.	.	.	.	.	−2
X.46	.	.	.	.	−4	4	4	−4	−4	4	4	.	−4	.	.	−4	−4	−4	.	.	.	4	.	4	.	−4	.
X.47	6	6	6	6	.	.	4	.	.	4	.	6	4	6	−2	.	.	.	.	.	−2	−4	−2	.	−2	.	.
X.48	6	6	6	6	.	.	−4	.	.	−4	.	−6	4	−6	2	.	.	.	.	.	−2	4	−2	.	−2	.	.
X.49	−6	−6	−6	−6	.	.	−4	.	.	−4	.	6	4	6	−2	.	.	.	.	.	2	4	2	.	2	.	.
X.50	−6	−6	−6	−6	.	.	4	.	.	4	.	−6	4	−6	2	.	.	.	.	.	2	−4	2	.	2	.	.
X.51	.	.	.	.	4	−4	4	4	4	4	−4	.	−4	.	.	4	4	4	.	.	.	4	.	−4	.	4	.
X.52	4	−4	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	.	4	.	−4	.	.
X.53	−4	−4	4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	4	.	−4	.	.
X.54	4	4	−4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	.	−4	.	4	.	.
X.55	−4	4	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	−4	.	4	.	.
X.56	8	−8	8	−8	.	.	.	.	.	.	.	−8	8	8	.	.	.	.	.	.	.	.	.	.	.	.	.
X.57	−2	2	2	−2	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	−2	−2	2	.	−2	.	2	.	2
X.58	.	.	.	.	.	.	.	.	.	.	.	4	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.
X.59	−6	−6	6	6	4	.	.	.	.	.	.	.	.	.	.	−4	.	.	−2	−2	−2	.	−2	.	2	.	2
X.60	−2	−2	2	2	4	.	.	.	.	.	.	.	.	.	.	−4	.	.	2	2	2	.	2	.	−2	.	−2
X.61	8	−8	8	−8	.	.	.	.	.	.	.	8	−8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.
X.62	6	6	−6	−6	4	.	.	.	.	.	.	.	.	.	.	−4	.	.	−2	−2	2	.	2	.	−2	.	2
X.63	−6	6	6	−6	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	2	2	−2	.	2	.	−2	.	−2
X.64	.	.	.	.	.	.	4	.	.	−4	.	.	.	.	.	.	.	.	4	−4	.	.	.	4	.	.	−4
X.65	−8	8	−8	8	.	.	.	.	.	.	.	−8	−8	8	.	.	.	.	.	.	.	.	.	.	.	.	.
X.66	2	−2	−2	2	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	−2	−2	−2	.	2	.	−2	.	2
X.67	.	.	.	.	.	.	4	.	.	−4	.	.	.	.	.	.	.	.	−4	4	.	.	.	4	.	.	4
X.68	6	−6	−6	6	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	2	2	2	.	−2	.	2	.	−2
X.69	.	.	.	.	.	.	.	.	.	.	.	−4	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.
X.70	.	.	.	.	.	.	4	.	.	−4	.	.	.	.	.	.	.	.	4	−4	.	.	.	−4	.	.	−4
X.71	−8	8	−8	8	.	.	.	.	.	.	.	8	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.
X.72	2	2	−2	−2	4	.	.	.	.	.	.	.	.	.	.	−4	.	.	2	2	−2	.	−2	.	2	.	−2
X.73	.	.	.	.	.	.	4	.	.	−4	.	.	.	.	.	.	.	.	−4	4	.	.	.	−4	.	.	4
X.74	.	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	8	.	.	−4	−4	.	.	.	.	.	.	4
X.75	.	.	.	.	.	.	.	−8	8	.	.	.	.	.	.	.	.	.	4	4	.	.	.	.	.	.	−4
X.76	.	.	.	.	.	8	.	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.77	.	.	.	.	.	8	.	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.78	.	.	.	.	.	−8	.	.	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.79	.	.	.	.	.	−8	.	.	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.80	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	4	−4
X.81	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.	.	.	4	−4
X.82	.	.	.	.	.	−4	.	.	.	.	4	−2	4	2	10	.	4	−4	2	2	4	.	−4	.	−4	.	2
X.83	.	.	.	.	.	12	.	.	.	.	12	−4	.	−4	4	.	.	.	.	.	.	.	.	−4	.	.	.
X.84	.	.	.	.	.	4	.	.	.	.	−4	2	−4	−2	6	.	−4	4	2	2	−4	.	4	.	4	.	2
X.85	.	.	.	.	.	12	.	.	.	.	12	4	.	4	−4	.	.	.	.	.	.	.	.	−4	.	.	.
X.86	.	.	.	.	.	4	.	.	.	.	−4	−2	4	2	2	.	−4	4	−2	−2	−4	.	4	.	4	.	−2
X.87	.	.	.	.	.	.	.	.	.	.	.	−4	.	−4	−4	.	4	4	.	.	.	.	.	.	.	.	.
X.88	.	.	.	.	.	.	.	.	.	.	.	−4	.	−4	−4	.	4	4	.	.	.	.	.	.	.	.	.
X.89	.	.	.	.	.	−4	.	.	.	.	4	2	−4	−2	−2	.	4	−4	−2	−2	4	.	−4	.	−4	.	−2
X.90	.	.	.	.	−4	.	.	12	12	.	.	.	.	.	8	−4	.	.	.	.	.	.	.	.	.	−4	.
X.91	.	.	.	.	12	.	.	−4	−4	.	.	.	.	.	−8	12	.	.	.	.	.	.	.	.	.	−4	.
X.92	.	.	.	.	.	−4	.	.	.	.	−4	4	.	4	−4	.	.	.	.	.	.	.	.	−4	.	.	.
X.93	.	.	.	.	.	−4	.	.	.	.	4	−2	−4	2	2	.	4	−4	−2	−2	−4	.	4	.	4	.	−2

Character table of $D(2\,\mathrm{Fi}_{22})$ *(continued)*

2	9	9	9	9	9	9	8	8	8	8	8	8	8	8	8	7	7	7	7	7	3	6	6	6	6	6	6	6
3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	1	1
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
	4_{40}	4_{41}	4_{42}	4_{43}	4_{44}	4_{45}	4_{46}	4_{47}	4_{48}	4_{49}	4_{50}	4_{51}	4_{52}	4_{53}	4_{54}	4_{55}	4_{56}	4_{57}	4_{58}	4_{59}	$5a$	$6a$	$6b$	$6c$	$6d$	$6e$	$6f$	$6g$
$2P$	2_{15}	2_{14}	2_{11}	2_{13}	2_{16}	2_{10}	2_{16}	2_{20}	2_{19}	2_{20}	2_{20}	2_{23}	2_{16}	2_{19}	2_{23}	2_{27}	2_{28}	2_{27}	2_{20}	2_{28}	$5a$	$3a$	$3a$	$3a$	$3a$	$3a$	$3a$	$3a$
$3P$	4_{40}	4_{41}	4_{42}	4_{43}	4_{44}	4_{45}	4_{46}	4_{47}	4_{48}	4_{49}	4_{50}	4_{51}	4_{52}	4_{53}	4_{54}	4_{55}	4_{56}	4_{57}	4_{58}	4_{59}	$5a$	2_{15}	2_{16}	2_{2}	2_{17}	2_{3}	2_{1}	2_{18}
$5P$	4_{40}	4_{41}	4_{42}	4_{43}	4_{44}	4_{45}	4_{46}	4_{47}	4_{48}	4_{49}	4_{50}	4_{51}	4_{52}	4_{53}	4_{54}	4_{55}	4_{56}	4_{57}	4_{58}	4_{59}	$1a$	$6a$	$6b$	$6c$	$6d$	$6e$	$6f$	$6g$
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1	1	−1	−1	−1	1	−1	1	1	1	1	1	1	1	1
$X.3$	2	.	2	.	.	2	.	2	2	.	2	.	.	2	.	.	.	.	.	.	−1	1	1	1	1	1	1	1
$X.4$	−2	.	−2	.	.	−2	.	−2	−2	.	−2	.	.	−2	.	.	.	.	.	.	−1	1	1	1	1	1	1	1
$X.5$	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1	1	1	1	1	1	1	.	−1	−1	−1	−1	−1	−1	−1
$X.6$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	1	−1	.	−1	−1	−1	−1	−1	−1	−1
$X.7$	−1	1	1	1	−1	1	−1	−3	−3	1	1	1	−1	1	1	−1	−1	−1	1	−1	.	−2	−2	2	−2	2	2	−2
$X.8$	1	1	−1	1	−1	−1	−1	3	3	1	−1	1	−1	−1	1	1	1	1	1	1	.	−2	−2	2	−2	2	2	−2
$X.9$	.	−2	.	−2	−2	.	−2	.	.	−2	.	−2	−2	.	−2	.	.	.	−2	.	1	.	.	.	.	.	.	.
$X.10$	2	2	.	−2	.	.	.	−4	−4	2	.	2	.	.	2	.	.	.	2	.	.	1	1	1	1	1	1	1
$X.11$	.	−2	2	2	.	2	.	−2	−2	−2	2	−2	.	2	−2	.	.	.	−2	.	.	1	1	1	1	1	1	1
$X.12$	−2	2	.	−2	.	.	.	4	4	2	.	2	.	.	2	.	.	.	2	.	.	1	1	1	1	1	1	1
$X.13$	.	−2	−2	2	.	−2	.	2	2	−2	−2	−2	.	−2	−2	.	.	.	−2	.	.	1	1	1	1	1	1	1
$X.14$	.	2	.	2	−2	.	−2	.	.	2	.	2	−2	.	2	.	.	.	2	.	.	2	2	−2	2	−2	−2	2
$X.15$	−1	−1	1	−1	1	1	1	−3	−3	−1	1	−1	1	1	−1	1	1	1	−1	1	.	.	.	.	.	.	.	.
$X.16$	1	−1	−1	−1	1	−1	1	3	3	−1	−1	−1	1	−1	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.
$X.17$	−3	−1	−3	−1	−1	−3	−1	1	1	3	1	−1	−1	1	−1	1	1	1	−1	1	.	.	.	.	.	.	.	.
$X.18$	3	−1	3	−1	−1	3	−1	−1	−1	3	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.
$X.19$	−1	−1	1	−1	1	1	1	1	1	3	−3	−1	1	−3	−1	1	1	1	−1	1	.	.	.	.	.	.	.	.
$X.20$	1	−1	−1	−1	1	−1	1	−1	−1	3	3	−1	1	3	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.
$X.21$	1	3	−1	3	−3	−1	−3	−1	−1	−1	3	−1	−3	3	−1	1	1	1	−1	1	.	.	.	.	.	.	.	.
$X.22$	−1	3	1	3	−3	1	−3	1	1	−1	−3	−1	−3	−3	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.
$X.23$	3	3	3	3	3	3	3	−1	−1	−1	−1	−1	3	−1	−1	1	1	1	−1	1	.	.	.	.	.	.	.	.
$X.24$	−3	3	−3	3	3	−3	3	1	1	−1	1	−1	3	1	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.
$X.25$	−2	.	.	.	2	.	2	.	.	.	.	.	−2	.	.	2	−2	−2	.	2	1	1	1	1	−1	−1	−1	−1
$X.26$	2	.	.	.	−2	.	2	.	.	.	.	.	−2	.	.	2	2	−2	.	−2	1	1	1	1	−1	−1	−1	−1
$X.27$	2	.	.	.	2	.	2	.	.	.	.	.	−2	.	.	−2	2	2	.	−2	1	1	1	1	−1	−1	−1	−1
$X.28$	−2	.	.	.	−2	.	2	.	.	.	.	.	−2	.	.	−2	−2	2	.	2	1	1	1	1	−1	−1	−1	−1
$X.29$	2	.	−2	.	.	−2	.	−2	−2	.	−2	.	.	−2	.	.	.	.	.	.	.	−1	−1	−1	−1	−1	−1	−1
$X.30$	−2	.	2	.	.	2	.	2	2	.	2	.	.	2	.	.	.	.	.	.	.	−1	−1	−1	−1	−1	−1	−1
$X.31$	2	−2	−4	2	.	−4	.	.	.	−2	4	2	.	4	2	.	.	.	2	.	.	.	.	.	.	.	.	.
$X.32$	−2	−2	4	2	.	4	.	.	.	−2	−4	2	.	−4	2	.	.	.	2	.	.	.	.	.	.	.	.	.
$X.33$	.	−2	.	−2	2	.	2	.	.	−2	.	2	2	.	2	.	.	.	2	.	.	.	.	.	.	.	.	.
$X.34$	.	−2	.	−2	−2	.	−2	.	.	−2	.	2	−2	.	2	.	.	.	2	.	.	.	.	.	.	.	.	.
$X.35$	4	2	−2	−2	.	−2	.	2	2	2	−2	−2	.	−2	−2	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.36$	−4	2	2	−2	.	2	.	−2	−2	2	2	−2	.	2	−2	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.37$	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	2	−2	2	2	−2	−2
$X.38$	.	.	2	.	−2	−2	2	−2	2	.	2	.	2	−2	.	.	.	.	.	.	.	1	1	1	1	1	1	1
$X.39$	.	.	−2	.	−2	2	2	2	−2	.	−2	.	2	2	.	.	.	.	.	.	.	1	1	1	1	1	1	1
$X.40$	.	.	−2	.	2	2	−2	2	−2	.	−2	.	−2	2	.	.	.	.	.	.	.	1	1	1	1	1	1	1
$X.41$	.	.	2	.	2	−2	−2	−2	2	.	2	.	−2	−2	.	.	.	.	.	.	.	1	1	1	1	1	1	1
$X.42$	.	.	2	.	2	−2	−2	−2	2	.	2	.	−2	−2	.	.	.	.	.	.	.	1	1	1	1	1	1	1
$X.43$	.	.	−2	.	2	2	−2	2	−2	.	−2	.	−2	2	.	.	.	.	.	.	.	1	1	1	1	1	1	1
$X.44$	.	.	−2	.	−2	2	2	2	−2	.	−2	.	2	2	.	.	.	.	.	.	.	1	1	1	1	1	1	1
$X.45$	.	.	2	.	−2	−2	2	−2	2	.	2	.	2	−2	.	.	.	.	.	.	.	1	1	1	1	1	1	1
$X.46$	.	4	.	−4	.	.	.	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.47$	−2	.	−2	.	.	−2	.	−2	−2	.	−2	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.48$	−2	.	2	.	.	2	.	2	2	.	2	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.49$	2	.	−2	.	.	−2	.	−2	−2	.	−2	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.50$	2	.	2	.	.	2	.	2	2	.	2	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.51$	.	−4	.	4	.	.	.	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.52$	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	1	−1	−1	−1	−1
$X.53$	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	1	−1	−1	−1	−1
$X.54$	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	1	−1	−1	−1	−1
$X.55$	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	1	−1	−1	−1	−1
$X.56$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	−1	−1	−1	−1
$X.57$	−2	.	.	.	2	.	2	.	.	.	.	.	−2	.	.	−2	2	2	.	−2	.	−1	−1	−1	1	1	1	1
$X.58$	.	.	−4	.	.	4	.	−4	4	.	−4	.	.	4	.	.	.	.	.	.	.	−2	−2	2	−2	2	2	−2
$X.59$	2	.	.	.	2	.	−2	.	.	.	.	.	2	.	.	2	2	−2	.	−2	.	−2	−2	2	2	−2	−2	2
$X.60$	−2	.	.	.	−2	.	2	.	.	.	.	.	−2	.	.	2	2	−2	.	−2	.	−1	−1	−1	1	1	1	1
$X.61$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	−1	−1	−1	−1
$X.62$	−2	.	.	.	2	.	−2	.	.	.	.	.	2	.	.	−2	−2	2	.	2	.	−2	−2	2	2	−2	−2	2
$X.63$	2	.	.	.	−2	.	−2	.	.	.	.	.	2	.	.	−2	2	2	.	−2	.	−2	−2	2	2	−2	−2	2
$X.64$	.	−4	.	.	4	.	.	.	.	.	.	−4	.	.	4	.	.	.	.	.	.	−2	2	−2	−2	−2	2	2
$X.65$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	−1	−1	−1	−1
$X.66$	2	.	.	.	2	.	2	.	.	.	.	.	−2	.	.	2	−2	−2	.	2	.	−1	−1	−1	1	1	1	1
$X.67$	.	−4	.	.	−4	.	.	.	.	.	.	−4	.	.	4	.	.	.	.	.	.	−2	2	−2	−2	−2	2	2
$X.68$	−2	.	.	.	−2	.	−2	.	.	.	.	.	2	.	.	2	−2	−2	.	2	.	−2	−2	2	2	−2	−2	2
$X.69$	.	.	4	.	.	−4	.	4	−4	.	4	.	.	−4	.	.	.	.	.	.	.	−2	−2	2	−2	2	2	−2
$X.70$	.	4	.	.	4	.	.	.	.	.	.	4	.	.	−4	.	.	.	.	.	.	−2	2	−2	−2	−2	2	2
$X.71$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	−1	−1	−1	−1
$X.72$	2	.	.	.	−2	.	2	.	.	.	.	.	−2	.	.	−2	−2	2	.	2	.	−1	−1	−1	1	1	1	1
$X.73$	.	4	.	.	−4	.	.	.	.	.	.	4	.	.	−4	.	.	.	.	.	.	−2	2	−2	−2	−2	2	2
$X.74$	.	.	.	.	4	.	−4	.	.	.	.	.	4	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
$X.75$	.	.	.	.	−4	.	−4	.	.	.	.	.	4	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
$X.76$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4	4	.	−4	1	.	.	.	.	.	.	.
$X.77$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	−4	.	4	1	.	.	.	.	.	.	.
$X.78$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
$X.79$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
$X.80$	.	.	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
$X.81$	.	.	.	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
$X.82$	4	.	2	.	2	−2	−2	−2	2	.	2	.	−2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.83$	.	−4	−4	.	.	−4	.	.	.	.	.	.	.	.	.	−2	−2	−2	.	−2	.	.	.	.	.	.	.	.
$X.84$	−4	.	6	.	2	−6	−2	2	−2	.	−2	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.85$	.	−4	4	.	.	4	.	.	.	.	.	.	.	.	.	2	2	2	.	2	.	.	.	.	.	.	.	.
$X.86$	−4	.	−6	.	−2	6	2	−2	2	.	2	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.87$	.	.	4	.	.	4	.	.	.	−4	.	−4	.	.	−4	.	.	.	4	.	.	.	.	.	.	.	.	.
$X.88$	.	.	4	.	.	4	.	.	.	−4	.	4	.	.	4	.	.	.	−4	.	.	.	.	.	.	.	.	.
$X.89$	4	.	−2	.	−2	2	2	2	−2	.	−2	.	2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.90$	.	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.91$	.	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.92$	.	−4	4	.	.	4	.	.	.	.	.	.	.	.	.	−2	−2	−2	.	−2	.	.	.	.	.	.	.	.
$X.93$	−4	.	2	.	−2	−2	2	−2	2	.	2	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $D(2\,\mathrm{Fi}_{22})$ (continued)

2	5	5	5	5	5	5	5	5	5	5	5	5	4	4	7	7	7	7	7	7	7	6	6	6	6	6	6	6	6
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	6h	6i	6j	6k	6l	6m	6n	6o	6p	6q	6r	6s	6t	6u	8a	8b	8c	8d	8e	8f	8g	8h	8i	8j	8k	8l	8m	8n	8o
2P	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	4_3	4_5	4_5	4_4	4_4	4_5	4_5	4_{17}	4_3	4_{18}	4_{17}	4_{19}	4_{20}	4_{19}	4_6
3P	2_4	2_{12}	2_{12}	2_{13}	2_7	2_6	2_{21}	2_5	2_{22}	2_9	2_8	2_{13}	2_{25}	2_{26}	8a	8b	8c	8d	8e	8f	8g	8h	8i	8p	8k	8l	8m	8n	8o
5P	6h	6j	6i	6s	6l	6m	6n	6o	6p	6q	6r	6k	6t	6u	8a	8b	8c	8d	8e	8f	8g	8h	8i	8j	8k	8l	8m	8n	8o
X.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	−1	1	−1
X.3	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	.	2	.	2	2	.	2	.	.	.	.	.	.	.	2
X.4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	.	−2	.	−2	−2	.	−2	.	.	.	.	.	.	.	−2
X.5	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	−1	1	−1	−1	1	−1	1	1	1	1	1	1	1	−1
X.6	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	1	1	1	−1	1	−1	−1	1	−1	1	1
X.7	2	.	.	.	.	.	−2	2	−2	.	.	.	.	.	1	3	−1	1	1	−1	−1	1	1	−1	1	−1	1	−1	−1
X.8	2	.	.	.	.	.	−2	2	−2	.	.	.	.	.	1	−3	−1	−1	−1	−1	1	−1	1	1	−1	−1	−1	−1	1
X.9	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	−2	.	.	−2	.	.	−2	.	.	−2	.	−2	.
X.10	1	−1	−1	−1	−1	−1	1	1	1	−1	−1	−1	−1	−1	−2	−2	.	.	.	.	2	.	−2	.	.	.	.	.	2
X.11	1	−1	−1	−1	−1	−1	1	1	1	−1	−1	−1	1	1	2	−4	.	2	2	.	.	.	2	.	.	.	.	.	.
X.12	1	−1	−1	−1	−1	−1	1	1	1	−1	−1	−1	1	1	−2	2	.	.	.	.	−2	.	−2	.	.	.	.	.	−2
X.13	1	−1	−1	−1	−1	−1	1	1	1	−1	−1	−1	−1	−1	2	4	.	−2	−2	.	.	.	2	.	.	.	.	.	.
X.14	−2	.	.	.	.	.	2	−2	2	.	.	.	.	.	2	.	−2	.	.	−2	.	.	2	.	.	−2	.	−2	.
X.15	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	3	1	1	1	1	−1	−1	−1	1	−1	1	−1	1	−1
X.16	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−3	1	−1	−1	1	1	1	−1	−1	1	1	1	1	1
X.17	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	1	3	1	1	3	1	1	−1	1	1	−1	1	−1	1
X.18	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	−1	3	−1	−1	3	−1	−1	−1	−1	−1	−1	−1	−1	−1
X.19	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	−1	−3	1	1	−3	3	−1	−1	1	−1	1	−1	1	−1
X.20	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	1	−3	−1	−1	−3	−3	1	−1	−1	1	1	1	1	1
X.21	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	−1	−1	1	−3	−1	−1	1	−1	1	−1	1	1
X.22	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	1	1	1	3	1	−1	−1	1	1	1	1	−1
X.23	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	−1	−1	−1	−1	−1	1	−1	1	1	−1	1	−1	−1
X.24	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	1	1	−1	1	−1	−1	−1	−1	−1	−1	−1	1
X.25	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	−1	.	.	.	.	.	.	.	−2	.	.	2	.	.	.	.
X.26	1	1	1	−1	1	1	1	−1	−1	−1	−1	−1	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.
X.27	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	.	.	.	.	.	.	.	2	.	.	−2	.	.	.	.
X.28	1	1	1	−1	1	1	1	−1	−1	−1	−1	−1	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.
X.29	−1	1	1	1	1	1	−1	−1	−1	1	1	1	1	1	.	2	.	−2	−2	.	2	.	.	.	.	.	.	.	2
X.30	−1	1	1	1	1	1	−1	−1	−1	1	1	1	−1	−1	.	−2	.	2	2	.	−2	.	.	.	.	.	.	.	−2
X.31	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	.	.	.	.	2	.	−2	.	.	.	.	.	−2
X.32	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	.	.	.	.	−2	.	−2	.	.	.	.	.	2
X.33	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	2	.	.	2	.	.	2	.	.	−2	.	−2	.
X.34	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	−2	.	.	−2	.	.	2	.	.	2	.	2	.
X.35	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	2	2	.	−4	.	2	.	.	.	.	.	.
X.36	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	−2	−2	.	4	.	2	.	.	.	.	.	.
X.37	.	.	.	.	2	−2	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.38	−1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	.	.	−2	2	−2	2	.	.	.	.	.	.	.	.	.
X.39	−1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	−1	−1	.	.	−2	−2	2	2	.	.	.	.	.	.	.	.	.
X.40	−1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	.	.	2	−2	2	−2	.	.	.	.	.	.	.	.	.
X.41	−1	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	−1	−1	.	.	2	2	−2	−2	.	.	.	.	.	.	.	.	.
X.42	−1	−1	−1	−1	1	1	−1	−1	−1	1	1	−1	1	1	.	.	2	−2	2	−2	.	.	.	.	.	.	.	.	.
X.43	−1	−1	−1	−1	1	1	−1	−1	−1	1	1	−1	−1	−1	.	.	2	2	−2	−2	.	.	.	.	.	.	.	.	.
X.44	−1	−1	−1	−1	1	1	−1	−1	−1	1	1	−1	1	1	.	.	−2	2	−2	2	.	.	.	.	.	.	.	.	.
X.45	−1	−1	−1	−1	1	1	−1	−1	−1	1	1	−1	−1	−1	.	.	−2	−2	2	2	.	.	.	.	.	.	.	.	.
X.46	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.47	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	2	2	.	−2	.	.	.	.	.	.	.	2
X.48	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	−2	−2	.	−2	.	.	.	.	.	.	.	2
X.49	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	2	2	.	2	.	.	.	.	.	.	.	−2
X.50	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	−2	−2	.	2	.	.	.	.	.	.	.	−2
X.51	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.52	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.53	1	1	1	−1	1	1	1	−1	−1	−1	−1	−1	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.54	1	1	1	−1	1	1	1	−1	−1	−1	−1	−1	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.55	−1	1	1	−1	−1	−1	−1	1	1	1	1	−1	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.56	1	−1	−1	−1	1	1	1	1	1	1	1	−1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.57	1	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1	−1	.	.	.	.	.	.	.	2	.	.	−2	.	.	.	.
X.58	−2	.	.	.	.	.	2	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.59	2	.	.	.	.	.	−2	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.
X.60	−1	−1	−1	1	−1	−1	−1	1	1	1	1	1	1	−1	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.
X.61	1	1	1	1	−1	−1	1	1	1	−1	−1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.62	2	.	.	.	.	.	−2	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.
X.63	−2	.	.	.	.	.	2	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	2	.	.	.	.
X.64	.	.	.	.	−2	2	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	−2	.
X.65	1	1	1	1	−1	−1	1	1	1	−1	−1	1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.66	1	−1	−1	1	1	1	1	−1	−1	−1	−1	1	−1	1	.	.	.	.	.	.	.	−2	.	.	2	.	.	.	.
X.67	.	.	.	.	2	−2	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	2	.
X.68	−2	.	.	.	.	.	2	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	−2	.	.	.	.
X.69	−2	.	.	.	.	.	2	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.70	.	.	.	.	2	−2	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	−2	.
X.71	1	−1	−1	−1	1	1	1	1	1	1	1	−1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.72	−1	−1	−1	1	−1	−1	−1	1	1	1	1	1	−1	1	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.
X.73	.	.	.	.	−2	2	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	2	.
X.74	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.75	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.76	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.77	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.78	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	A	.	.	.	.	.
X.79	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{A}$	.	.	.	.	.
X.80	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.81	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.82	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2	2	2	.	.	.	.	.	.	.	.	.
X.83	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.
X.84	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2	2	2	.	.	.	.	.	.	.	.	.
X.85	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.
X.86	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	2	−2	.	.	.	.	.	.	.	.	.
X.87	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.88	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.89	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2	2	−2	.	.	.	.	.	.	.	.	.
X.90	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	−2	.	2	.	.
X.91	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	−2	.	2	.	.
X.92	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.
X.93	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	−2	−2	.	.	.	.	.	.	.	.	.

Character table of $D(2\,\mathrm{Fi}_{22})$ *(continued)*

2	6	6	5	5	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5
3	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	1	.	.
5	.	.	.	.	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.
	$8p$	$8q$	$8r$	$8s$	$10a$	$10b$	$10c$	$10d$	$10e$	$10f$	$10g$	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$16a$	$16b$
$2P$	4_{18}	4_{20}	4_{29}	4_{30}	$5a$	$5a$	$5a$	$5a$	$5a$	$5a$	$5a$	$6a$	$6b$	$6c$	$6a$	$6c$	$6b$	$8a$	$8a$
$3P$	$8j$	$8q$	$8r$	$8s$	$10a$	$10c$	$10b$	$10d$	$10e$	$10g$	$10f$	4_{13}	4_{14}	4_1	4_{15}	4_2	4_{16}	$16a$	$16b$
$5P$	$8p$	$8q$	$8r$	$8s$	2_2	2_4	2_4	2_3	2_1	2_5	2_5	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$16b$	$16a$
$X.1$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$X.2$	−1	−1	−1	−1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
$X.3$	.	.	.	.	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	.	.
$X.4$	.	.	.	.	−1	−1	−1	−1	−1	−1	−1	1	1	1	1	1	1	.	.
$X.5$	1	1	1	1	.	.	.	.	.	.	.	−1	−1	−1	−1	−1	−1	1	1
$X.6$	−1	−1	−1	−1	.	.	.	.	.	.	.	1	1	1	1	1	1	−1	−1
$X.7$	−1	1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.8$	1	−1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.9$	.	.	.	.	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.
$X.10$	.	.	.	.	.	.	.	.	.	.	.	1	1	−1	1	−1	1	.	.
$X.11$	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	−1	1	−1	.	.
$X.12$	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	−1	1	−1	.	.
$X.13$	.	.	.	.	.	.	.	.	.	.	.	1	1	−1	1	−1	1	.	.
$X.14$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.15$	1	−1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.16$	−1	1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.17$	1	1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.18$	−1	−1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.19$	1	−1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.20$	−1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.21$	1	−1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.22$	−1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.23$	1	1	−1	−1	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.24$	−1	−1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.25$	.	.	.	.	1	−1	−1	−1	−1	1	1	1	−1	−1	−1	1	1	.	.
$X.26$	.	−2	.	.	1	1	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	.	.
$X.27$	.	.	.	.	1	−1	−1	−1	−1	1	1	−1	1	1	1	−1	−1	.	.
$X.28$	.	2	.	.	1	1	1	−1	−1	−1	−1	1	1	1	−1	−1	−1	.	.
$X.29$	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	−1	1	−1	.	.
$X.30$	.	.	.	.	.	.	.	.	.	.	.	1	1	−1	1	−1	1	.	.
$X.31$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.32$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.33$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.34$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.35$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.36$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.37$	.	.	.	.	−2	.	.	2	−2	.	.	.	.	.	.	.	.	.	.
$X.38$	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1	1	−1	−1	.	.
$X.39$	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	−1	1	1	.	.
$X.40$	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1	1	−1	−1	.	.
$X.41$	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	−1	1	1	.	.
$X.42$	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	−1	−1	1	.	.
$X.43$	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	1	1	−1	.	.
$X.44$	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	−1	−1	1	.	.
$X.45$	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	1	1	−1	.	.
$X.46$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.47$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.48$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.49$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.50$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.51$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.52$	.	.	.	.	−1	1	1	1	1	−1	−1	1	−1	−1	−1	1	1	.	.
$X.53$	.	.	.	.	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	1	.	.
$X.54$	.	.	.	.	−1	−1	−1	1	1	1	1	1	1	1	−1	−1	−1	.	.
$X.55$	.	.	.	.	−1	1	1	1	1	−1	−1	−1	1	1	1	−1	−1	.	.
$X.56$	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	−1	1	1	.	.
$X.57$	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1	−1	1	1	.	.
$X.58$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.59$	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.60$	.	−2	.	.	.	.	.	.	.	.	.	1	1	1	−1	−1	−1	.	.
$X.61$	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	−1	−1	1	.	.
$X.62$	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.63$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.64$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.65$	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	1	1	−1	.	.
$X.66$	.	.	.	.	.	.	.	.	.	.	.	−1	1	1	1	−1	−1	.	.
$X.67$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.68$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.69$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.70$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.71$	.	.	.	.	.	.	.	.	.	.	.	1	−1	−1	1	−1	−1	.	.
$X.72$	.	2	.	.	.	.	.	.	.	.	.	−1	−1	−1	1	1	1	.	.
$X.73$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.74$	.	.	.	.	1	1	1	−1	−1	−1	−1	.	.	.	.	.	.	.	.
$X.75$	.	.	.	.	1	−1	−1	−1	−1	1	1	.	.	.	.	.	.	.	.
$X.76$	.	.	.	.	1	−1	−1	1	1	−1	−1	.	.	.	.	.	.	.	.
$X.77$	.	.	.	.	1	−1	−1	1	1	−1	−1	.	.	.	.	.	.	.	.
$X.78$	$\bar{A}$	.	.	.	1	−1	−1	1	1	−1	−1	.	.	.	.	.	.	.	.
$X.79$	A	.	.	.	1	−1	−1	1	1	−1	−1	.	.	.	.	.	.	.	.
$X.80$	.	.	.	.	−1	B	$-B$	1	−1	B	$-B$	.	.	.	.	.	.	.	.
$X.81$	.	.	.	.	−1	$-B$	B	1	−1	$-B$	B	.	.	.	.	.	.	.	.
$X.82$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.83$	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.84$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.85$	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.86$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.87$	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.88$	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.89$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.90$	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.91$	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.92$	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.93$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $D(2\,\mathrm{Fi}_{22})$ (continued)

2	18	18	18	18	17	17	16	16	16	16	17	17	15	15	16	13	13	13	13	14	14
3	1	1	1	1	1	1	1	1	1	1	.	.	1	1	.	1	1	1	1	.	.
5	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	$1a$	2_1	2_2	2_3	2_4	2_5	2_6	2_7	2_8	2_9	2_{10}	2_{11}	2_{12}	2_{13}	2_{14}	2_{15}	2_{16}	2_{17}	2_{18}	2_{19}	2_{20}
$2P$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$
$3P$	$1a$	2_1	2_2	2_3	2_4	2_5	2_6	2_7	2_8	2_9	2_{10}	2_{11}	2_{12}	2_{13}	2_{14}	2_{15}	2_{16}	2_{17}	2_{18}	2_{19}	2_{20}
$5P$	$1a$	2_1	2_2	2_3	2_4	2_5	2_6	2_7	2_8	2_9	2_{10}	2_{11}	2_{12}	2_{13}	2_{14}	2_{15}	2_{16}	2_{17}	2_{18}	2_{19}	2_{20}
$X.94$	120	120	120	120	−120	−120	48	48	48	48	24	24	−48	−48	−24	12	12	12	12	−8	−8
$X.95$	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	−8	−8
$X.96$	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	−8	−8
$X.97$	120	120	120	120	−120	−120	−48	−48	−48	−48	24	24	48	48	−24	12	12	12	12	−8	−8
$X.98$	120	120	120	120	120	120	−24	−24	−24	−24	−8	−8	−24	−24	−8	.	.	.	.	8	8
$X.99$	120	120	120	120	−120	−120	48	48	48	48	24	24	−48	−48	−24	12	12	12	12	−8	−8
$X.100$	120	120	120	120	120	120	24	24	24	24	−8	−8	24	24	−8	.	.	.	.	8	8
$X.101$	120	120	120	120	120	120	−24	−24	−24	−24	−8	−8	−24	−24	−8	.	.	.	.	8	8
$X.102$	128	−128	−128	128	.	.	64	−64	64	−64	.	.	.	.	.	−32	32	32	−32	.	.
$X.103$	160	−160	160	−160	160	−160	−16	−16	16	16	.	.	−16	16	.	−8	−8	8	8	.	.
$X.104$	160	−160	160	−160	−160	160	16	16	−16	−16	.	.	−16	16	.	−8	−8	8	8	.	.
$X.105$	160	−160	−160	160	.	.	80	−80	80	−80	.	.	.	.	.	−40	40	40	−40	.	.
$X.106$	160	−160	160	−160	−160	160	16	16	−16	−16	.	.	−16	16	.	−8	−8	8	8	.	.
$X.107$	160	−160	160	−160	160	−160	48	48	−48	−48	.	.	48	−48	.	8	8	−8	−8	.	.
$X.108$	160	−160	160	−160	160	−160	−16	−16	16	16	.	.	−16	16	.	−8	−8	8	8	.	.
$X.109$	160	−160	160	−160	160	−160	−16	−16	16	16	.	.	−16	16	.	−8	−8	8	8	.	.
$X.110$	160	−160	160	−160	−160	160	16	16	−16	−16	.	.	−16	16	.	−8	−8	8	8	.	.
$X.111$	160	160	−160	−160	.	.	64	−64	−64	64	−32	32	.	.	.	−16	16	−16	16	−32	32
$X.112$	160	−160	160	−160	−160	160	16	16	−16	−16	.	.	−16	16	.	−8	−8	8	8	.	.
$X.113$	160	160	−160	−160	.	.	.	.	.	.	−32	32	.	.	.	16	−16	16	−16	−32	32
$X.114$	160	160	160	160	−160	−160	.	.	.	.	32	32	.	.	−32	−16	−16	−16	−16	32	32
$X.115$	160	−160	−160	160	.	.	−48	48	−48	48	.	.	.	.	.	−8	8	8	−8	.	.
$X.116$	160	−160	160	−160	160	−160	−16	−16	16	16	.	.	−16	16	.	−8	−8	8	8	.	.
$X.117$	160	−160	160	−160	−160	160	−48	−48	48	48	.	.	48	−48	.	8	8	−8	−8	.	.
$X.118$	160	160	−160	−160	.	.	−64	64	64	−64	−32	32	.	.	.	−16	16	−16	16	−32	32
$X.119$	160	−160	−160	160	.	.	−48	48	−48	48	.	.	.	.	.	−8	8	8	−8	.	.
$X.120$	160	−160	−160	160	.	.	−48	48	−48	48	.	.	.	.	.	−8	8	8	−8	.	.
$X.121$	160	160	−160	−160	.	.	.	.	.	.	−32	32	.	.	.	16	−16	16	−16	−32	32
$X.122$	160	160	−160	−160	.	.	.	.	.	.	−32	32	.	.	.	16	−16	16	−16	−32	32
$X.123$	192	192	−192	−192	.	.	.	.	.	.	64	−64	.	.	.	.	.	.	.	.	.
$X.124$	192	192	−192	−192	.	.	.	.	.	.	64	−64	.	.	.	.	.	.	.	.	.
$X.125$	240	240	240	240	240	240	48	48	48	48	−16	−16	48	48	−16	.	.	.	.	16	16
$X.126$	240	240	240	240	240	240	48	48	48	48	−16	−16	48	48	−16	.	.	.	.	−16	−16
$X.127$	240	240	−240	−240	.	.	96	−96	−96	96	−48	48	.	.	.	−24	24	−24	24	16	−16
$X.128$	240	240	−240	−240	.	.	−96	96	96	−96	−48	48	.	.	.	−24	24	−24	24	16	−16
$X.129$	240	−240	240	−240	−240	240	−72	−72	72	72	.	.	72	−72	.	12	12	−12	−12	.	.
$X.130$	240	−240	240	−240	240	−240	72	72	−72	−72	.	.	72	−72	.	12	12	−12	−12	.	.
$X.131$	240	−240	240	−240	240	−240	72	72	−72	−72	.	.	72	−72	.	12	12	−12	−12	.	.
$X.132$	240	240	240	240	−240	−240	.	.	.	.	48	48	.	.	−48	−24	−24	−24	−24	−16	−16
$X.133$	240	240	−240	−240	.	.	−96	96	96	−96	−48	48	.	.	.	−24	24	−24	24	16	−16
$X.134$	240	240	−240	−240	.	.	96	−96	−96	96	−48	48	.	.	.	−24	24	−24	24	16	−16
$X.135$	240	−240	240	−240	−240	240	−72	−72	72	72	.	.	72	−72	.	12	12	−12	−12	.	.
$X.136$	240	240	240	240	240	240	−48	−48	−48	−48	−16	−16	−48	−48	−16	.	.	.	.	16	16
$X.137$	240	240	240	240	−240	−240	.	.	.	.	48	48	.	.	−48	−24	−24	−24	−24	−16	−16
$X.138$	240	240	240	240	240	240	−48	−48	−48	−48	−16	−16	−48	−48	−16	.	.	.	.	−16	−16
$X.139$	240	240	240	240	240	240	−48	−48	−48	−48	−16	−16	−48	−48	−16	.	.	.	.	−16	−16
$X.140$	320	−320	−320	320	.	.	32	−32	32	−32	.	.	.	.	.	16	−16	−16	16	.	.
$X.141$	320	−320	320	−320	320	−320	−32	−32	32	32	.	.	−32	32	.	−16	−16	16	16	.	.
$X.142$	320	−320	320	−320	−320	320	32	32	−32	−32	.	.	−32	32	.	−16	−16	16	16	.	.
$X.143$	320	−320	−320	320	.	.	32	−32	32	−32	.	.	.	.	.	16	−16	−16	16	.	.
$X.144$	320	−320	320	−320	−320	320	32	32	−32	−32	.	.	−32	32	.	−16	−16	16	16	.	.
$X.145$	320	−320	320	−320	320	−320	−32	−32	32	32	.	.	−32	32	.	−16	−16	16	16	.	.
$X.146$	384	384	384	384	−384	−384	.	.	.	.	−128	−128	.	.	128	.	.	.	.	.	.
$X.147$	384	384	−384	−384	.	.	.	.	.	.	128	−128	.	.	.	.	.	.	.	.	.
$X.148$	384	384	−384	−384	.	.	.	.	.	.	128	−128	.	.	.	.	.	.	.	.	.
$X.149$	480	−480	−480	480	.	.	−144	144	−144	144	.	.	.	.	.	−24	24	24	−24	.	.
$X.150$	480	480	−480	−480	.	.	.	.	.	.	−96	96	.	.	.	48	−48	48	−48	32	−32
$X.151$	640	−640	−640	640	.	.	64	−64	64	−64	.	.	.	.	.	32	−32	−32	32	.	.

Character table of $D(2\,\mathrm{Fi}_{22})$ (continued)

2	12	12	13	13	11	11	12	12	12	12	12	11	11	10	6	11	11	12	12	11	11	11	11	11
3	1	1	.	.	1	1	.	.	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	2_{21}	2_{22}	2_{23}	2_{24}	2_{25}	2_{26}	2_{27}	2_{28}	2_{29}	2_{30}	2_{31}	2_{32}	2_{33}	2_{34}	$3a$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9
$2P$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$1a$	$3a$	2_2	2_2	2_1	2_1	2_{10}	2_{10}	2_{10}	2_{11}	2_{10}
$3P$	2_{21}	2_{22}	2_{23}	2_{24}	2_{25}	2_{26}	2_{27}	2_{28}	2_{29}	2_{30}	2_{31}	2_{32}	2_{33}	2_{34}	$1a$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9
$5P$	2_{21}	2_{22}	2_{23}	2_{24}	2_{25}	2_{26}	2_{27}	2_{28}	2_{29}	2_{30}	2_{31}	2_{32}	2_{33}	2_{34}	$3a$	4_1	4_2	4_3	4_4	4_5	4_6	4_7	4_8	4_9
$X.94$	−12	−12	8	.	6	6	−4	−4	−4	−4	.	4	4	2	.	−6	−6	.	.	−4	4	−6	6	−6
$X.95$	.	.	−8	−8	.	.	.	.	.	.	−8	.	.	−8	.	.	.	8	8	.	.	8	8	8
$X.96$	.	.	−8	−8	.	.	.	.	.	.	−8	.	.	8	.	.	.	8	8	.	.	−8	−8	−8
$X.97$	−12	−12	8	.	−6	−6	−4	−4	−4	−4	.	4	4	6	.	6	6	.	.	−4	4	−2	2	−2
$X.98$	.	.	8	8	−12	−12	.	.	.	.	−8	.	.	4	.	−12	−12	8	−8	.	.	4	4	4
$X.99$	−12	−12	8	.	6	6	4	4	4	4	.	−4	−4	10	.	−6	−6	.	.	−4	4	2	−2	2
$X.100$	.	.	8	−8	12	12	12	12	12	12	8	12	12	4	.	12	12	−8	−8	.	.	4	4	4
$X.101$	.	.	8	8	−12	−12	.	.	.	.	−8	.	.	4	.	−12	−12	8	−8	.	.	4	4	4
$X.102$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.
$X.103$	−8	8	.	.	8	−8	−8	−8	8	8	.	−8	8	.	1	8	−8	.	.	.	.	8	8	−8
$X.104$	8	−8	.	.	16	−16	8	8	−8	−8	.	−8	8	.	1	−16	16	.	.	.	.	.	.	.
$X.105$	.	.	.	.	.	.	8	−8	8	−8	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.106$	8	−8	.	.	8	−8	−8	−8	8	8	.	8	−8	.	1	−8	8	.	.	.	.	8	−8	−8
$X.107$	8	−8	.	.	.	.	8	8	−8	−8	.	8	−8	.	−2	.	.	.	.	.	.	.	.	.
$X.108$	−8	8	.	.	16	−16	8	8	−8	−8	.	8	−8	.	1	16	−16	.	.	.	.	.	.	.
$X.109$	−8	8	.	.	−16	16	8	8	−8	−8	.	8	−8	.	1	−16	16	.	.	.	.	.	.	.
$X.110$	8	−8	.	.	−8	8	−8	−8	8	8	.	8	−8	.	1	8	−8	.	.	.	.	−8	8	8
$X.111$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.112$	8	−8	.	.	−16	16	8	8	−8	−8	.	−8	8	.	1	16	−16	.	.	.	.	.	.	.
$X.113$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	.	.	.	.	.	.	.	.
$X.114$	16	16	−32	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	−16	16	.	.	.
$X.115$	.	.	.	.	.	.	8	−8	8	−8	.	.	.	.	4	.	.	.	.	.	.	.	.	.
$X.116$	−8	8	.	.	−8	8	−8	−8	8	8	.	−8	8	.	1	−8	8	.	.	.	.	−8	−8	8
$X.117$	−8	8	.	.	.	.	8	8	−8	−8	.	−8	8	.	−2	.	.	.	.	.	.	.	.	.
$X.118$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.119$	.	.	.	.	.	.	8	−8	8	−8	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.120$	.	.	.	.	.	.	8	−8	8	−8	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.121$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.122$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.
$X.123$	.	.	.	.	.	.	−16	16	16	−16	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.124$	.	.	.	.	.	.	16	−16	−16	16	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.125$	.	.	16	−16	.	.	−8	−8	−8	−8	16	−8	−8	.	.	.	.	−16	−16	.	.	.	.	.
$X.126$	.	.	−16	−16	.	.	.	.	.	.	−16	.	.	.	.	.	.	16	16	.	.	.	.	.
$X.127$	.	.	.	.	.	.	8	−8	−8	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.128$	.	.	.	.	.	.	8	−8	−8	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.129$	−12	12	.	.	12	−12	−4	−4	4	4	.	4	−4	.	.	−12	12	.	.	.	.	4	−4	−4
$X.130$	12	−12	.	.	−12	12	−4	−4	4	4	.	−4	4	.	.	−12	12	.	.	.	.	−4	−4	4
$X.131$	12	−12	.	.	12	−12	−4	−4	4	4	.	−4	4	.	.	12	−12	.	.	.	.	4	4	−4
$X.132$	24	24	16	.	12	12	.	.	.	.	.	.	.	−4	.	−12	−12	.	.	8	−8	−4	4	−4
$X.133$	.	.	.	.	.	.	−8	8	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.134$	.	.	.	.	.	.	−8	8	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.135$	−12	12	.	.	−12	12	−4	−4	4	4	.	4	−4	.	.	12	−12	.	.	.	.	−4	4	4
$X.136$	.	.	16	16	.	.	.	.	.	.	−16	.	.	.	.	.	.	16	−16	.	.	.	.	.
$X.137$	24	24	16	.	−12	−12	.	.	.	.	.	.	.	4	.	12	12	.	.	8	−8	4	−4	4
$X.138$	.	.	−16	16	.	.	.	.	.	.	16	.	.	.	.	.	.	−16	16	.	.	.	.	.
$X.139$	.	.	−16	16	.	.	.	.	.	.	16	.	.	.	.	.	.	−16	16	.	.	.	.	.
$X.140$	.	.	.	.	.	.	−16	16	−16	16	.	.	.	.	2	.	.	.	.	.	.	.	.	.
$X.141$	−16	16	.	.	8	−8	.	.	.	.	.	.	.	.	−1	8	−8	.	.	.	.	−8	−8	8
$X.142$	16	−16	.	.	8	−8	.	.	.	.	.	.	.	.	−1	−8	8	.	.	.	.	−8	8	8
$X.143$	.	.	.	.	.	.	16	−16	16	−16	.	.	.	.	2	.	.	.	.	.	.	.	.	.
$X.144$	16	−16	.	.	−8	8	.	.	.	.	.	.	.	.	−1	8	−8	.	.	.	.	8	−8	−8
$X.145$	−16	16	.	.	−8	8	.	.	.	.	.	.	.	.	−1	−8	8	.	.	.	.	8	8	−8
$X.146$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.147$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.148$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.149$	.	.	.	.	.	.	−8	8	−8	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.150$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.151$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	.	.	.	.	.	.	.

Character table of $D(2\,\mathrm{Fi}_{22})$ *(continued)*

2	11	11	11	9	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10	9	9	9	9	9	9	9
3	.	.	.	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}
$2P$	2_{11}	2_{11}	2_{11}	2_{15}	2_{16}	2_{15}	2_{16}	2_{6}	2_{10}	2_{14}	2_{8}	2_{8}	2_{14}	2_{10}	2_{11}	2_{11}	2_{10}	2_{2}	2_{6}	2_{19}	2_{19}	2_{16}	2_{16}	2_{15}	2_{11}	2_{16}
$3P$	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}
$5P$	4_{10}	4_{11}	4_{12}	4_{13}	4_{14}	4_{15}	4_{16}	4_{17}	4_{18}	4_{19}	4_{20}	4_{21}	4_{22}	4_{23}	4_{24}	4_{25}	4_{26}	4_{27}	4_{28}	4_{29}	4_{30}	4_{31}	4_{32}	4_{33}	4_{34}	4_{35}
$X.94$	6	−4	−4	.	.	.	.	.	4	.	.	.	.	−4	2	4	−2	−2	.	−4	4	−2	−2	4	.	−4
$X.95$	8	.	.	.	.	.	.	−4	.	.	12	12	.	.	.	.	.	−8	−4	.	.	.	.	.	.	.
$X.96$	−8	.	.	.	.	.	.	12	.	.	−4	−4	.	.	.	.	.	8	12	.	.	.	.	.	.	.
$X.97$	2	4	4	.	.	.	.	.	4	.	.	.	.	−4	−2	−4	2	−6	.	−4	4	2	2	4	.	−4
$X.98$	4	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	4	4	.	4	4	.	.	.	.	.
$X.99$	−2	−4	−4	.	.	.	.	.	−4	.	.	.	.	4	2	4	−2	−10	.	4	−4	2	2	−4	.	4
$X.100$	4	.	.	.	.	.	.	.	−4	.	.	.	.	−4	−4	.	−4	4	.	.	.	.	.	.	.	.
$X.101$	4	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	4	4	.	4	4	.	.	.	.	.
$X.102$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.103$	−8	.	.	−8	−8	8	8	8	.	.	.	.	.	.	.	.	.	.	−8	.	.	.	.	.	.	.
$X.104$	.	.	.	−4	4	4	−4	.	.	.	−8	8	.	.	.	.	.	.	.	.	.	.	.	−4	.	4
$X.105$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4	.	.	.
$X.106$	8	.	.	−8	8	8	−8	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.107$	.	.	.	.	.	.	.	8	.	.	.	.	.	.	.	.	.	.	−8	.	.	−4	−4	.	.	.
$X.108$	.	.	.	−4	−4	4	4	−8	.	.	.	.	.	.	.	.	.	.	8	.	.	.	.	−4	.	−4
$X.109$	.	.	.	4	4	−4	−4	−8	.	.	.	.	.	.	.	.	.	.	8	.	.	.	.	4	.	4
$X.110$	−8	.	.	8	−8	−8	8	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.111$	.	−16	16	.	.	.	.	.	.	8	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.112$	.	.	.	4	−4	−4	4	.	.	.	−8	8	.	.	.	.	.	.	.	.	.	.	.	4	.	−4
$X.113$	.	.	.	.	.	.	.	.	.	−8	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.114$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.115$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.
$X.116$	8	.	.	8	8	−8	−8	8	.	.	.	.	.	.	.	.	.	.	−8	.	.	.	.	.	.	.
$X.117$	.	.	.	.	.	.	.	.	.	.	8	−8	.	.	.	.	.	.	.	.	.	4	4	.	.	.
$X.118$	.	16	−16	.	.	.	.	.	.	8	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.119$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.
$X.120$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	4	.	.	.
$X.121$	.	.	.	.	.	.	.	.	.	−8	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.122$	.	.	.	.	.	.	.	.	.	−8	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.123$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.124$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.125$	.	.	.	.	.	.	.	.	−8	.	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.
$X.126$	.	.	.	.	.	.	.	−8	.	.	−8	−8	.	.	.	.	.	.	−8	.	.	.	.	.	.	.
$X.127$	.	8	−8	.	.	.	.	.	.	−4	.	.	4	.	.	.	.	.	.	.	.	−4	4	.	.	.
$X.128$	.	−8	8	.	.	.	.	.	.	−4	.	.	4	.	.	.	.	.	.	.	.	4	−4	.	.	.
$X.129$	4	.	.	6	−6	−6	6	.	.	.	−4	4	.	.	.	.	.	.	.	.	.	−2	−2	2	.	−2
$X.130$	4	.	.	−6	−6	6	6	−4	.	.	.	.	.	.	.	.	.	.	4	.	.	2	2	−2	.	−2
$X.131$	−4	.	.	6	6	−6	−6	−4	.	.	.	.	.	.	.	.	.	.	4	.	.	2	2	2	.	2
$X.132$	4	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	−4	4	.	.	.	.	.	.	.	.
$X.133$	.	−8	8	.	.	.	.	.	.	−4	.	.	4	.	.	.	.	.	.	.	.	−4	4	.	.	.
$X.134$	.	8	−8	.	.	.	.	.	.	−4	.	.	4	.	.	.	.	.	.	.	.	4	−4	.	.	.
$X.135$	−4	.	.	−6	6	6	−6	.	.	.	−4	4	.	.	.	.	.	.	.	.	.	−2	−2	−2	.	2
$X.136$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−8	−8	.	.	.	.	.
$X.137$	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	.	4	−4	.	.	.	.	.	.	.	.
$X.138$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.139$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.140$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.141$	8	.	.	4	4	−4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	.	−4
$X.142$	−8	.	.	4	−4	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−4	.	4
$X.143$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.144$	8	.	.	−4	4	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	−4
$X.145$	−8	.	.	−4	−4	4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	.	4
$X.146$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.147$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.148$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.149$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4	.	.	.
$X.150$	.	.	.	.	.	.	.	.	.	8	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.151$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $D(2\,\mathrm{Fi}_{22})$ *(continued)*

2	9	9	9	9	9	9	9	9	9	9	8	8	8	8	8	8	8	8	8	7	7	7	7	7	3	6	6
3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.
	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}	4_{41}	4_{42}	4_{43}	4_{44}	4_{45}	4_{46}	4_{47}	4_{48}	4_{49}	4_{50}	4_{51}	4_{52}	4_{53}	4_{54}	4_{55}	4_{56}	4_{57}	4_{58}	4_{59}	$5a$	$6a$	$6b$
$2P$	2_{14}	2_{16}	2_{13}	2_{16}	2_{15}	2_{14}	2_{11}	2_{13}	2_{16}	2_{10}	2_{16}	2_{20}	2_{19}	2_{20}	2_{20}	2_{23}	2_{16}	2_{19}	2_{23}	2_{27}	2_{28}	2_{27}	2_{20}	2_{28}	$5a$	$3a$	$3a$
$3P$	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}	4_{41}	4_{42}	4_{43}	4_{44}	4_{45}	4_{46}	4_{47}	4_{48}	4_{49}	4_{50}	4_{51}	4_{52}	4_{53}	4_{54}	4_{55}	4_{56}	4_{57}	4_{58}	4_{59}	$5a$	2_{15}	2_{16}
$5P$	4_{36}	4_{37}	4_{38}	4_{39}	4_{40}	4_{41}	4_{42}	4_{43}	4_{44}	4_{45}	4_{46}	4_{47}	4_{48}	4_{49}	4_{50}	4_{51}	4_{52}	4_{53}	4_{54}	4_{55}	4_{56}	4_{57}	4_{58}	4_{59}	$1a$	$6a$	$6b$
$X.94$	.	−4	.	−2	4	.	6	.	−2	−6	2	2	−2	.	−2	.	2	2	.	.	.	.	.	.	.	.	.
$X.95$	.	.	−4	.	.	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.96$	.	.	−4	.	.	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.97$	.	−4	.	2	4	.	−6	.	2	6	−2	−2	2	.	2	.	−2	−2	.	.	.	.	.	.	.	.	.
$X.98$	.	.	.	.	.	.	−4	.	.	−4	.	.	.	−4	.	−4	.	.	−4	.	.	.	4	.	.	.	.
$X.99$	.	4	.	2	−4	.	−2	.	2	2	−2	2	−2	.	−2	.	−2	2	.	.	.	.	.	.	.	.	.
$X.100$	−4	.	.	.	.	−4	−4	.	.	−4	.	.	.	.	.	.	.	.	.	2	2	2	.	2	.	.	.
$X.101$	.	.	.	.	.	.	−4	.	.	−4	.	.	.	−4	.	4	.	.	4	.	.	.	−4	.	.	.	.
$X.102$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	−2	2
$X.103$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.104$	.	−4	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.105$	.	.	−4	4	.	.	.	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2
$X.106$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.107$	.	.	.	4	.	.	.	.	4	.	−4	.	.	.	.	.	4	.	.	.	.	.	.	.	.	2	2
$X.108$	.	4	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.109$	.	−4	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.110$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.111$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2
$X.112$	.	4	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.113$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4
$X.114$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2
$X.115$	.	.	−4	−4	.	.	.	4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	4	−4
$X.116$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1
$X.117$	.	.	.	−4	.	.	.	.	−4	.	−4	.	.	.	.	.	4	.	.	.	.	.	.	.	.	2	2
$X.118$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2
$X.119$	.	.	−4	−4	.	.	.	4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.120$	.	.	−4	−4	.	.	.	4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.121$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.122$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.123$	−8	.	.	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.
$X.124$	8	.	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.
$X.125$	8	.	.	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.126$	.	.	8	.	.	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.127$	−4	.	.	4	.	4	.	.	−4	.	.	.	.	.	.	−4	.	.	4	.	.	.	.	.	.	.	.
$X.128$	−4	.	.	−4	.	4	.	.	4	.	.	.	.	.	.	−4	.	.	4	.	.	.	.	.	.	.	.
$X.129$	.	2	.	2	−2	.	.	.	2	.	2	.	.	.	.	.	−2	.	.	−2	2	2	.	−2	.	.	.
$X.130$	.	2	.	−2	2	.	.	.	−2	.	2	.	.	.	.	.	−2	.	.	−2	−2	2	.	2	.	.	.
$X.131$	.	−2	.	−2	−2	.	.	.	−2	.	2	.	.	.	.	.	−2	.	.	2	2	−2	.	−2	.	.	.
$X.132$	.	.	.	.	.	.	−4	.	.	4	.	4	−4	.	4	.	.	−4	.	.	.	.	.	.	.	.	.
$X.133$	4	.	.	4	.	−4	.	.	−4	.	.	.	.	.	.	4	.	.	−4	.	.	.	.	.	.	.	.
$X.134$	4	.	.	−4	.	−4	.	.	4	.	.	.	.	.	.	4	.	.	−4	.	.	.	.	.	.	.	.
$X.135$	.	−2	.	2	2	.	.	.	2	.	2	.	.	.	.	.	−2	.	.	2	−2	−2	.	2	.	.	.
$X.136$	.	.	.	.	.	.	.	.	.	.	.	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.137$	.	.	.	.	.	.	4	.	.	−4	.	−4	4	.	−4	.	.	4	.	.	.	.	.	.	.	.	.
$X.138$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.139$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.140$	.	.	−8	.	.	.	.	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.141$	.	4	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.142$	.	−4	.	.	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.143$	.	.	8	.	.	.	.	−8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2
$X.144$	.	4	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.145$	.	−4	.	.	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1
$X.146$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.
$X.147$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.
$X.148$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	.	.
$X.149$	.	.	4	4	.	.	.	−4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.150$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.151$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	−2

Character table of $D(2\,\mathrm{Fi}_{22})$ (continued)

	6c	6d	6e	6f	6g	6h	6i	6j	6k	6l	6m	6n	6o	6p	6q	6r	6s	6t	6u	8a	8b	8c	8d	8e	8f	8g	8h	8i	8j	8k
2	6	6	6	6	6	5	5	5	5	5	5	5	5	5	5	5	5	4	4	7	7	7	7	7	7	7	6	6	6	6
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2P	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	3a	4_3	4_5	4_5	4_4	4_4	4_5	4_5	4_{17}	4_3	4_{18}	4_{17}
3P	2_2	2_{17}	2_3	2_1	2_{18}	2_4	2_{12}	2_{12}	2_{13}	2_7	2_6	2_{21}	2_5	2_{22}	2_9	2_8	2_{13}	2_{25}	2_{26}	8a	8b	8c	8d	8e	8f	8g	8h	8i	8p	8k
5P	6c	6d	6e	6f	6g	6h	6j	6i	6s	6l	6m	6n	6o	6p	6q	6r	6k	6t	6u	8a	8b	8c	8d	8e	8f	8g	8h	8i	8j	8k
X.94	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	2	−2	−2	.	.	.	.	.
X.95	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	2
X.96	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	2
X.97	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2	−2	2	.	.	.	.	.
X.98	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.99	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	2	−2	2	.	.	.	.	.
X.100	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.
X.101	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.102	−2	2	2	−2	−2	.	.	.	.	2	−2	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.103	1	−1	−1	−1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	1	−1	1	.	.	.	.	.	.	.	.	.	.	.
X.104	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	1	−1	.	.	.	.	.	.	.	.	.	.	.
X.105	2	−2	−2	2	2	.	.	.	.	−2	2	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.106	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	−1	1	.	.	.	.	.	.	.	.	.	.	.
X.107	−2	−2	2	2	−2	−2	.	.	.	.	.	2	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.108	1	−1	−1	−1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	1	1	−1	.	.	.	.	.	.	.	.	.	.	.
X.109	1	−1	−1	−1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	1	−1	1	.	.	.	.	.	.	.	.	.	.	.
X.110	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	1	−1	.	.	.	.	.	.	.	.	.	.	.
X.111	2	2	2	−2	−2	.	.	.	.	2	−2	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.112	1	−1	−1	−1	−1	−1	−1	−1	1	1	1	−1	1	1	−1	−1	1	−1	1	.	.	.	.	.	.	.	.	.	.	.
X.113	−4	4	−4	4	−4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.114	−2	2	−2	−2	2	2	.	.	.	.	.	−2	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.115	−4	−4	4	−4	4	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.116	1	−1	−1	−1	−1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	1	1	−1	.	.	.	.	.	.	.	.	.	.	.
X.117	−2	−2	2	2	−2	2	.	.	.	.	.	−2	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.118	2	2	2	−2	−2	.	.	.	.	−2	2	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.119	2	2	−2	2	−2	.	C	$\bar{C}$	C	.	.	.	.	.	.	.	$\bar{C}$	.	.	.	.	.	.	.	.	.	.	.	.	.
X.120	2	2	−2	2	−2	.	$\bar{C}$	C	$\bar{C}$	.	.	.	.	.	.	.	C	.	.	.	.	.	.	.	.	.	.	.	.	.
X.121	2	−2	2	−2	2	.	$\bar{C}$	C	C	.	.	.	.	.	.	.	$\bar{C}$	.	.	.	.	.	.	.	.	.	.	.	.	.
X.122	2	−2	2	−2	2	.	C	$\bar{C}$	$\bar{C}$	.	.	.	.	.	.	.	C	.	.	.	.	.	.	.	.	.	.	.	.	.
X.123	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.124	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.125	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.126	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.127	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.128	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.129	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−2	.	.	2
X.130	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.131	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.132	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.133	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.134	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.135	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	−2
X.136	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.137	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.138	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.139	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.140	−2	2	2	−2	−2	.	.	.	.	−2	2	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.141	−1	1	1	1	1	−1	1	1	−1	1	1	−1	1	1	−1	−1	−1	−1	1	.	.	.	.	.	.	.	.	.	.	.
X.142	−1	1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	1	1	−1	−1	1	.	.	.	.	.	.	.	.	.	.	.
X.143	−2	2	2	−2	−2	.	.	.	.	−2	2	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.144	−1	1	1	1	1	1	1	1	−1	−1	−1	1	−1	−1	1	1	−1	1	−1	.	.	.	.	.	.	.	.	.	.	.
X.145	−1	1	1	1	1	−1	1	1	−1	1	1	−1	1	1	−1	−1	−1	1	−1	.	.	.	.	.	.	.	.	.	.	.
X.146	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.147	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.148	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.149	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.150	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X.151	2	−2	−2	2	2	.	.	.	.	2	−2	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Character table of $D(2\,\mathrm{Fi}_{22})$ *(continued)*

2	6	6	6	6	6	6	5	5	3	3	3	3	3	3	3	4	4	4	4	4	4	5	5
3	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	1	1	1	1	.	.
5	.	.	.	.	.	.	.	.	1	1	1	1	1	1	1	.	.	.	.	.	.	.	.
	$8l$	$8m$	$8n$	$8o$	$8p$	$8q$	$8r$	$8s$	$10a$	$10b$	$10c$	$10d$	$10e$	$10f$	$10g$	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$16a$	$16b$
$2P$	4_{19}	4_{20}	4_{19}	4_6	4_{18}	4_{20}	4_{29}	4_{30}	$5a$	$5a$	$5a$	$5a$	$5a$	$5a$	$5a$	$6a$	$6b$	$6c$	$6a$	$6c$	$6b$	$8a$	$8a$
$3P$	$8l$	$8m$	$8n$	$8o$	$8j$	$8q$	$8r$	$8s$	$10a$	$10c$	$10b$	$10d$	$10e$	$10g$	$10f$	4_{13}	4_{14}	4_1	4_{15}	4_2	4_{16}	$16a$	$16b$
$5P$	$8l$	$8m$	$8n$	$8o$	$8p$	$8q$	$8r$	$8s$	2_2	2_4	2_4	2_3	2_1	2_5	2_5	$12a$	$12b$	$12c$	$12d$	$12e$	$12f$	$16b$	$16a$
$X.94$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.95$	.	−2	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.96$	.	−2	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.97$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.98$	.	.	.	.	.	.	2	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.99$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.100$	.	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.101$	.	.	.	.	.	.	−2	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.102$	.	.	.	.	.	.	.	.	2	.	.	−2	2	.	.	.	.	.	.	.	.	.	.
$X.103$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	−1	−1	1	−1	.	.
$X.104$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	1	1	−1	.	.
$X.105$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.106$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	−1	−1	1	.	.
$X.107$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.108$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	1	−1	1	.	.
$X.109$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	−1	−1	1	−1	.	.
$X.110$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	1	1	−1	.	.
$X.111$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.112$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	−1	−1	1	.	.
$X.113$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.114$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.115$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.116$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	1	−1	1	.	.
$X.117$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.118$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.119$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.120$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.121$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.122$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.123$	.	.	.	.	.	.	.	.	−2	.	.	−2	2	.	.	.	.	.	.	.	.	.	.
$X.124$	.	.	.	.	.	.	.	.	−2	.	.	−2	2	.	.	.	.	.	.	.	.	.	.
$X.125$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.126$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.127$	2	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.128$	−2	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.129$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.130$	.	2	.	.	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.131$	.	−2	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.132$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.133$	2	.	−2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.134$	−2	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.135$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.136$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.137$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.138$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	D	$\bar{D}$
$X.139$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$\bar{D}$	D
$X.140$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.141$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	−1	−1	1	−1	.	.
$X.142$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	−1	1	−1	−1	1	.	.
$X.143$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.144$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	1	−1	1	1	−1	.	.
$X.145$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−1	−1	1	1	−1	1	.	.
$X.146$	.	.	.	.	.	.	.	.	−1	1	1	−1	−1	1	1	.	.	.	.	.	.	.	.
$X.147$	.	.	.	.	.	.	.	.	1	B	$-B$	1	−1	$-B$	B	.	.	.	.	.	.	.	.
$X.148$	.	.	.	.	.	.	.	.	1	$-B$	B	1	−1	B	$-B$	.	.	.	.	.	.	.	.
$X.149$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.150$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$X.151$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

where $A = -4\zeta(4)_4, B = -2\zeta(5)_5^3 - 2\zeta(5)_5^2 - 1, C = -4\zeta(3)_3 - 2, D = 2\zeta(8)_8^3 + 2\zeta(8)_8$.

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