7880 Applied Logic, Fall 2009

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Logical Definability and Random Graphs

The theme of this course is that logical definability can be used to understand combinatorial properties of randomly chosen finite graphs.

An idealisation of a real network is the random graph model advanced by Erdős and Rényi. In this model a random graph $G(n, p)$ is an undirected graph on n labelled vertices with each edge chosen independently with probability p (in general p may depend on n).

Combinatorists ask questions such as: For a given property X of graphs (connected, contains a triangle, ...) what is the probability that $G(n, p)$ has property X? When $p = \frac{1}{2}$ this probability is the proportion of graphs for which X holds. What happens as n tends to infinity? It was noticed, for instance, that for many properties this limit exists and is either 0 or 1 - loosely then, larger and larger randomly chosen graphs begin to look alike!

Consequently, logicians generalised the probabilistic arguments and showed theorems of the following form: Every property X that can be *defined* in a certain logical language already ensures that the limit is 0 or 1. The main tools come from an area of mathematical logic called finite model theory which also has applications in database theory, artificial intelligence, and computational complexity theory.

I hope this course will appeal to those with backgrounds in any of the following: logic, probability, combinatorics, or theoretical computer science. The prerequisites for this course are familiarity with the content of upper-level undergraduate courses in logic and probability. Although we will need some notions usually covered in a first graduate course in logic, the makeup of the class will determine how we cover missing background. There will be no exam. I will ask each student to present a recent paper or chapter from a book.

Keywords: evolution of random graphs, threshold function, zero-one law, convergence law, infinitary logic, almost-sure theory, finite model theory.